Effects of modelling order policies in production networks by potential functions

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Outline

- Introduction
- Modelling
- Introduction of potential function
- Analysis of single node
- Analysis of supply chain
- Summary and outlook

Introduction

- Regarding discontinuities in the processes and generally a non-synchronous flow of material and information in logistic networks, these systems are highly nonlinear
- Several studies found oscillatory and even chaotic behaviour in different models and case studies.
- Most results for supply networks were obtained on linearised dynamic equations
- Focus on the effects of nonlinearities in supply chains
- These nonlinearities give rise to a rich variety of bifurcations

Scenario

- Supply chain of k nodes
- Represents storage for one product, with stock size N_i
- No production, only delivery, ordering and stock keeping
- Flow oriented model with inflow and outflow

$$\dot{N}_i = Q_i^{in} - Q_i^{out}$$

Flow oriented model

- The order rate is usually given by an ordering or stock keeping policy with smooth adaptation:

$$\dot{Q}_i^{in} = \frac{1}{\tau_i}\left(\sigma_i F_i(N_i) - Q_i^{in}\right).$$

- Both equations form the well known ODE:

$$\ddot{N}_i + \frac{1}{\tau_i}\dot{N}_i + \frac{\sigma_i}{\tau_i}F_i(N_i) = -\frac{1}{\tau_i}Q_i^{out} - \dot{Q}_i^{out}$$

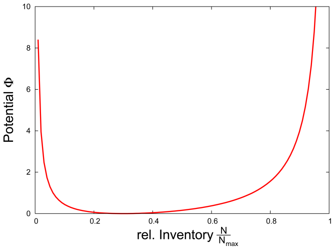
- Function $F_i(N_i)$ represents the stock keeping policy
- Many different functions possible, e.g. forecasting methods

Introduction of Potential function

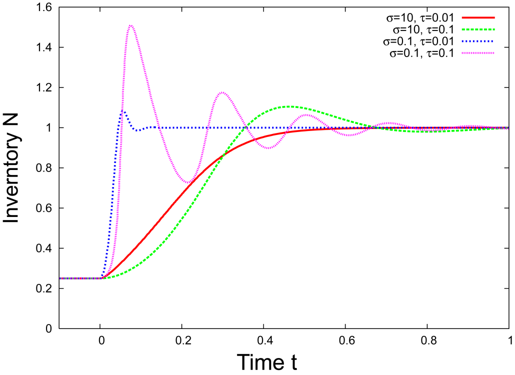
- In general such a policy can be formulated in terms of a potential function $F = -\frac{d\Phi_i}{dN_i}$

$$\Phi_i(N_i) = \frac{(N_i - N_i^{opt})^2}{(N_i - N_i^{min})(N_i^{max} - N_i)}$$

- Corridor policy which tries to hold the stock on a desired level N_i^{opt}
- Prevents the inventory from falling below a minimal level N_i^{min} and exceeding the storage capacity N_i^{max}

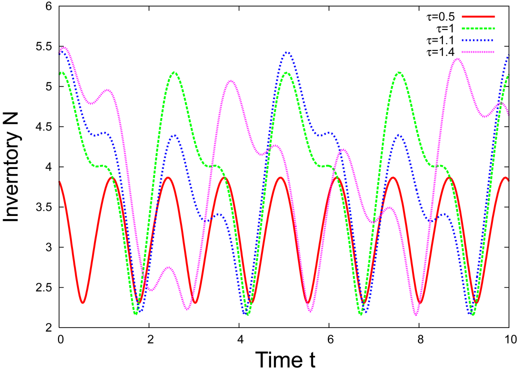


Step function



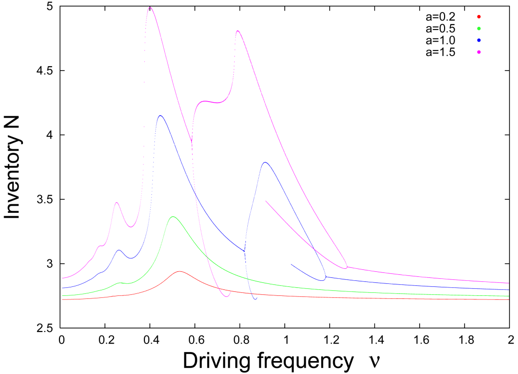
- Parameters τ and σ can be interpreted as values for stability and flexibility

Period doubling



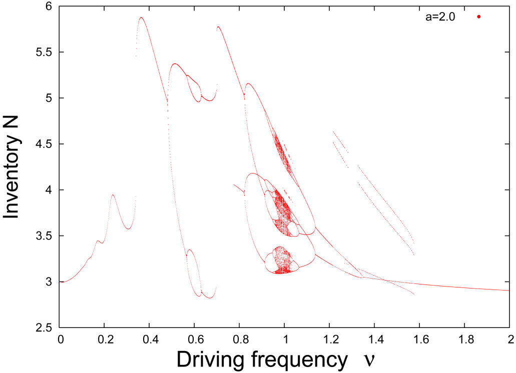
- Simulation of one node with sinusoidal demand and unlimited resources.
- All parameters were kept constant. Only time-constant τ was varied.

