Deutsch-Französische Sommerschule

Ecole d'Eté Franco-Allemande

Franco-German Summer School

# Inverse Problems and Partial Differential Equations

**October 7 - 11, 2013**
Universität Bremen

**Lecturers:**
Thorsten Hohage (University of Göttingen)
Patrick Joly (INRIA Saclay)
Juliette Leblond (INRIA Sophia-Antipolis)
Peter Maass (University of Bremen)
Peter Monk (University of Delaware)
Karim Ramdani (INRIA Nancy)
Andreas Rieder (KIT)
Samuli Siltanen (University of Helsinki)

www.math.uni-bremen.de/zetem/ip-school2013

**Organizers:**
Houssem Haddar (INRIA Saclay)
Armin Lechleiter (University of Bremen)

Universität Bremen

Université franco-allemande
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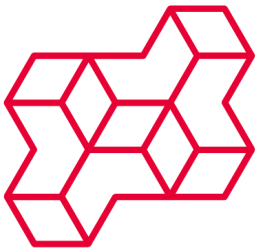

Zentrum für Technomathematik
Universität Bremen


INRIA Saclay–Ile-de-France

Workgroup on Inverse Problems (ZETEM)

Project-Team DeFI (INRIA)

Franco-German Summer School

# Inverse Problems and Partial Differential Equations

University of Bremen, October 6–11, 2013

Organized by

Houssem Haddar and Armin Lechleiter




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# Welcome from the Organizers

Inverse problems for partial differential equations are a relatively young research area within the big field of applied mathematics. We are very happy that so many young researchers from different disciplines joined us for this Franco-German summer school dealing with this topic. Admittedly, a summer school in October should rather be called an autumn school, in particular if it takes place in Northern Germany — we hope that the weather is keeping up. Anyway, it will be a pleasure for us to spend this week with you in the inspiring scientific environment of the University of Bremen.

The organization of a summer school strongly depends on sponsors and it is a pleasure for us to acknowledge generous financial support of the Franco-German University (Université Franco-Allemande UFA/Deutsch-Französische Hochschule DFH) and of the French embassy in Berlin. Further, we acknowledge financial support of the research institute INRIA Saclay–Ile-de-France and of the Center for Industrial Mathematics ZETEM at the University of Bremen.

Numerous persons at the Center for Industrial Mathematics supported this event. We would like to thank Ebba Feldmann for organizing the non-scientific part of the summer school, Judith Barthel for designing the summer school's poster and cup, and Matthias Knauer for supporting us with hints and tricks for setting up the web site, as well as everybody in the lab helping to create a comfortable atmosphere for our visitors.

To have an excellent time together we would like to briefly indicate to you three important points (more indications, e.g., on lunch and dinner, can be found on page 9).

**Posters:** A number of young researchers contribute to this workshop by presenting a poster. Posters are shown inside the room we use for coffee breaks and dinner, MZH 1460. On Monday evening before the dinner buffet starts, we will briefly introduce these young researchers and their posters. After dinner (and, of course, during the entire week) there is plenty of time to look at and to discuss about the posters.

**Teaching Material:** Slides of talks will be made available online via the conference web site http://www.math.uni-bremen.de/zetem/ip-school2013.

**Wireless LAN:** The university network is accessible via guest accounts. The account information can be found in the folder you received upon your arrival.

We hope that you enjoy this summer school and have a great time in Bremen!

Houssem Haddar and Armin Lechleiter

# Summer School Program

## Sunday, October 6

**18h30–21h00 — Registration in front MZH 1460**

**19h00–21h00 — Dinner (MZH 1460)**

## Monday, October 7

**9h00-10h30 — Session 1 (MZH 1470)**

An introduction to scattering theory — Peter Monk, University of Delaware (United States)

**10h30-11h00 — Coffee break (MZH 1460)**

**11h00-12h30 — Session 2 (MZH 1470)**

Sampling methods (Part I) — Peter Monk, University of Delaware (United States)

**12h30-14h30 — Lunch Break**

**14h30-16h00 — Session 3 (MZH 1470)**

Newton-like solvers for nonlinear ill-posed problems (Part I) — Andreas Rieder, KIT – Karlsruher Institut für Technologie (Germany)

