Optimal Preemptive Resume Priorities for Repair of Unreliable Machines in Two-Phase Production System

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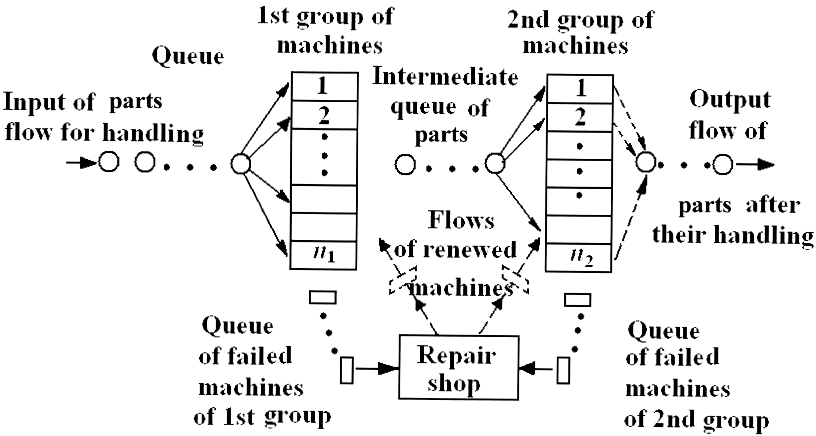
Problem description



Scheme of production system with two groups of unreliable machines in parallel

Notation

λ input rate

n_i number of machines at stage i

μ_i production rate of a machine of the stage i

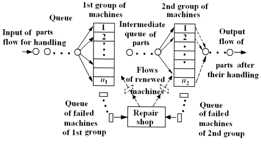
a_i rate of failures flow of a machine at stage i

$R > \max\{n_1, n_2\}$ capacity of the plant

$1/b_i$ mean repair time of a machine at stage i

$N_i(t)$ number of parts at the stage i at the moment t

$\nu_i(t)$ number of failed machines at stage i at the moment t



Scheme of production system with two groups of unreliable machines in parallel

Finite Markov chain

$(N_1(t), N_2(t), \nu_1(t), \nu_2(t))$ is a homogeneous Markov chain on

$\Omega = \{(k_1, k_2, l_1, l_2) :$
$k_1, k_2 \geq 0; k_1 + k_2 \leq R; 0 \leq l_1 \leq \min(k_1, n_1); 0 \leq l_2 \leq \min(k_2, n_2)\}$

$$p(k_1, k_2, l_1, l_2, t) = Pr\{N_1(t) = k_1, N_2(t) = k_2, \nu_1(t) = l_1, \nu_2(t) = l_2\}. \tag{1}$$

We are looking for the statistical equilibrium distribution

$$p(k_1, k_2; l_1, l_2) = \lim_{t \to \infty} p(k_1, k_2; l_1, l_2, t), \quad (k_1, k_2; l_1, l_2) \in \Omega \tag{2}$$

Kolmogorov equations

$$
\begin{aligned}
(\lambda(1-u(R-k_1-k_2))+\sum_{i=2}^{2}(\mu_i(k_i,l_i)+a_i(k_i,n_i)) \\
+b_1u(l_1)+b_2u(l_2)(1-u(l_1)))p(k_1,k_2;l_1,l_2) \\
=\lambda p(k_1-1,k_2;l_1,l_2)u(k_1)u(R-k_1-k_2) \\
+\mu_1(k_1,i_1)p(k_1+1,k_2-1,l_1,l_2)u(k_2) \\
+\mu_2(k_2+1,l_2)p(k_1,k_2+1,i_1,i_2)u(R-k_1-k_2) \\
+a_1(k_1,l_1-1)p(k_1,k_2;l_1-1,l_2)u(l_1) \\
+a_2(k_2,l_2-1)p(k_1,k_2;l_1,l_2-1)u(l_2) \\
+b_1p(k_1,k_2;l_1+1,l_2)u(n_1-l_1)+b_2p(k_1,k_2;0,l_2+1) \\
u(n_2-l_2)(1-u(l_1)),(k_1,k_2,;l_1,l_2)\in\Omega
\end{aligned}
\tag{3}
$$

$$
\sum_{(k_1,k_2,;l_1,l_2)\in\Omega} p(k_1,k_2,;l_1,l_2)=1, \tag{4}
$$

$u(0)=0,\ u(k)=1$ for $k>0,\ \mu_j(k_j,l_j)=\mu_j\min\{n_j-l_j,k_j\}$,
$a_j(k_j,l_j)=a_j\min(k_j-l_j,n_j-l_j),\ j=1,2.$

Objective functional

$S(j_1, j_2)$ objective functional defined on the set of pairs (j_1, j_2), $j_1, j_2 = 1, 2,\ j_1 \neq j_2$,
"penalty for being at a state $(k_1, k_2, ; l_1, l_2)$:"

$$S(1,2) := \sum_{(k_1,k_2,;l_1,l_2)\in\Omega} s(k_1, k_2, ; l_1, l_2) p(k_1, k_2, ; l_1, l_2). \tag{5}$$

For example

$$M(1,2) := \sum_{(k_1,k_2,;l_1,l_2)\in\Omega} \mu_2(k_2, l_2) p(k_1, k_2, ; l_1, l_2). \tag{6}$$

maximizes the outgoing flow rate

Statistic equilibrium probabilities

How to find $p(k_1, k_2; l_1, l_2)$?

