Efficient reconstruction algorithm for 3D tomographic imaging

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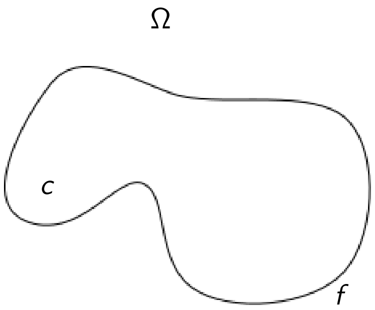
Joint work with A. Hannukainen, L. Harhanen, N. Hyvönen, S. Staboulis and T. Tarvainen

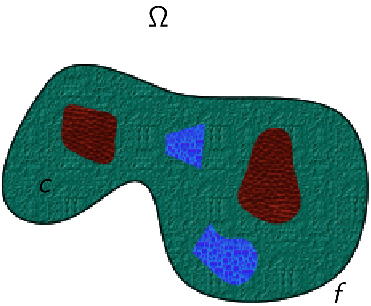
A parameter estimation problem



Consider

- domain $\Omega \subset \mathbb{R}^d$, $d = 2, 3$,
- parameter $c(x) \in C$, $x \in \Omega$,
- measurement $f \in F$ (on $\partial\Omega$),
- *forward operator* $\mathcal{F} : C \to F$.

A parameter estimation problem



Consider

- domain $\Omega \subset \mathbb{R}^d$, $d = 2, 3$,
- parameter $c(x) \in C$, $x \in \Omega$,
- measurement $f \in F$ (on $\partial\Omega$),
- *forward operator* $\mathcal{F} : C \to F$.

Assume that c contains distinct inclusions in a constant background.

Given a measurement $f \in F$, the *inverse problem* is to find such $c \in C$ that $\mathcal{F}(c) \approx f$.

Outline

Likelihood

We discretize the forward problem according to a FE mesh of Ω with piecewise linear basis functions ϕ_n, and write

$$c(x) = \sum_{n=1}^{N} c_n \phi_n(x).$$

Assuming an additive Gaussian noise, the data $\mathcal{V} = \mathcal{F}(c) + \eta$, where $\eta \sim \mathcal{N}(0, \Gamma)$, we get the likelihood function

$$p(\mathcal{V} \,|\, c) \, \propto \, \exp\Big(-\frac{1}{2}(\mathcal{V} - \mathcal{F}(c))^{\mathsf{T}} \Gamma^{-1} (\mathcal{V} - \mathcal{F}(c)) \Big).$$

Prior

To quantify the *a priori* information, the (discretized) parameter is given a prior

$$p(c) \propto \exp\big(-a\,R(c)\big),$$

where $a > 0$ is a free parameter and R is of the form

$$R(c) = \int_\Omega r\big(|\nabla c(x)|\big)\,dx$$

with $r : \mathbb{R}_+ \to \mathbb{R}_+$ being a suitable edge-preferring function, e.g.

$$r(t) = \sqrt{T^2 + t^2} \approx |t| \quad \text{or} \quad r(t) = \frac{1}{2}\,T^2 \log\left(1 + (t/T)^2\right).$$

Posterior

From the Bayes' formula we get the posterior density

$$
\begin{aligned}
p(c \,|\, \mathcal{V}) &\propto p(\mathcal{V} \,|\, c)\, p(c) \\
&\propto \exp\Big(-\frac{1}{2}\big(\mathcal{V} - \mathcal{F}(c)\big)^{\mathsf{T}} \Gamma^{-1} \big(\mathcal{V} - \mathcal{F}(c)\big) - a\, R(c) \Big).
\end{aligned}
$$

Finding the MAP estimate corresponding to the posterior distribution is equivalent to minimizing the Tikhonov functional

$$
G(c) := \frac{1}{2}\big(\mathcal{V} - \mathcal{F}(c)\big)^{\mathsf{T}} \Gamma^{-1} \big(\mathcal{V} - \mathcal{F}(c)\big) + a\, R(c).
$$

Linearization of the forward model

Starting from the initial guess $c^{(0)} = c_0$ we linearize $\mathcal{F}(c)$ around the current iterate $c^{(l)}$ to get a sequence of functionals

$$G^{(l)}(c) := \frac{1}{2}(y^{(l)} - J^{(l)}c)^{\mathsf{T}}\Gamma^{-1}(y^{(l)} - J^{(l)}c) + a\,R(c),$$

where $J^{(l)}$ is the Jacobian of the map $c \mapsto \mathcal{F}(c)$ evaluated at the current iterate $c^{(l)}$ and $y^{(l)} = \mathcal{V} - \mathcal{F}(c^{(l)}) + J^{(l)}c^{(l)}$.