**16h00-16h30 — Coffee Break (MZH 1460)**

**16h30-18h00 — Session 4 (MZH 1470)**

Inverse problems with sparsity constraints (Part I) — Peter Maass, Universität Bremen (Germany)

**18h30-21h00 — Dinner (MZH 1460)**

## Tuesday, October 8

### 9h00-10h30 — Session 5 (MZH 1470)

Sampling methods (Part II) — Peter Monk, University of Delaware (United States)

### 10h30-11h00 — Coffee break (MZH 1460)

### 11h00-12h30 — Session 6 (MZH 1470)

Modeling of piezo-electric sensors — Sébastien Imperiale, INRIA Saclay–Ile-de-France (France)

### 12h30-14h30 — Lunch Break

### 14h30-16h00 — Session 7 (MZH 1470)

Inverse problems with sparsity constraints (Part II) — Peter Maass, Universität Bremen (Germany)

### 16h00-16h30 — Coffee Break (MZH 1460)

### 16h30-18h00 — Session 8 (MZH 1470)

Newton-like solvers for nonlinear ill-posed problems (Part II) — Andreas Rieder, KIT – Karlsruher Institut für Technologie (Germany)

### 18h30-21h00 — Dinner (MZH 1460)

## Wednesday, October 9

### 9h00-10h30 — Session 9 (MZH 1470)

Inverse source problems (Part I) — Juliette Leblond, INRIA Sophia Antipolis (France)

### 10h30-11h00 — Coffee break (MZH 1460)

### 11h00-12h30 — Session 10 (MZH 1470)

Initial data inverse problems (Part I) — Karim Ramdani, INRIA Nancy–Grand Est (France)

### 12h30-14h00 (!!!) — Lunch Break

Please note that the lunch break on Wednesday takes only 90 minutes.

### 14h00-15h15 — Session 11 (MZH 1470)

14h00-14h25 Asymptotic formula for a Stokes transmission problem caused by a small interface change of inclusion — Thi Hong Cam Luong, Université de Cergy-Pontoise (France)

14h25-14h50 The Foldy-Lax approximation of the acoustic scattered waves by small bodies — Durga Prasad Challa, Radon Institut (RICAM) Linz (Austria)

14h50-15h15 Direct and inverse acoustic scattering by extended and point-like scatterers — Guanghui Hu, Weierstraß Institut (WIAS) Berlin (Germany)

See also the abstracts on page 5.

### 16h00-18h00 — Guided City Tour

### 18h15-22h00 — Summer School Dinner (Downtown)

See page 9 for more information on the tour and the dinner.

## Thursday, October 10

### 9h00-10h30 — Session 12 (MZH 1470)

Initial data inverse problems (Part II) — Karim Ramdani, INRIA Nancy–Grand Est (France)

### 10h30-11h00 — Coffee break (MZH 1460)

### 11h00-12h30 — Session 13 (MZH 1470)

Inverse source problems (Part II) — Juliette Leblond, INRIA Sophia Antipolis (France)

### 12h30-14h30 — Lunch Break

### 14h30-16h00 — Session 14 (MZH 1470)

Variational regularization for stochastic noise models (Part I) — Thorsten Hohage, Universität Göttingen (Germany)

### 16h00-16h30 — Coffee Break (MZH 1460)

### 16h30-18h00 — Session 15 (MZH 1470)

Nonlinear inversion methods (Part I) — Samuli Siltanen, University of Helsinki (Finland)

### 18h30-21h00 — Dinner (MZH 1460)

# Friday, October 11

## 9h00-10h40 — Session 16 (MZH 1470)

| | |
|---|---|
| 9h00-9h25 | Numerical analysis of the factorization method for EIT in uncertain backgrounds — Giovanni Migliorati, Ecole Polytechnique Fédérale de Lausanne (Switzerland) |
| 9h25-9h50 | Qualitative non-destructive testing of concrete-like materials: A sampling method for differential measurements — Lorenzo Audibert, Ecole Polytechnique (France) |
| 9h50-10h15 | Comparative SVD-analysis of standard L2-Full Waveform Inversion and its MBTT reformulation — Kirill Gadylshin, Novosibirsk State University (Russia) |
| 10h15-10h40 | Improving seismic imaging in poorly resolved regions – an efficient way to estimate the stochastic Hessian — Lucas Abraham Willemsen, MIT (USA) |

See also the abstracts on page 5.

