An anti-windup control of manufacturing lines: performance analysis

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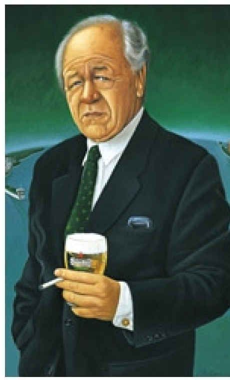
Introduction



Customer Supply chain Manufacturer

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This presentation focuses on manufacturing systems.

Related problems:

- Supply chains
- Parallel calculations
- Multiprocessor embedded systems
- Biological networks

Background

$$\dot{x} = f(x, w)$$

$x(t) \in \mathbb{R}^n$, $w(t) \in \mathbb{R}^m$.

Following ideas of Demidovich (1967)

The system is uniformly convergent for a class of inputs $\mathscr{W}$ if for every $w(\cdot) \in \mathscr{W}$ there is a solution $\bar{x}(t, t_0, x_0)$ such that

- $\bar{x}(t)$ is bounded on $(-\infty, +\infty)$
- $\bar{x}(t)$ is globally uniformly asymptotically stable

Att: $\bar{x}(t)$ is defined on the whole time axis

Properties of uniformly convergent systems

- $\bar{x}(t)$ is unique (due to uniformity)
- if $w(t)$ is periodic, so is $\bar{x}(t)$
- a cascade of uniformly convergent systems is uniformly convergent (due to boundedness assumption); even though each system is quadratically convergent the cascade is not necessarily quadratically convergent

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A continuous-time model of manufacturing machines



Customer   Supply chain   Manufacturer

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Let $v(t)$ be the production speed and $y(t)$ is the cumulative output

$$\dot{y} = v$$

Constraints:

- $v(t) \geq 0$
- $v(t) \leq v_{\max}$
- in a line: $y_{i-1} \geq y_i$, otherwise $v_i = 0$ (the buffer in front of ith machine should be nonempty)

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Control of manufacturing machines
Problem statement

The output of the system should follow demand $y_d(t)$.