For the linearization, we need to approximate (using a FEM)

- the forward solution $\mathcal{F}(c^{(l)})$,
- the Jacobian $J^{(l)}$.

Linearization of the forward model

To minimize, the necessary condition $\nabla G^{(l)} = 0$ gives the equation

$$(J^{(l)})^{\mathsf{T}}\Gamma^{-1}J^{(l)}c + a\,(\nabla R)(c) = \;(J^{(l)})^{\mathsf{T}}\Gamma^{-1}y^{(l)}.$$

The gradient of R has the representation $(\nabla R)(c) = M(c)c$, with

$$M_{i,j}(c) := \int_{\Omega} \frac{r'(|\nabla c(x)|)}{|\nabla c(x)|}\;\nabla\phi_i(x)\cdot\nabla\phi_j(x)\,dx, \quad i,j = 1,\dots,N,$$

so we can write the equation in the form

$$\Big((J^{(l)})^{\mathsf{T}}\Gamma^{-1}J^{(l)} + a\,M(c)\Big)\,c \;=\; (J^{(l)})^{\mathsf{T}}\Gamma^{-1}y^{(l)}.$$

Single lagged diffusivity step

To get rid of the nonlinearity of $M(c)$, we evaluate the M-matrix at $c^{(l)}$ and, denoting

$$A = \Gamma^{-1/2} J^{(l)}, \quad M = M(c^{(l)}), \quad \tilde{y} = \Gamma^{-1/2} y^{(l)},$$

we obtain the normal equation

$$\left(A^{\mathsf{T}} A + aM\right) c \; = \; A^{\mathsf{T}} \tilde{y}, \qquad a > 0.$$

The solution of this equation is equivalent to the minimizer of

$$\|Ac - \tilde{y}\|^2 + a\, c^{\mathsf{T}} M c\,.$$

Priorconditioning

We *formally* factorize $M = L^\mathsf{T} L$ to get the transformed equation

$$\left((L^{-1})^\mathsf{T} A^\mathsf{T} A L^{-1} + aI\right)\tilde{c} = (L^{-1})^\mathsf{T} A^\mathsf{T} \tilde{y},$$

where $\tilde{c} = Lc$. Set $a = 0$ and apply LSQR combined with an early stopping rule. Starting the iteration from $\tilde{c} = 0$, the solution space spanned by the Krylov solver after m iterations is

$$\mathcal{K}_m = \mathrm{span}\left\{M^{-1}A^\mathsf{T}\tilde{y}, \ldots, \left(M^{-1}A^\mathsf{T}A\right)^{m-1} M^{-1}A^\mathsf{T}\tilde{y}\right\},$$

so, for all m, the solution is of form $c_m = M^{-1}\alpha$ for some $\alpha \in \mathbb{R}^N$.

- We need to ensure the existence of M^{-1}.

Stopping conditions

We define the general stopping conditions for the iterations

$$\mathcal{C}_{\text{out}}, \mathcal{C}_{\text{in}} : c \mapsto \{\text{true}, \text{false}\}$$

using the corresponding residuals

$$E(c) = \|\Gamma^{-1/2} (\mathcal{V} - \mathcal{F}(c))\|, \quad e(c) = \|Ac - \tilde{y}\|.$$

For example, based on the Morozov discrepancy principle, we terminate the iteration, when the residual reaches the noise level ε,

$$\mathcal{C}_{\text{out}}(c^{(l)}) := E(c^{(l)}) \leq \varepsilon \quad \text{and} \quad \mathcal{C}_{\text{in}}(c_m) := e(c_m) \leq \varepsilon.$$

The algorithm

Define the stopping conditions $\mathcal{C}_{\text{in}}(c), \mathcal{C}_{\text{out}}(c)$.
Choose the prior $r(t)$ and the free parameter $T > 0$.

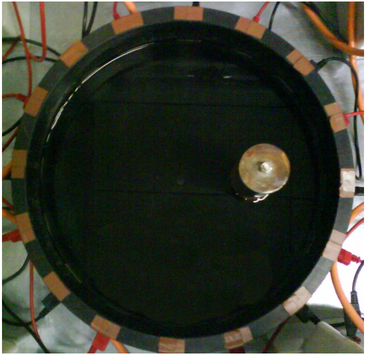
Minimize $E(c)$ over $c \in \mathbb{R}$ to determine c_0. (G-N)
Set $l = 0$, $c^{(0)} = c_0$ and $\mathcal{F} = \mathcal{F}(c^{(0)})$.