## 10h40-11h00 — Coffee break (MZH 1460)

## 11h00-12h30 — Session 17 (MZH 1470)

Variational regularization for stochastic noise models (Part II) — Thorsten Hohage, Universität Göttingen (Germany)

## 12h30-13h45 (!!!) — Lunch Break

Please note that the lunch break on Friday takes only 75 minutes.

## 13h45-15h15 — Session 18 (MZH 1470)

Nonlinear inversion methods (Part II) — Samuli Siltanen, University of Helsinki (Finland)

## 15h15-16h00 — Coffee Break & Closing (MZH 1460)

## 18h00-20h00 — Dinner (MZH 1460)

If enough participants register for dinner, there will be a buffet on Friday evening, too.

# Abstracts of Talks

## Asymptotics formula for Stokes transmission problem caused by a small interface change of inclusion

**Thi Hong Cam LUONG**
Université de Cergy-Pontoise (France)

We consider the transmission of Stokes problem in the case there is a small interface change of the inclusion. The asymptotic expansion is derived due to the theory of layer potentials and Fredholm's alternative. Firstly, we prove that the solution of the problem is continuous with respect to this small perturbation. Secondly, we derive the first-order term in the displacement field. It is worth emphasizing that even though only the first-order term is given, our method enables us to derive higher-order terms as well. The derivation is rigorous and based on layer potential techniques.

## The Foldy-Lax approximation of the acoustic scattered waves by small bodies

**Durga Prasad Challa**
Austrian Academy of Sciences, RICAM (Austria)

We are concerned with the acoustic scattering problem by many small rigid obstacles of arbitrary shapes. We give a sufficient condition on the number $M$ and the diameter $a$ of the obstacles as well as the minimum distance $d$ between them under which the Foldy-Lax approximation is valid. As an application, we study the inverse scattering by the small obstacles in the presence of multiple scattering.

## Direct and Inverse Acoustic Scattering by Extended and Point-Like Scatterers

**Guanghui Hu**
Weierstraß-Institut für Angewandte Analysis und Stochastik WIAS (Germany)

We are concerned with the acoustic scattering by an extended obstacle surrounded by point-like obstacles. The extended obstacle is supposed to be sound-soft while the point-like obstacles are modeled by refraction indices of Dirac-like forms multiplied by refraction coefficients. In the

first part, we consider the forward problem and show that the scattered field is a sum of two fields; one is due to the diffusion by the extend obstacle and the other one is a linear combination of the interactions between the point-like obstacles and the interaction between the point-like obstacles with the extended one. This is done following two approaches: the Foldy formal method and the Krein resolvent method. In the second part, we deal with an inverse problem which consists of reconstructing both the extended and the point-like obstacles from the corresponding far field map. To solve this problem, we describe and justify the factorization method of Kirsch. Using this method, we provide several numerical results and discuss the multiple scattering effect reflecting the interactions between the point-like obstacles and also between these obstacles with the extended one.

## Numerical analysis of the Factorization Method for EIT in uncertain backgrounds

**Giovanni Migliorati**
Ecole Polytechnique Fédérale de Lausanne (Switzerland)

We address the Factorization Method (FM) applied to the continuous model for Electrical Impedance Tomography (EIT). Most of the works on the FM in the literature treat the case of a homogeneous background. Our work is mostly related to numerical issues associated with EIT in inhomogeneous and uncertain backgrounds. We propose a numerical scheme to solve the dipole-like Neumann boundary-value problem when the background conductivity coefficient is inhomogeneous and deterministic, and use this scheme to design an efficient implementation of the FM algorithm for an inhomogeneous but deterministic background. We then discuss the case where the background conductivity coefficient is piecewise constant with a known spatial distribution but unknown parameter values. We propose several variants of the FM to cope with this configuration. Our approach relies on an optimization scheme motivated by the structure of the sampling operator and the outcome of the FM, to simultaneously recover the background parameters and the location of the inclusion. Numerical results showing the capabilities of the proposed approach will be presented as well.