- tracking

$$\lim_{t\to\infty} |y(t) - y_d(t)| = 0$$

- approximate tracking

$$\limsup_{t\to\infty} |y(t) - y_d(t)| \leq \Delta$$

  for some “accuracy” $\Delta > 0$

$y_d(t)$ is the current demand

$$y_d(t) = u_d t + y_{d0} + r(t)$$

where $0 \leq u_d \leq v_{\max}$ and $r(t)$ is the fluctuation on the market

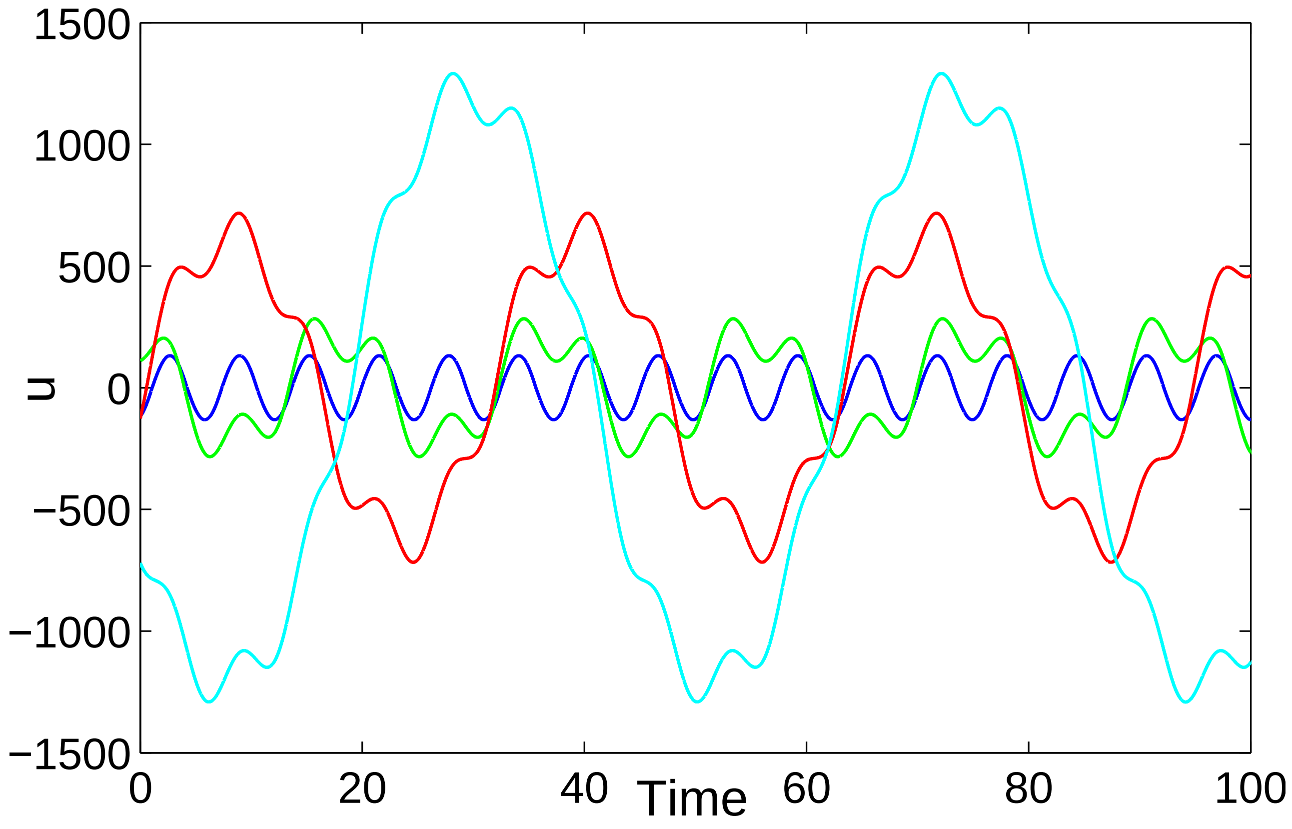
PI control

$$v(t) = -k_p \left(y(t) - y_d(t)\right) - k_i \int_0^t \left(y(s) - y_d(s)\right) ds$$

If a control contains an integral, there is no static error, i.e. if $r(t) \equiv 0$ the tracking goal is achieved asymptotically; if $r(t) \neq 0$ the tracking properties can be analyzed by linear control theory (Bode plots).

Saturation $\Longrightarrow$ integrator windup

Problem: more than one “steady state” solutions can co-exist



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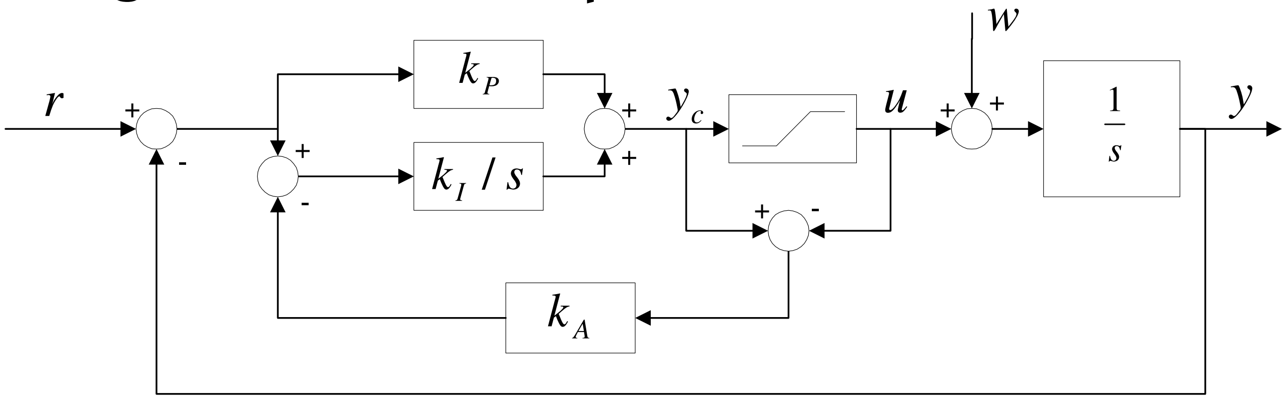
Integrator anti-windup