Note that the number of equations in (3) grows very fast with R, n_1, n_2.

Possible approach:

$$p(k_1, k_2; l_1, l_2) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} c_{ij}(k_1, k_2; l_1, l_2) a_1^i a_2^j \qquad (7)$$

note: $c_{00}(k_1, k_2; 0, 0)$ probabilities for absolutely reliable machines
a recurrent formula for $c_{ij}(k_1, k_2; l_1, l_2)$ may be derived

Example

Let $R = 2, n_1 = n_2 = 1$ and
machines of the <u>first</u> stage have the highest priority
then the system (3),(4) is

$$\lambda p(0,0,0,0) = \mu_2 p(0,1,0,0), \tag{8}$$

$$(\lambda+\mu_1+a_1)p(1,0,0,0) = \lambda p(0,0,0,0)+\mu_2 p(1,1;0,0)+b_1 p(1,0;1,0), \tag{9}$$

$$(\lambda+\mu_2+a_2)p(0,1,0,0) = \mu_1 p(1,0,0,0)+\mu_2 p(0,2;0,0)+b_2 p(0,1;0,1), \tag{10}$$

$$(\mu_1+\mu_2+a_1+a_2)p(1,1;0,0) = \lambda p(0,1;0,0)+\mu_1 p(2,0;0,0) + b_1 p(1,1;1,0)+b_2 p(1,1;0,1), \tag{11}$$

$$(\lambda+b_1)p(1,0;1,0) = a_1 p(1,0;0,0)+\mu_2 p(1,1;,0), \tag{12}$$

$$(\lambda+b_1)p(0,1;0,1) = a_2 p(0,1;0,0), \tag{13}$$

$$(\mu_1+a_1)p(2,0;0,0) = \lambda p(1,0;0,0)+b_1 p(2,0;1,0), \tag{14}$$

$$(mu_2+a_2)p(0,2;0,0) = \mu_1 p(1,1;0,0)+b_2 p(0,2;0,1), \tag{15}$$

$$(\mu_2+b_1+a_2)p(1,1;1,0) = a_1 p(1,1;0,0), \tag{16}$$

Example

$$(\mu_1+b_2+a_1)p(1,1;0,1) = a_2p(1,1;0,0)+b_1p(1,1;1,1)+\lambda p(0,1;0,1), \tag{17}$$

$$b_1p(2,0;1,0) = \lambda p(1,0;1,0) + a_1p(2,0;0,0), \tag{18}$$

$$b_2p(0,2;0,1) = \mu_1p(1,1;0,1) + a_2p(0,2;0,0), \tag{19}$$

$$b_1p(1,1;1,1) = a_1p(1,1;0,1) + a_2p(1,1;1,0), \tag{20}$$

$$\begin{aligned} p(0,0;0,0) + p(1,0;0,0) + p(0,1;0,0) + p(1,1;0,0) + p(1,0;1,0) \\ +p(0,1;0,1) + p(2,0;0,0) + p(0,2;0,0) + p(1,1;1,0) \\ +p(1,1;0,1) + p(2,0;1,0) + p(0,2;0,1) + p(1,1;1,1) = 1 \end{aligned} \tag{21}$$

system of this algebraic equations can be solved easily.
$M(1,2)$ can be then calculated.

Example

In case $R = 2, n_1 = n_2 = 1$ and
machines of the <u>second</u> stage have the highest priority
the equations (16),(17),(24) should be changed to

$$(\mu_2 + b_1 + a_2)p(1,1;1,0) = a_1 p(1,1;0,0) + b_2 p(1,1;1,1), \quad (22)$$

$$(\mu_1 + b_2 + a_1)p(1,1;0,1) = a_2 p(1,1;0,0) + \lambda p(0,1;0,1), \quad (23)$$

$$b_2 p(1,1;1,1) = a_1 p(1,1;0,1) + a_2 p(1,1;1,0), \quad (24)$$

Then $M(2,1)$ can be calculated and compared with $M(1,2)$
This will show which priority is better in the sense of maximal outgoing rate.

Conclusions

- We have considered a simple production plant with two stages
- There two possible priorities for the repair allocations are considered
- The equilibrium probabilities for the corresponding Markov chain can be found
- Comparison of the objective functionals for both priorities shows the effectiveness of priorities
- This is a first step on the way of modelling of production networks with unreliable machines