## Qualitative non-destructive testing of concrete-like materials: A sampling method for differential measurements

**Lorenzo Audibert**
Ecole Polytechnique and EDF (France)

We propose a new sampling method to image the difference between two medium made of inhomogeneous inclusions. The novelty of this method lies on the fact that it does not require the knowledge of the Green's function of one of the two heterogeneous media but only the far or near field operator of each medium. Numerical simulations motivated by ultrasound non-destructive testing of concrete-like material will be presented.

## Comparative SVD-analysis of standard $L^2$ Full Waveform Inversion and its MBTT reformulation

**Kirill Gadylshin**
Mechanics and Mathematics Department, Novosibirsk State University (Russia)

The common knowledge now is that standard least squares Full Waveform Inversion is unable to reconstruct macrovelocity for a reasonable frequency band of input data but claims unpractically low time frequencies. There are a range of different approaches to overcome this weakness and among them the migration based travel time (MBTT) reformulation of the cost function. Here we compare SVD for linear approximation of standard least squares Full Waveform Inversion with its migration based travel time reformulation. In order to do that we start with linearization of both nonlinear forward maps with subsequent computations of singular spectra (singular values and right/left singular vectors) for corresponding linear operators. The next step is to construct the two families of linear spans of right singular vectors corresponding to a set of fixed values of the condition number and to analyze their mutual disposition. Our computations demonstrate the reliable reconstruction of the smooth velocity component by Full Waveform Inversion in migration based travel-time formulation.

## Improving seismic imaging in poorly resolved regions. An efficient way to estimate the stochastic Hessian

**Lucas A. (Bram) Willemsen**
Department of Earth, Atmospheric and Planetary Sciences, M.I.T. (United States)

Full Waveform Inversion (FWI) is a method that attempts to use all the information contained in the recorded seismic waveforms. This stands in contrast to traveltime tomography and seismic migration which only use part of the waveform. Using the additional information contained in the full waveform enables us to generate higher quality images of the earth. FWI is computationally more expensive than the traditional methods, and suffers from convergence issues if an unsuitable misfit functional between data and forward model is used. In this paper I will focus on frequency domain least squares, which is a popular misfit measure. The gradient, with respect to the model parameters, of this objective function has a strong connection with Reverse Time Migration (RTM) and is primarily sensitive to the shallow Earth. If a gradient-descent approach is used in the minimization of the misfit functional, we are almost exclusively trying to find a shallow Earth structure that explains the data well. Details in deeper layers or poorly illuminated areas will not be resolved properly. The impedance contrast between salt and sedimentary layers causes most of the incoming signal to reflect. The recorded amplitude of subsalt reflections is therefore very low, which results in low sensitivity. Because of this, subsalt imaging is a notoriously difficult job. The Hessian contains important information about the sensitivity of the data to the model parameters and the way in which two simultaneous model perturbations influence each other. Incorporating the Hessian in the inversion strategy therefore allows the model update to be corrected at areas of low sensitivity. Due to the quadratic nature of Hessian based inversion it also speeds up convergence to a (local) minimum. The Hessian can be written as the sum of two parts. The first part is the approximate Hessian which contains products of first order scattering. The other part contains second order scattering and is often neglected. In this paper a method is presented that allows arbitrary elements of the approximate Hessian to be estimated

simultaneously. This is done by placing virtual white-noise sources at the true source and receiver locations and doing an ensemble average of the product of recordings at the locations of interest. Elements of the approximate Hessian for least squares inversion can be written in terms of these ensemble averages. Preliminary theoretical and numerical investigations suggest that the number of forward models required for this procedure does not increase with the number of shots. As the number of shots increases this means that the cost of estimating these approximate Hessian entries becomes negligible relative to the cost of calculating the gradient. The most obvious application would be to estimate the diagonal of the approximate Hessian. This can then be used as a very inexpensive preconditioner for optimization procedures, such as the truncated Newton method. Off-diagonal elements of the Hessian contain important information about how a perturbation at one point changes the gradient of the objective functional at another point. This valuable trade-off information can be used to improve the descent speed of the inversion process. The algorithm presented allows the diagonal and off-diagonal elements of the Hessian to be approximated for a specific region of interest within the model. This freedom of only approximating the relevant part of the Hessian allows the image to be enhanced locally without having to compute large quantities of less interesting Hessian entries.