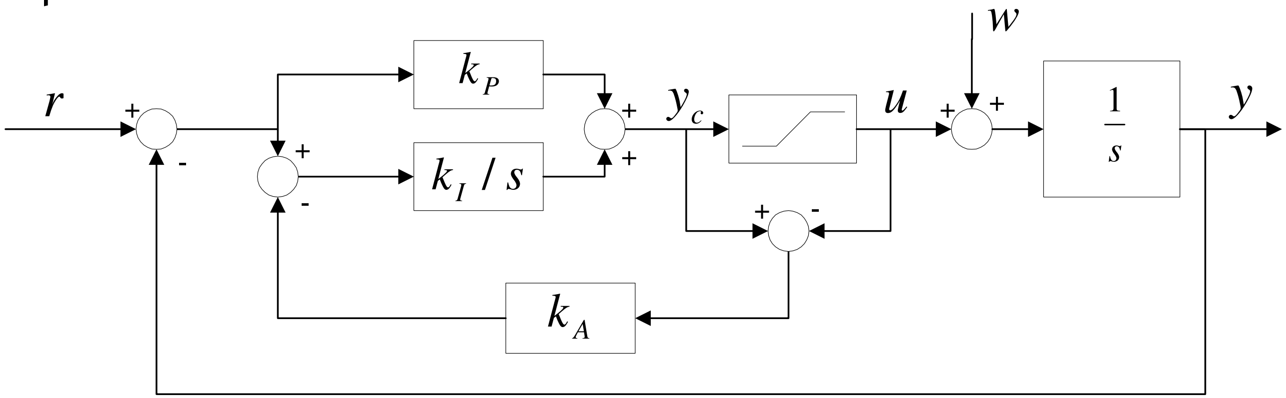
If the saturation is active, an extra signal prevents from the windup.

What is an anti-windup?

Different problem statements co-exist (finite $\mathcal{L}_2$ gain, finite incremental $\mathcal{L}_2$ gain, etc.)

Our focus: the system should be uniformly convergent with (r, w) as an input.

Important observation: the original model with constraints on v can be represented in the form



with $w = v_{max}/2 - u_d$ and the saturation nonlinearity

$$u = \frac{v_{\max}}{2}\text{sat}(y_c), \quad \text{sat}(y_c) := \text{sign}(y_c)\min\{1, |y_c|\}$$

The control problem: $y(t)$ approximately follows $r(t)$ (market fluctuation).

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If $k_I, k_p, k_a > 0$, $r(t)$ is uniformly continuous, $|w(t)| < 1$ and

$$v_{\max} k_a k_p > 2$$

the system is uniformly convergent.

The proof is based on the Lypunov function $V = (x_1 - x_2)^\top P (x_1 - x_2)$ with positive definite $P = P^\top > 0$, and $x_1(t), x_2(t)$ being two different solutions (in an appropriate coordinate system)

The derivative of V satisfies

$$\dot{V} \leq -\alpha(t) V, \quad \int_{t_0}^{t_0+T} \alpha(s) ds > 0$$

α is integrally separated from zero uniformly in t_0 and uniformly with respect to the initial conditions from any given compact set.

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Performance analysis of Lur'e systems

If a system is uniformly convergent it has a unique bounded (on the whole time axis) GUAS solution $\bar{x}(t)$.

It allows to pose a problem of performance analysis

- transient performance: how fast any $x(t)$ converges to $\bar{x}(t)$
- steady state performance: properties of $\bar{x}(t)$

We focus on the steady state performance with harmonic $r(t) = b\sin(\omega t)$

- computer simulation (accurate, numerically inefficient)
- describing functions, Galerkin approximation (numerically efficient, approximate)

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Lur’e system:

$$\begin{cases} \dot{x} &= Ax - B\phi(y) + Fu \\ y &= Cx + Du \end{cases}$$

Incremental sector condition

$$0 \le \frac{\phi(y_1) - \phi(y_2)}{y_1 - y_2} \le \mu$$

An approximate system

$$\begin{cases} \dot{\xi} &= A\xi - BK\zeta + Fu \\ \zeta &= C\xi + Du \end{cases}$$

The gain K is to be chosen to minimize

$$J := \frac{1}{T}\int_0^T [\phi(\bar{\zeta}(t)) - K\bar{\zeta}(t)]^2 dt,$$

that is

$$K^* = \left(\int_0^T \bar{\zeta}^2(t)dt\right)^{-1} \int_0^T \phi(\bar{\zeta}(t))\bar{\zeta}(t)dt.$$