1. Evaluate the Jacobian $J^{(l)}$ and $M = M(c^{(l)})$.
2. Form A, $\tilde{y}$ and $L^{\mathsf{T}}L = M$. Use LSQR to solve $c^{(l+1)}$ from

$$(L^{-1})^{\mathsf{T}} A^{\mathsf{T}} A L^{-1} \tilde{c} = (L^{-1})^{\mathsf{T}} A^{\mathsf{T}} \tilde{y}, \qquad c = L^{-1}\tilde{c}$$

   starting from $\tilde{c} = 0$ and stopping when $\mathcal{C}_{\text{in}}(c_m)$.
3. Compute $\mathcal{F} = \mathcal{F}(c^{(l+1)})$. If $\mathcal{C}_{\text{out}}(c^{(l+1)})$, exit with $c^{(l+1)}$ as reconstruction, otherwise set $l \leftarrow l + 1$ and return to step 1.

Electrical impedance tomography



- Attached to the boundary: electrodes $e_k \subset \partial\Omega$, $k = 1, \ldots K$.
- Input currents $I = [I_k]_{k=1}^K$, $I_k \in \mathbb{R}$ and measure the corresponding voltages $U = [U_k]_{k=1}^K$, $U_k \in \mathbb{R}$.
- Determine the conductivity $\sigma(x) \in L^\infty(\Omega)$, $\sigma > 0$.
- Unknown contact resistances $z_k \in \mathbb{R}_+$, $k = 1, \ldots K$.
- Measuring with $K - 1$ linearly independent current patterns, we get $\mathcal{F}(\sigma, z) = U \in \mathbb{R}^{K(K-1)}$.

Complete electrode model

Pair $(u, U) \in H^1(\Omega) \oplus \mathbb{R}^K_\diamond$ is the unique solution of

$$
\begin{aligned}
\nabla \cdot (\sigma \nabla u) &= 0 && \text{in } \Omega, \\
\sigma \frac{\partial u}{\partial \nu} &= 0 && \text{on } \partial\Omega \setminus \cup_{k=1}^K e_k, \\
u + z_k \sigma \frac{\partial u}{\partial \nu} &= U_k && \text{on } e_k, \quad k = 1, \dots, K, \\
\int_{e_k} \sigma \frac{\partial u}{\partial \nu} \, dS &= I_k, && k = 1, \dots, K,
\end{aligned}
$$

for a given current pattern $I \in \mathbb{R}^K_\diamond$, with $\nu : \partial\Omega \to \mathbb{R}^d$ denoting the exterior unit normal of $\partial\Omega$.

The algorithm for EIT

Assuming an additive Gaussian noise, an edge-preferring prior for σ and an uninformative prior for z, we end up minimizing

$$G(\sigma, z) = \frac{1}{2}\big(\mathcal{V} - \mathcal{F}(\sigma, z)\big)^{\mathsf{T}}\Gamma^{-1}\big(\mathcal{V} - \mathcal{F}(\sigma, z)\big) + a\,R(\sigma).$$

After linearization and the lagged diffusivity step, denoting $J^{(l)} = \left[J_\sigma^{(l)}, J_z^{(l)}\right]$ and $M = M(\sigma^{(l)})$, we get the equation

$$\begin{bmatrix} \left(J_\sigma^{(l)}\right)^{\mathsf{T}}\Gamma^{-1}J_\sigma^{(l)} + aM & \left(J_\sigma^{(l)}\right)^{\mathsf{T}}\Gamma^{-1}J_z^{(l)} \\ \left(J_z^{(l)}\right)^{\mathsf{T}}\Gamma^{-1}J_\sigma^{(l)} & \left(J_z^{(l)}\right)^{\mathsf{T}}\Gamma^{-1}J_z^{(l)} \end{bmatrix} \begin{bmatrix} \sigma \\ z \end{bmatrix} = \left(J^{(l)}\right)^{\mathsf{T}}\Gamma^{-1}y^{(l)}.$$

The algorithm for EIT

Solving the second line for z and denoting $B = \Gamma^{-1/2} J_z^{(l)}$, we get

$$z = \left(B^\mathsf{T} B\right)^{-1} B^\mathsf{T} \left(\Gamma^{-1/2} y^{(l)} - \Gamma^{-1/2} J_\sigma^{(l)} \sigma\right). \tag{1}$$