# Lunch, Dinner and Excursion

## Lunch

In the folder you received when registrating you will find vouchers for the university restaurant that is called "Mensa". The lunch plan of the Mensa can be found either online (http://www.studentenwerk.bremen.de/files/main_info/essen/plaene/uniessen.php) or on the information board next to the lecture hall. For the voucher you get

- one main dish – you can take any main dish from any line, alternatively a big salad or a soufflé or a big soup
- one starter or, alternatively, one desert
- one drink

When you got your food you have to hand in the voucher at any of the pay desks.

## Tab water in restaurants

Note that in Germany free tab water is *completely unusual and hardly known* in any type of restaurant. If you want a glass of water you have to buy it.

## Dinner (except Wednesday)

From Sunday to Thursday (and, if necessary, on Friday, too) we organize a buffet for all workshop participants taking place in room MZH 1460 on the first floor of the MZH building.

## Excursion on Wednesday

On Wednesday afternoon you can take part in a guided city tour through the historic center of Bremen. (Unfortunately, the town hall is not open for public visits during the week the summer school takes place.) We will leave the university after the afternoon session at 15h15 by tramway (line 6). The meeting point for the excursion is in front of the main entrance of the cathedral ("Dom") at 16h00. For the excursion and the way back you will need two tramway tickets. There will be a list lying out until Wednesday morning where you can mark if you want us to buy tickets for you (the price for a single ticket will then be 2,10 € which is cheaper than buying it inside the tramway).

## Summer School Dinner on Wednesday Evening

On Wednesday after the excursion we will have dinner in a restaurant in downtown called "Die Schmidt". The location of the restaurant is Wernerstraße 38, 28203 Bremen. If you will not take part in the guided city tour you should be at the restaurant at about 18h15.

Since our reservation of the place ends at 22h00 that will be the time to leave the place. The fastest way back to campus is either to walk to the bus stop "Klinikum Bremen Mitte" and take bus 25 to the central railway station, or to walk to the tramway stop "Brunnenststraße" and take a tramway line to the central railway station (see the map below). From the central railway station you reach campus using the tramway line 6.

# Participants

| | | |
|---|---|---|
| Abeer | Aldoghaither | King Abdullah University of Science and Technology |
| Akindeinde | Saheed Ojo | Julius- Maximilians-Universität Würzburg |
| Audibert | Lorenzo | Ecole Polytechnique/INRIA/EDF R&D |
| Bakker | Jenneke | ETH Zurich, Institute of Geophysics |
| Belonosov | Mikhail | IPGG SB Russian Academy of Sciences |
| Cheikh | Fatma | Ecole Nationale d'Ingénieurs de Tunis & INRIA Rocquencourt |
| Berndt Hildebrandt | Christina | Technical University of Denmark |
| Bondarenko | Oleksandr | KIT – Karlsruher Institut für Technologie |
| Bürgel | Florian | Universität Bremen, ZETEM |
| Challa | Durga Prasad | RICAM — Austrian Academy of Sciences |
| Dahmani | Zoubir | University of Mostaganem, UMAB |
| Daskalakis | Emmanouil | University Of Crete, Department of Applied Mathematics |
| Dollé | Guillaume | Université de Strasbourg |
| Feldmann | Ebba | ZETEM, Universität Bremen |
| Foged Schmidt | Marie | Technical University of Denmark |
| Fotopoulos | George | University of Oulu, Department of Mathematical Sciences |
| Gadylshin | Kirill | Novosibirsk State University,<br>Mechanics and Mathematics Department |
| Garmatter | Dominik | University of Stuttgart,<br>Chair of Optimization and inverse Problems |
| Gilliocq-Hirtz | Diane | Université de Haute Alsace |
| Glasse | Benjamin | University of Bremen & IWT Bremen |
| Gunel | Korhan | Adnan Menderes University, Department of Mathematics |
| Haddar | Houssem | INRIA Saclay–Ile-de-France |
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| Kluth | Tobias | Universität Bremen, Kognitive Neuroinformatik |
| Kruschel | Christian | TU Braunschweig, Institute for Analysis and Algebra |

| | | |
|---|---|---|
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| Luong | Thi Hong Cam | Université de Cergy-Pontoise, Laboratoire AGM |
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| Zaitseva | Anastassiya | Novosibirsk State University, Mechanics and Mathematics Department |