If $u = b\sin\omega t$, $\bar{\zeta}(t) = a\sin(\omega t + \psi)$

If ϕ is odd,

$$K(a) = \frac{2}{\pi a}\int_0^\pi \phi(a\sin\theta)\sin\theta d\theta.$$

Approximation:

$$\begin{cases} \dot{\xi} = A\xi - BK(a)\zeta + Fb\sin\omega t \\ \zeta = C\xi + Du \end{cases}$$

Harmonic balance equation (HBE):

$$|1 + K(a)G(i\omega)|^2 a^2 = |C(i\omega I_n - A)^{-1}F + D|^2 b^2,$$

where $G(i\omega) = C(i\omega I_n - A)^{-1}B$.

Question: given b, ω, is the amplitude a determined in a unique way?

Answer: check the FDI for the frequency of excitation:

$$\mu \mathrm{Re}\, G(i\omega) > -1.$$

Idea of the proof: the left hand side of HBE should be a monotonically increasing function of a.

FDI should be satisfied for **one** frequency.

Accuracy of harmonic linearization

Problem: estimate the difference between

$$\overline{z}(t) = H\overline{x}(t) \quad \text{and} \quad \overline{\eta}(t) = H\overline{\xi}(t)$$

$$\rho_1 := \sup_{k=3,5,\ldots} |C(ik\omega I_n - A + \frac{\mu}{2}BC)^{-1}B|$$

$$\rho_2 := \sup_{k=3,5,\ldots} |H(ik\omega I_n - A + \frac{\mu}{2}BC)^{-1}B|$$

- (A, B) is controllable, (A, C) is observable.
- HBE has a unique positive real solution $a(b, \omega)$
- $\rho_1 \mu < 2$
- ϕ is an odd function

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Then

- $\bar{x}(t)$ is the only $2\pi/\omega$ periodic solution
- the following estimate is true

$$\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\left[\bar{z}(t)-\bar{\eta}(t)\right]^2 dt \le \gamma^2 v^2(a(b,\omega)),$$

where

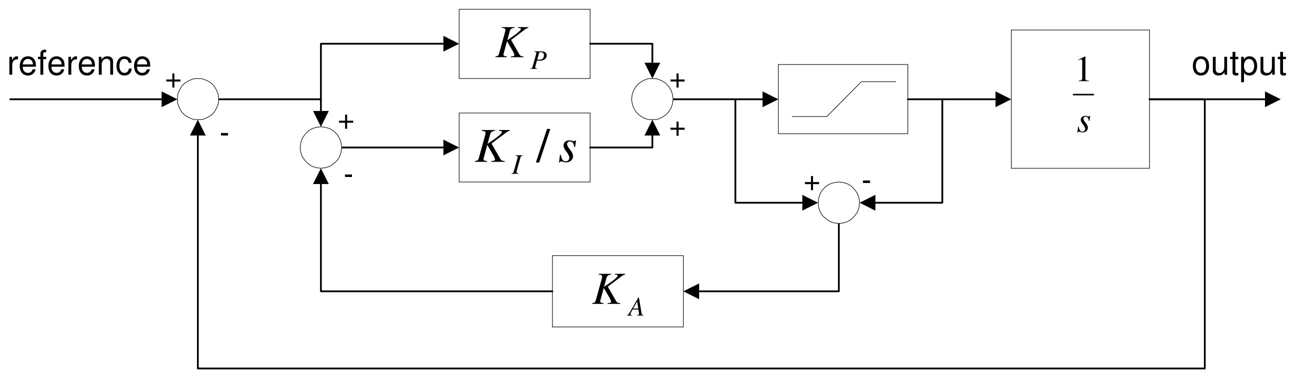
$$v^2(a)=\frac{1}{2\pi}\int_0^{2\pi}\left[\frac{2}{\pi}\int_0^{\pi}\phi(a\sin\theta)\sin\theta d\theta\cdot\sin\vartheta-\phi(a\sin\vartheta)\right]^2 d\vartheta$$