Substituting this to the first line, we obtain

$$(A^\mathsf{T} A + aM)\sigma = A^\mathsf{T} \tilde{y},$$

where

$$A = Q\Gamma^{-1/2} J_\sigma^{(l)}, \quad \tilde{y} = Q\Gamma^{-1/2} y^{(l)}, \quad Q = \mathrm{I} - P, \quad P = B(B^\mathsf{T} B)^{-1} B^\mathsf{T}.$$

Fix $\sigma = \sigma_0$ on $\cup_{k=1}^K e_k$ and apply priorconditioned LSQR to solve σ elsewhere. Substitute the solution to (1) to get z.

The algorithm for EIT

Define the stopping conditions $\mathcal{C}_{\text{in}}(\sigma, z), \mathcal{C}_{\text{out}}(\sigma, z)$.
Choose the prior $r(t)$ and the free parameter $T > 0$.

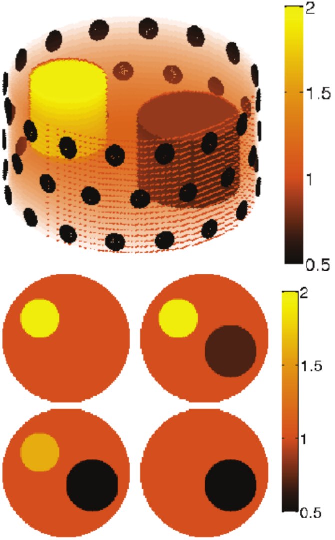
Let $(\sigma_0, z_0) = \arg\min_{\sigma, z \in \mathbb{R}_+} E(\sigma, z)$.
Set $l = 0$, $\sigma^{(0)} = \sigma_0$, $z^{(0)} = z_0$ and $\mathcal{F} = \mathcal{F}(\sigma^{(0)}, z^{(0)})$.

1. Evaluate $J^{(l)} = [J_\sigma^{(l)}, J_z^{(l)}]$ and $M = M(\sigma^{(l)})$.
2. Form A, $\tilde{y}$ and $L^\mathsf{T} L = M$. Use LSQR to solve $\sigma^{(l+1)}$ from

$$(L^{-1})^\mathsf{T} A^\mathsf{T} A L^{-1} \tilde{\sigma} = (L^{-1})^\mathsf{T} A^\mathsf{T} \tilde{y}, \qquad \sigma = L^{-1} \tilde{\sigma}$$

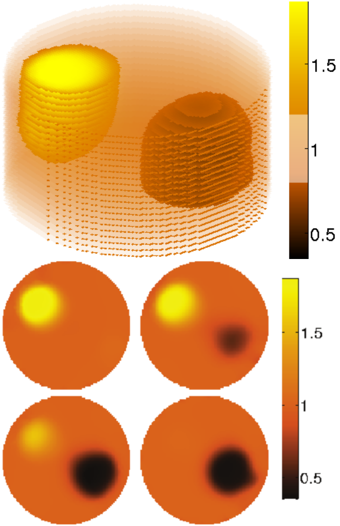
   starting from $\tilde{\sigma} = 0$, until $\mathcal{C}_{\text{in}}(\sigma_m)$. Calculate $z^{(l+1)}$.
3. Compute $\mathcal{F} = \mathcal{F}(\sigma^{(l+1)}, z^{(l+1)})$. If $\mathcal{C}_{\text{out}}(\sigma^{(l+1)}, z^{(l+1)})$, exit with $\sigma^{(l+1)}, z^{(l+1)}$, otherwise set $l \leftarrow l + 1$ and return to 1.

Case 1: EIT, cylinder (simulated data)



- $\mu_{\text{bg}} = 1$
- $K = 48$ electrodes
- $z_k \sim \mathcal{N}(2 \times 10^{-3}, (5 \times 10^{-4})^2)$, $k = 1, \ldots, K$
- FEM: $N = 103\,507$
- $\mathcal{F}(\sigma, z) \in \mathbb{R}^{2256}$
- 0.4% of noise

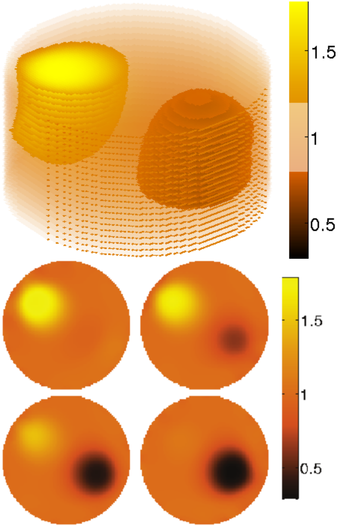
Case 1: reconstruction with Perona-Malik prior