Bifurcation diagram



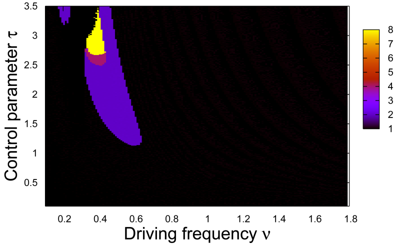
- All parameters were kept constant. Only driving frquency ν was varied for four different values of amplitude *a*.
- Typical non-linear behaviour with multiple resonances and period doublings.

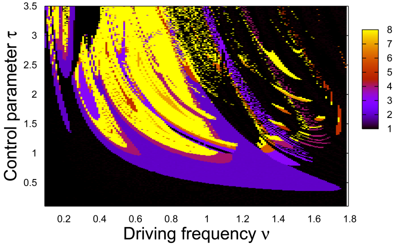
Bifurcation diagram



- All parameters were kept constant. Only driving frquency ν was varied for an amplitude $a = 2.0$.
- Now, additionally choat-like behaviour and diffentent attractor visible.

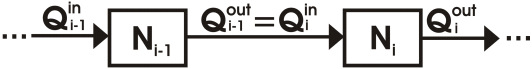
Arnolds tongue





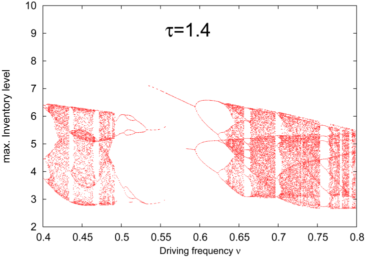
- All parameter were kept constant. Only driving frequency ν and time constant τ were varied for two different values of σ.
- The number of the period relative to the driving frequency is color coded and typical Arnold tongues can be found.

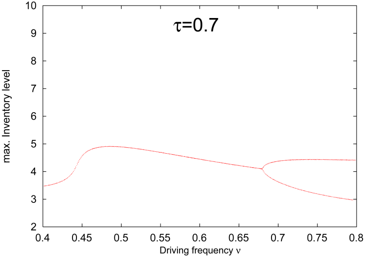
Second scenarion: Supply chain



- Unidirectional coupling: inflow is the outflow of upstream node
- Last node with sinusoidal demand, dynamics do not change
- First node with unlimited resources

Bifurcation diagrams without coupling





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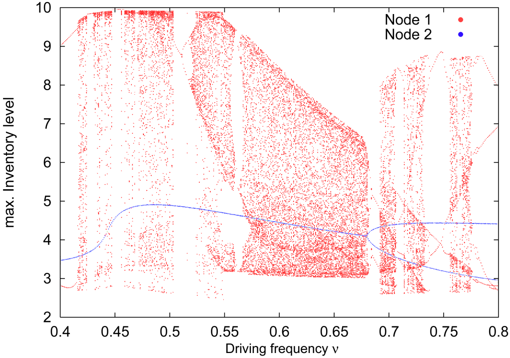
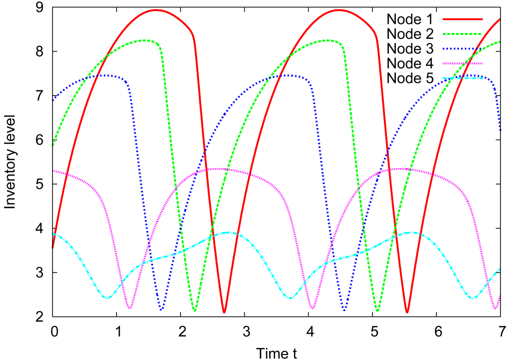
New diagrams after coupling



- Node 1: $\tau = 1.4$
- Node 2: $\tau = 0.7$
- Dynamics of last node do not change, but of upstream nodes

Bullwhip effect



- Amplification of oscillary amplitudes along the supply chain

Summary and outlook

- New approach to model order policies with potential function
- Parameters σ and τ represent flexibility and stability
- Highly nonlinear behaviour
  - Period doublings, up to chaos-like oscillations
  - Multiple resonances
  - Co-existing attractors

- Extension to networks
- Analyis of different topologies, not only chains
- Can also be applied to adaption of production rate
- Bidirectional coupling

Thank you for your attention

Contact

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