and

$$\gamma=\frac{2\rho_2}{2-\mu\rho_1}.$$

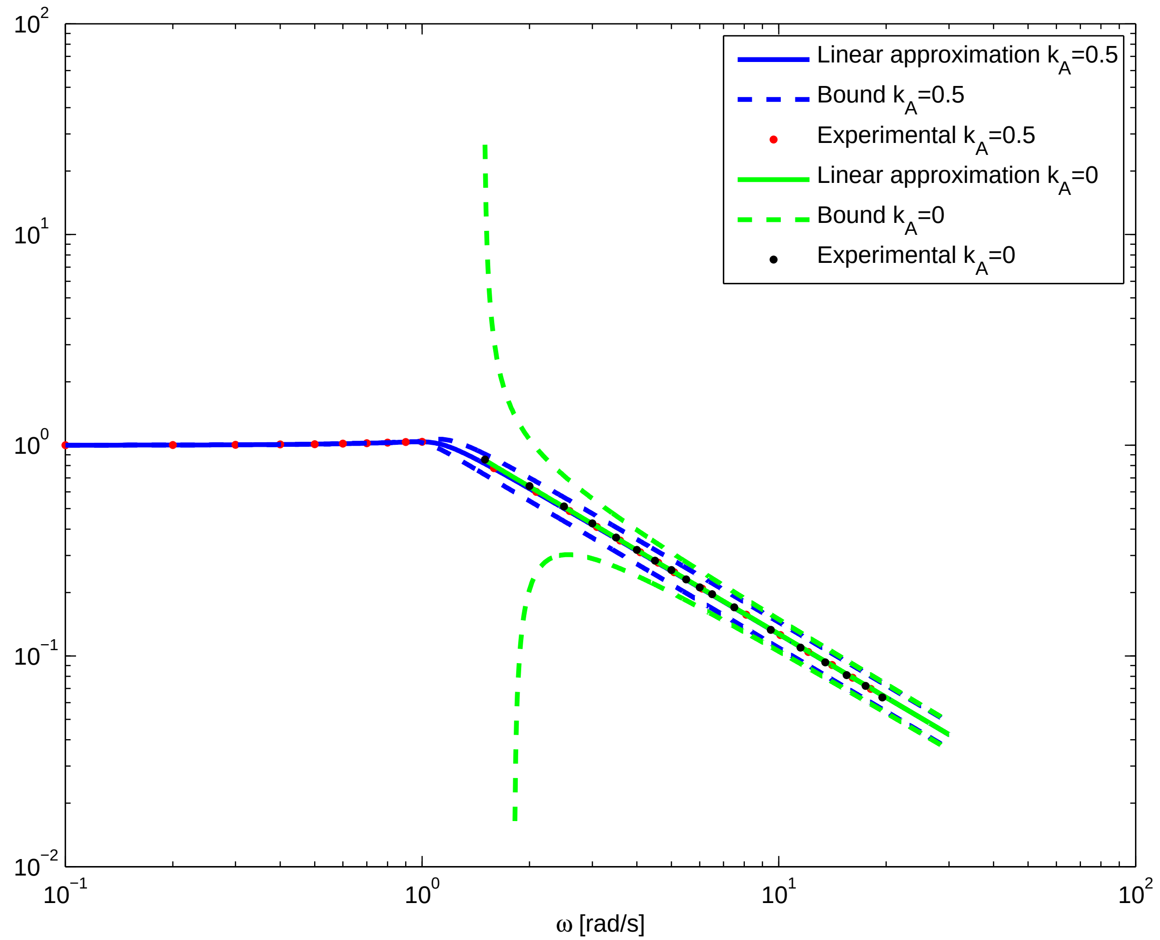
Idea of the proof: contraction mapping argument