- P-M prior, $T = 5 \times 10^{-3}$
- FEM: $N = 17\,031$

- $\sigma_0 = 0.93$
- running time $\sim$ 3 minutes (LSQR rounds: 32+24+18)

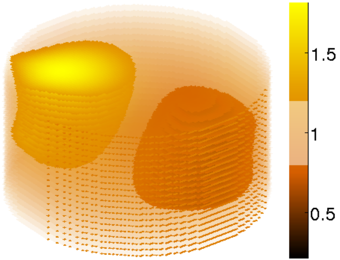
Case 1: reconstruction with total variation prior

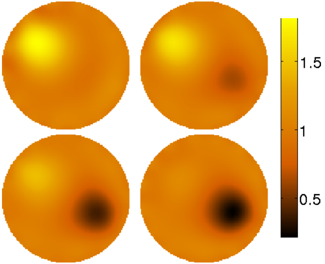


- TV prior, $T = 10^{-6}$
- FEM: $N = 17\,031$

- $\sigma_0 = 0.93$
- running time $\sim$ 3 minutes (LSQR rounds: 32+28+22)

Case 1: reconstruction with a Tikhonov prior

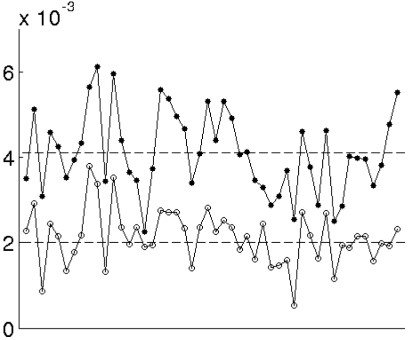




- $r(t) = \frac{1}{2}t^2$
- FEM: $N = 17\,031$

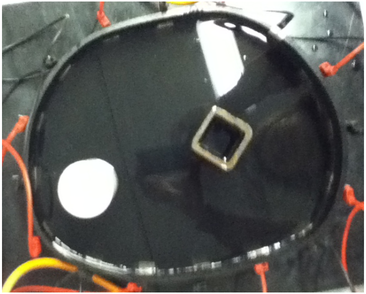
- $\sigma_0 = 0.93$
- running time $\sim$ 3 minutes (LSQR rounds: 32+33+33)

Case 1: reconstruction of z



The reconstructed contact resistances (filled) are systematically too high compared to the exact ones.

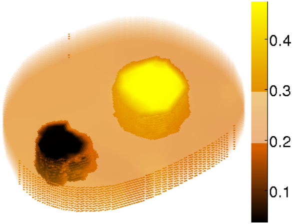
Case 2: EIT, water tank (measured data)

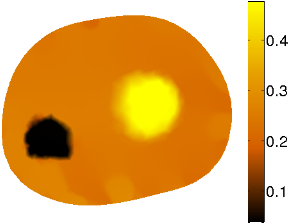


- thorax-shaped water tank with circumference 106 cm
- one plastic and one metallic inclusion cylinder
- $K = 16$ rectangular electrodes of width 2 cm and height 5 cm
- $\mathcal{V} \in \mathbb{R}^{240}$

The measurements were performed using the *Kuopio impedance tomography* (KIT4) device.

Case 2: reconstruction

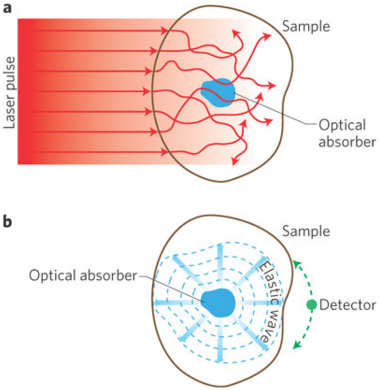




- $\Gamma = (\gamma \max_{i,j} |\mathcal{V}_i - \mathcal{V}_j|)^2 \, \mathrm{I}$,
  $\gamma = 3.7 \times 10^{-4}$
- P-M prior, $T = 5 \times 10^{-3}$
- FEM: $N = 27\,276$

- $\sigma_0 = 0.25$
- running time $\sim$ 4 minutes
  (LSQR rounds: 63+37+31+26+24)

Quantitative photoacoustic tomography



- Light pulse through part(s) of the boundary $s_k \subset \partial\Omega$, $k = 1, \ldots, K$.
- Absorbers transform light into acoustic wave.
- Ultrasound sensors detect the wave on the surface.
- Using the surface measurements, reconstruct $H_k(x) \in L^2(\Omega)$.
- Reconstruct the diffusivity and the absorption $\kappa(x), \mu(x) \in L^\infty_+(\Omega, \mathbb{R})$.
- Internal measurements $\mathcal{F}(\kappa, \mu) = H \in \mathbb{R}^{KN}$.

