Local and Global Dynamics of Production and Supply Networks under Mixed Production Strategies

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Motivation

- General problem: Planning of production processes in the presence of supply and demand
  - Production breakdowns due to lack of material
  - Production breakdowns due to high inventories

Example: US factories of DaimlerChrysler

Summer 2006: breakdown of sellings, continuation of production

Result: 100,000 unsold new cars on stock

Consequence: breakdown of production for up to four weeks in December / January 2007

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  - simple enough to be studied analytically (system of ODEs)

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  here: generalisation of Helbing's input-output model of commodity flows
  (D. Helbing *et al.*, Phys. Rev. E, 2004)

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- Understand dynamic factors that determine stability of production networks on both local and global level
- Derive strategies to minimize undesired phenomena like Bullwhip effects

Contents

Contents

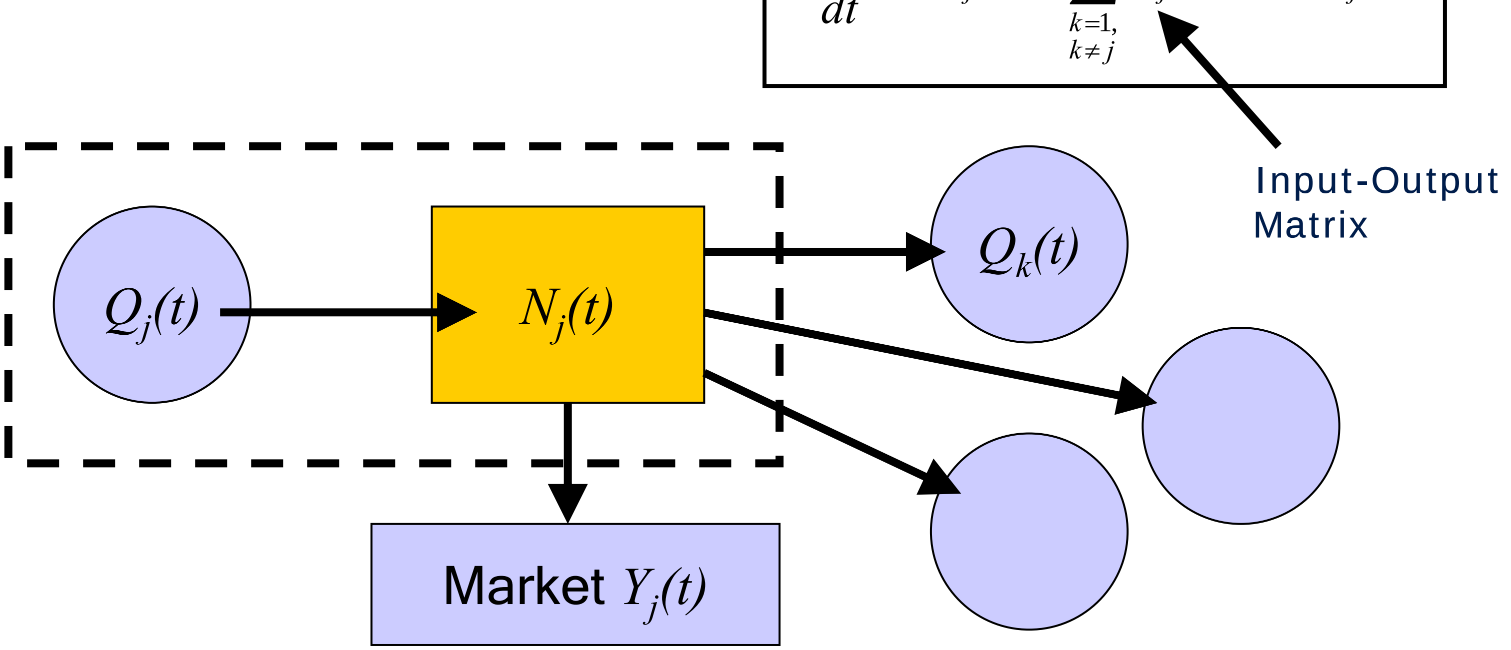
2. Description of the Model

Basic model:

D. Helbing, S. Lämmer, U. Witt, and T. Brenner: Network-induced oscillatory behavior in material flow networks and irregular business cycles. Phys Rev E 70: 056118 (2004)

analyzed for pull systems only!