Illustrative example



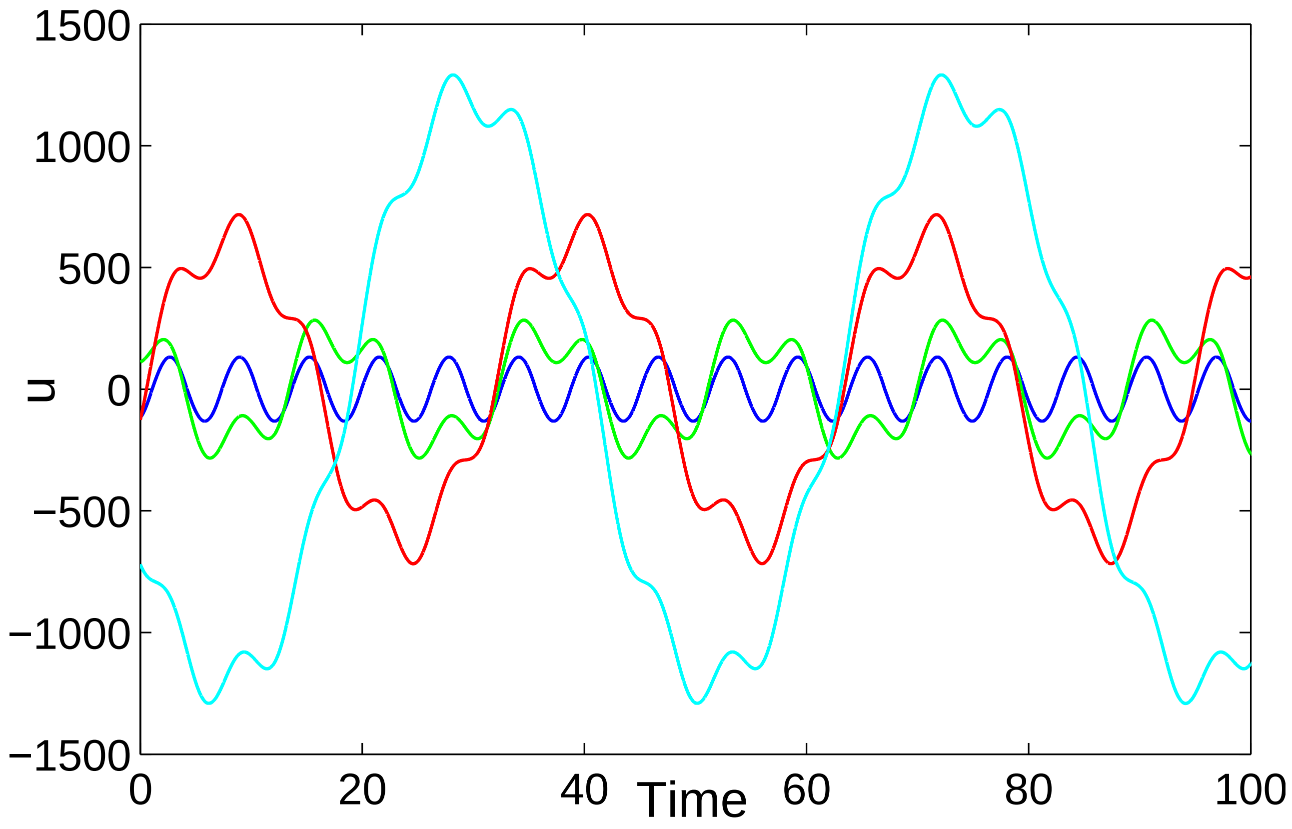
$K_i = 20$, $K_p = 10$, $K_a = 0$, $\omega = 1$

$$K(a) = \begin{cases} 1, & a \leq 1 \\ \frac{2}{\pi}\left(\sin^{-1}\left(\frac{1}{a}\right) + \frac{1}{a}\sqrt{1 - \frac{1}{a^2}}\right), & a > 1 \end{cases}$$



Accuracy of harmonic linearization. Blue – AW, green – no AW.

If the conditions of theorem are not satisfied the describing function method can be misleading due to possible subharmonic solutioins



Control of manufacturing lines

The last machine (N) in the line should follow $y_d(t)$.

The jth machine should follow

$$y_d(t) + \gamma_j(y_d(t) - y_{j+1}(t))$$

With such a coupling neglecting positivity constraints imposed on the buffers one gets a cascade system.

Recall that a cascade of uniformly convergent systems is uniformly convergent, hence the analysis of the manufacturing line can be performed *mutatis mutandis* .