Diffusion approximation of the radiative transfer equation

According to the DA, for $\Omega \subset \mathbb{R}^3$, the photon density $\varphi_k \in H^1(\Omega)$ is the unique solution of

$$\begin{cases} -\nabla \cdot (\kappa \nabla \varphi_k) + \mu \varphi_k = 0 & \text{in } \Omega, \\ \frac{1}{4}\varphi_k + \frac{1}{2}\nu \cdot \kappa \nabla \varphi_k = \Phi_k & \text{on } \partial\Omega. \end{cases}$$

And the available photoacoustic measurement (the absorbed energy density) is defined as

$$H_k(x) = \mu(x)\varphi_k(x) \in L^2(\Omega).$$

The algorithm for QPAT

Using the transformations $\tilde{\kappa} = \log(\kappa/\kappa_0)$, $\tilde{\mu} = \log(\mu/\mu_0)$ and assuming an additive Gaussian noise and edge-preferring priors for $\tilde{\kappa}$ and $\tilde{\mu}$, we end up minimizing

$$G(\tilde{\kappa}, \tilde{\mu}) = \frac{1}{2}\big(\mathcal{V} - \mathcal{F}(\tilde{\kappa}, \tilde{\mu})\big)^{\mathrm{T}} \Gamma^{-1} \big(\mathcal{V} - \mathcal{F}(\tilde{\kappa}, \tilde{\mu})\big) + a\, R(\tilde{\kappa}) + b\, R(\tilde{\mu}).$$

And after linearization and the lagged diffusivity step, we get

$$\left(A^{\mathrm{T}} A + aM\right) \begin{bmatrix} \tilde{\kappa} \\ \tilde{\mu} \end{bmatrix} = A^{\mathrm{T}} \tilde{y},$$

where

$$A = \Gamma^{-1/2} J^{(l)}, \quad M = \begin{bmatrix} M(\tilde{\kappa}^{(l)}) & 0 \\ 0 & \frac{b}{a}\, M(\tilde{\mu}^{(l)}) \end{bmatrix}, \quad \tilde{y} = \Gamma^{-1/2} y^{(l)}.$$

The algorithm for QPAT

Approximate matrix M by the pos. def. matrix

$$M_\delta = M + \delta I, \qquad \delta > 0.$$

Set $\varepsilon = \sqrt{KN}$ and define the stopping conditions as

$$\mathcal{C}_{\text{in}}(\tilde{\kappa}_m, \tilde{\mu}_m) := e(\tilde{\kappa}_m, \tilde{\mu}_m) \leq \varepsilon \text{ OR } \left(1 - \frac{e(\tilde{\kappa}_m, \tilde{\mu}_m)}{e(\tilde{\kappa}_{m-d}, \tilde{\mu}_{m-d})}\right) \leq \tau,$$

where $d \in \mathbb{N}$ and $\tau > 0$ and

$$\mathcal{C}_{\text{out}}(\tilde{\kappa}^{(l)}, \tilde{\mu}^{(l)}) := E(\tilde{\kappa}^{(l)}, \tilde{\mu}^{(l)}) \leq \varepsilon \text{ OR } E(\tilde{\kappa}^{(l)}, \tilde{\mu}^{(l)}) \geq E(\tilde{\kappa}^{(l-1)}, \tilde{\mu}^{(l-1)}).$$

The algorithm for QPAT

To initialize, solve $\varphi_k^{(0)} = \varphi_k(\kappa_0, \mu_0)$ for $k = 1, \dots, K$ and use

$$\mu_{\text{init}} = \frac{1}{K} \sum_{k=1}^{K} \frac{\mathcal{V}_k}{\varphi_k^{(0)}}$$

to get $\tilde{\mu}_{\text{init}} = \log(\mu_{\text{init}}/\mu_0)$. Let $\mathcal{F} = \mathcal{F}(0, \tilde{\mu}_{\text{init}})$, form the Jacobian $J = J_{\tilde{\kappa}}(0)$, set $\hat{y} = J^{\mathsf{T}}\Gamma^{-1}(\mathcal{V} - \mathcal{F})$ and build $M_\delta(0) = L^{\mathsf{T}}L$. Apply the LSQR algorithm to solve $\tilde{\kappa}_{\text{init}}$ from

$$(L^{-1})^{\mathsf{T}} A^{\mathsf{T}} A L^{-1} \hat{\kappa} = (L^{-1})^{\mathsf{T}} A^{\mathsf{T}} \hat{y}, \qquad \tilde{\kappa} = L^{-1}\hat{\kappa}$$

starting from $\hat{\kappa} = 0$ and stopping when $\mathcal{C}_{\text{in}}(\tilde{\kappa}_m)$.

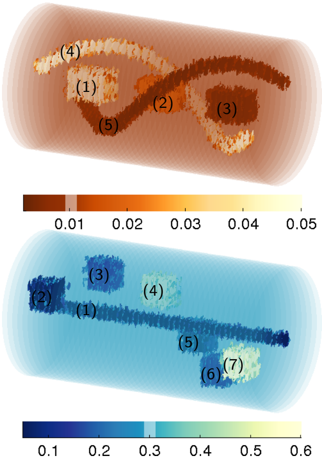
The algorithm for DOT

Define the stopping conditions $\mathcal{C}_{\text{in}}(\tilde\kappa, \tilde\mu), \mathcal{C}_{\text{out}}(\tilde\kappa, \tilde\mu)$.
Choose $r(t)$ and the free parameters $T > 0$ and b/a.

Let $(\kappa_0, \mu_0) = \arg\min_{\kappa,\mu\in\mathbb{R}_+} E(\log\kappa, \log\mu)$.
Solve $\{\varphi_k^{(0)}\}_{k=1}^K$ and initialize $\tilde\mu_{\text{init}}$. Solve $\tilde\kappa_{\text{init}}$.
Set $l = 0$, $\tilde\kappa^{(0)} = \tilde\kappa_{\text{init}}$, $\tilde\mu^{(0)} = \tilde\mu_{\text{init}}$ and $\mathcal{F} = \mathcal{F}(\tilde\kappa^{(0)}, \tilde\mu^{(0)})$.

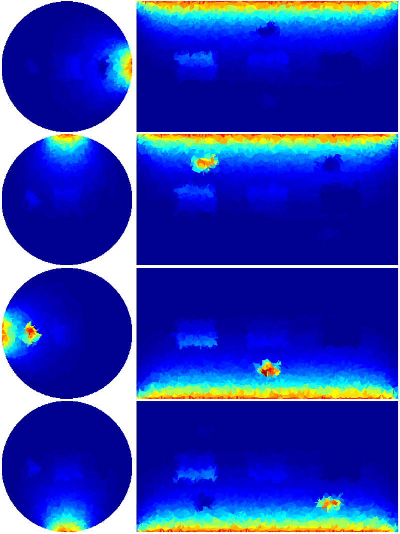
1. Evaluate $J^{(l)}$ and $M = M(\tilde\kappa^{(l)}, \tilde\mu^{(l)})$.
2. Form A, $\tilde y$ and $L^{\mathrm{T}}L = M$. Use LSQR to solve $\begin{bmatrix}\tilde\kappa^{(l+1)}\\ \tilde\mu^{(l+1)}\end{bmatrix}$ from
$$(L^{-1})^{\mathrm{T}}A^{\mathrm{T}}AL^{-1}\tilde\beta = (L^{-1})^{\mathrm{T}}A^{\mathrm{T}}\tilde y, \qquad [\tilde\kappa^{\mathrm{T}}, \tilde\mu^{\mathrm{T}}]^{\mathrm{T}} = L^{-1}\tilde\beta$$
starting from $\tilde\beta = 0$ and stopping when $\mathcal{C}_{\text{in}}(\tilde\kappa_m, \tilde\mu_m)$.
3. Compute $\mathcal{F} = \mathcal{F}(\tilde\kappa^{(l+1)}, \tilde\mu^{(l+1)})$. If $\mathcal{C}_{\text{out}}(\tilde\kappa^{(l+1)}, \tilde\mu^{(l+1)})$, exit with $\kappa(\tilde\kappa^{(l+1)}), \mu(\tilde\mu^{(l+1)})$, otherwise set $l \leftarrow l+1$ and go to 1.