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Implementation issue

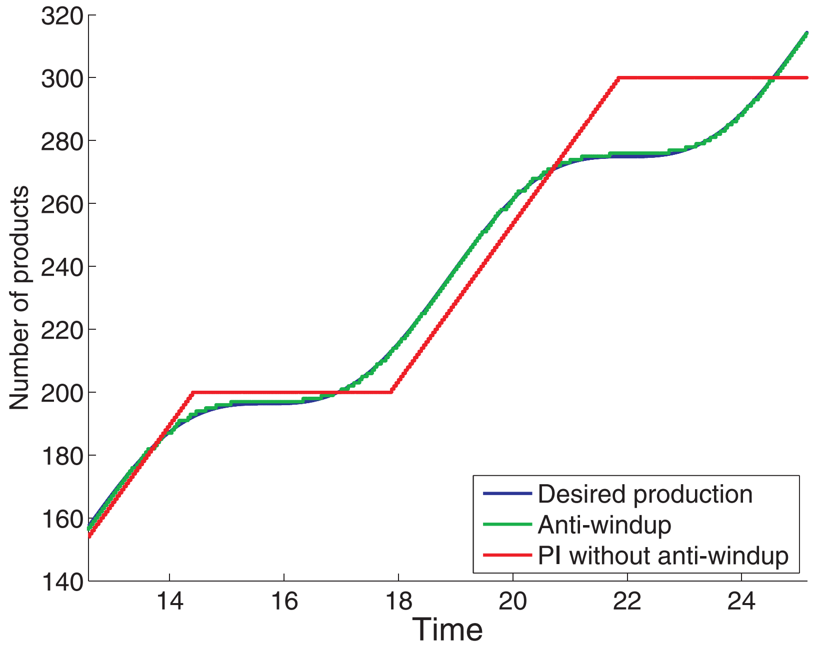
The controller produces a continuous command $v_j(t)$ (the speed of production, jth machine).

This signal should be converted into on-off form $v_{PWMj}(t)$ (similarly to pulse-width modulation) so that

$$\int_0^T v_j(t)dt \approx \int_0^T v_{PWMj}(t)dt$$

A minimal time for an "on"-phase is t_{0j}- the time required for the jth machine to process a lot.

Results of computer simulation (4 machines in a line)



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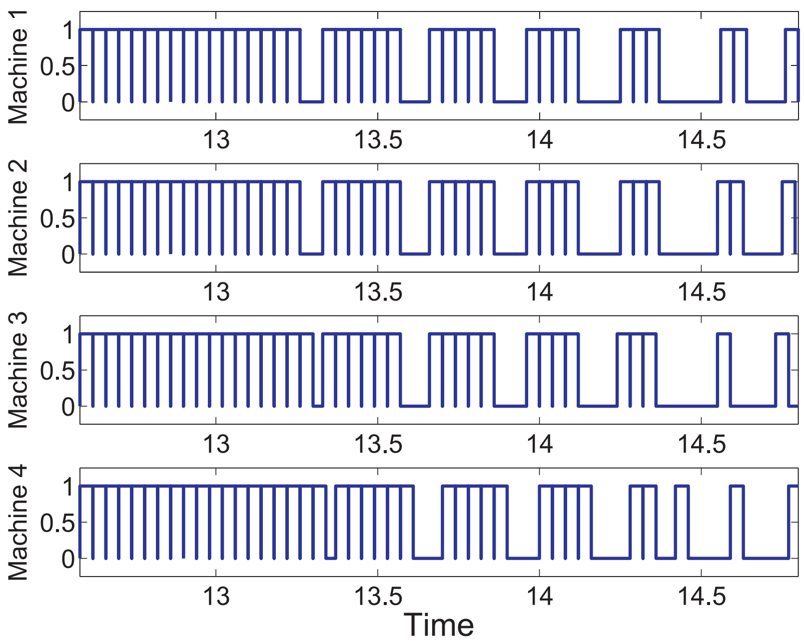