Case 3: QPAT, cylinder



absorption		diffusivity	
bg	0.01	bg	0.3
(1)	0.05	(1)	0.05
(2)	0.02	(2)	0.05
(3)	0.002	(3)	0.15
(4)	0.05	(4)	0.6
(5)	0.002	(5)	0.05
		(6)	0.15
		(7)	0.6

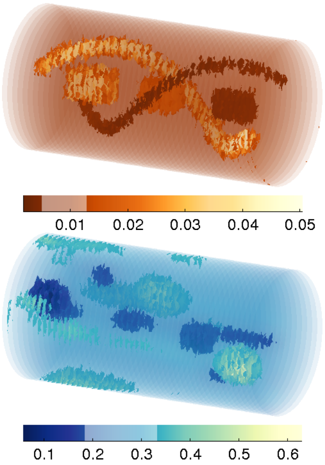
Case 3: measurements



- $K = 4$ illuminations
- FEM: $N = 130\,091$
- $\mathcal{F}(\kappa, \mu) \in \mathbb{R}^{520\,364}$

- FEM: $N = 51\,794$
- $\mathcal{V}_k = PH_k + \eta_k,\ k = 1, \ldots K,$ where $P \in \mathbb{R}^{51\,794 \times 130\,091}$
- $\mathcal{V} \in \mathbb{R}^{207\,176}$
- 1% of noise

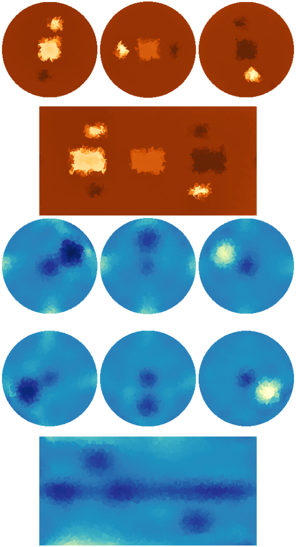
Case 3: initial guesses



- P-M prior, $T = 5 \cdot 10^{-3}$
- $\delta = 10^{-6}$, $b/a = 1$
- $d = 10$, $\tau = 10^{-2}$

- $\mu_0 = 0.0087$, $\kappa_0 = 0.26$

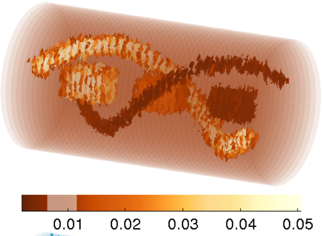
Case 3: initial guesses

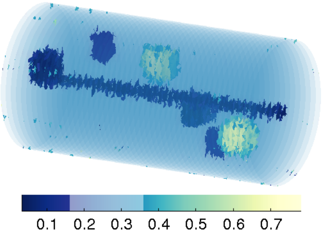


- P-M prior, $T = 5 \cdot 10^{-3}$
- $\delta = 10^{-6}$, $b/a = 1$
- $d = 10$, $\tau = 10^{-2}$

- $\mu_0 = 0.0087$, $\kappa_0 = 0.26$

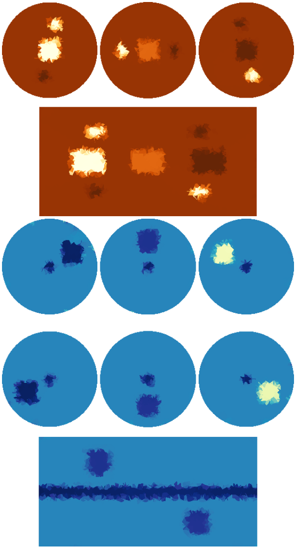
Case 3: reconstructions





	mean absorption		
	μ_{target}	μ_{rec}	$P\mu_{\text{target}}$
bg	0.01	0.00996	0.0101
(1)	0.05	0.0456	0.0459
(2)	0.02	0.0188	0.0189
(3)	0.002	0.00263	0.00266
(4)	0.05	0.0425	0.0432
(5)	0.002	0.00335	0.00338

Case 3: reconstructions



mean diffusivity

	κ_{target}	κ_{rec}	$P\kappa_{\text{target}}$
bg	0.3	0.304	0.300
(1)	0.05	0.0708	0.0874
(2)	0.05	0.0628	0.0668
(3)	0.15	0.165	0.163
(4)	0.6	0.595	0.579
(5)	0.05	0.0764	0.0792
(6)	0.15	0.174	0.169
(7)	0.6	0.576	0.573

- running time $\sim$ 12 minutes, (five LSQR rounds)

References

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THANK
YOU