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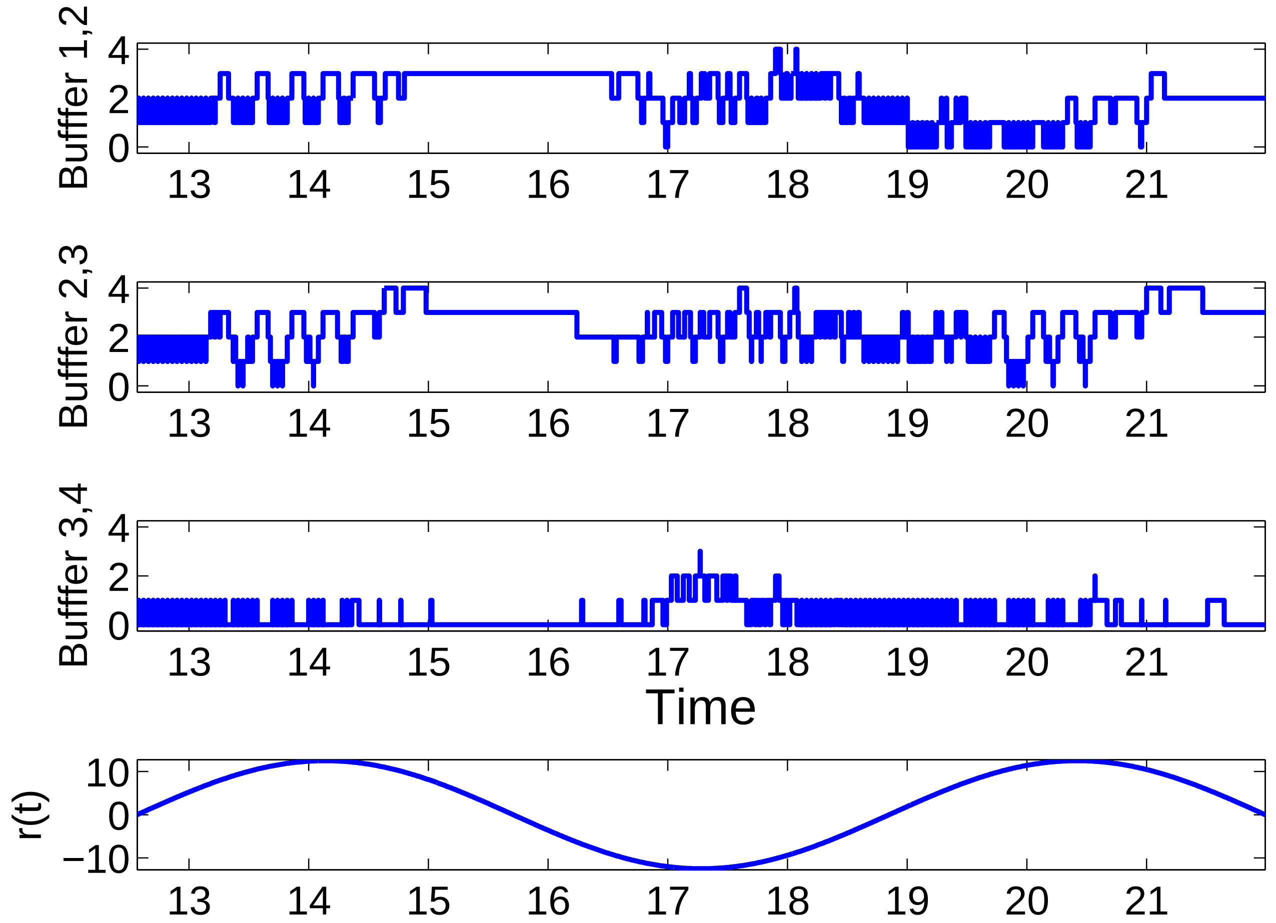
Results of computer simulation (4 machines in a line)



Future research. From manufacturing lines towards manufacturing networks

Assembling, parallel machines:

- To study separately topology of the network and individual machine dynanics
- Passivity-based approach, similarly to
  1. A. Pogromsky, G. Santoboni and H. Nijmeijer, Partial synchronization: from symmetry towards stability, *Physica D*, 2002
  2. A. Pogromsky, A partial synchronization theorem, submitted

Constraints on communication between the machines:

- Discretization, batching, finite capacity of the information channels, drops
- Shannon-like theorems for control of networks (See A. Matveev, A. Savkin, *Estimation and Control over Communication Networks*, Birkhäuser Boston, 2008)

Reentrant systems:

- To extend results of J.A.W.M. van Eekelen, A.A.J. Lefeber, J.E. Rooda

Conclusions

- A simple continuous-time model of a manufacturing machine
- An anti-windup control of systems with saturation
- Performance analysis of systems with saturation in frequency domain
- Extension to manufacturing lines
- Computer simulation for a more detailed model