$$\frac{dN_j}{dt} = Q_j(t) - \sum_{k=1,\, k\neq j}^{m} C_{jk} Q_k(t) - Y_j(t)$$



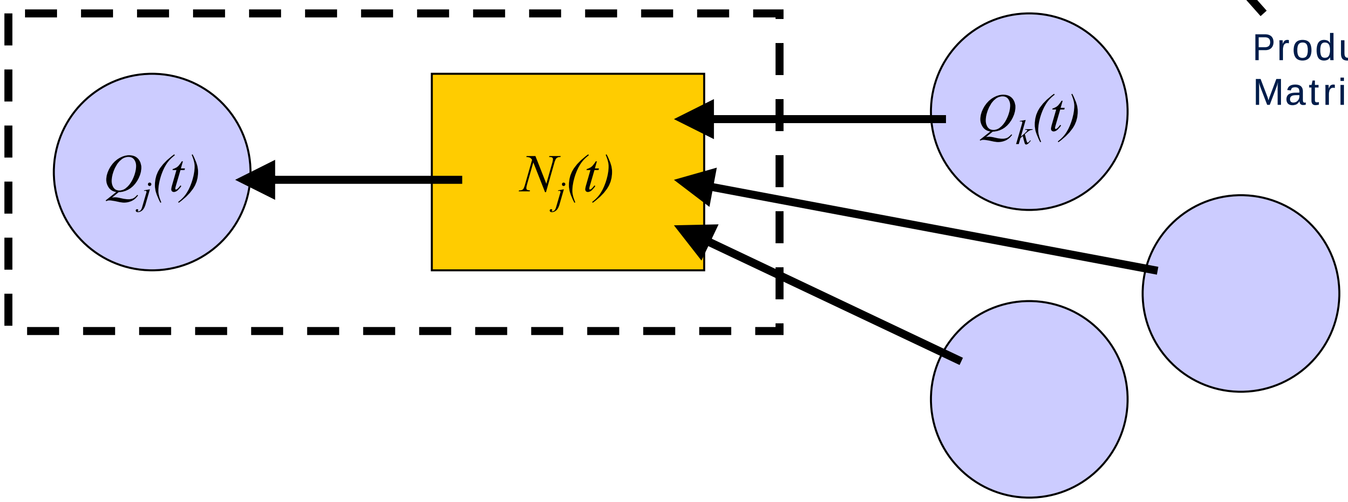
2. Description of the Model

Modifications for a push system:

consider input instead of output buffers
external market does not play a role any more
change production strategy

$$\frac{dN_j}{dt} = \sum_{\substack{k=1,\\ k\neq j}}^{m} T_{jk} Q_k(t) - Q_j(t)$$

Production Matrix



Production strategy:

optimize stocks on a given optimal buffer level N^0 and avoid fluctuations

	Pull strategy	Push strategy
Low stocks	Increase production	Decrease production
High stocks	Decrease production	Increase production

Pull:

$$\frac{1}{Q_j(t)}\frac{dQ_j}{dt} = \hat{\nu}_j\left(\frac{N_j^0}{N_j(t)} - 1\right) - \hat{\mu}_j \frac{1}{N_j(t)}\frac{dN_j}{dt}$$

Push:

$$\frac{1}{Q_j(t)}\frac{dQ_j}{dt} = -\hat{\nu}_j\left(\frac{N_j^0}{N_j(t)} - 1\right) + \hat{\mu}_j \frac{1}{N_j(t)}\frac{dN_j}{dt}$$

Additional optimization goal:

desired production rate Q^0 (e.g., according to optimal machine capacity)

$\Rightarrow$ relevant for CONWIP and related strategies

CONWIP:

$$\frac{1}{Q_j(t)}\frac{dQ_j}{dt} = \alpha_j \left(\frac{Q_j^0}{Q_j(t)} - 1 \right)$$

Pull:

$$\frac{1}{Q_j(t)}\frac{dQ_j}{dt} = \hat{\nu}_j \left(\frac{N_j^0}{N_j(t)} - 1 \right) - \hat{\mu}_j \frac{1}{N_j(t)}\frac{dN_j}{dt}$$

Push:

$$\frac{1}{Q_j(t)}\frac{dQ_j}{dt} = -\hat{\nu}_j \left(\frac{N_j^0}{N_j(t)} - 1 \right) + \hat{\mu}_j \frac{1}{N_j(t)}\frac{dN_j}{dt}$$

Price adjustments:

	Pull strategy	Push strategy
Low stocks	Increase price	Increase price
High stocks	Decrease price	Decrease price

In any case:

$$\frac{1}{P_j(t)}\frac{dP_j}{dt} = \nu_j\left(\frac{N_j^0}{N_j(t)} - 1\right) - \mu_j \frac{1}{N_j(t)}\frac{dN_j}{dt}$$

$\Rightarrow$ Dynamic price-production feedback that acts differently in case of push and pull strategy

Linearization of this set of equations:

$$N_j(t) = n_j(t) + N_j^0$$

$$P_j(t) = p_j(t) + P_j^0$$

$$Q_j(t) = q_j(t) + Q_j^0$$

$$Y_j(t) = \xi_j(t) + Y_j^0$$

First-order solution (pull):

$$\frac{dn_j}{dt} = q_j(t) - \sum_{\substack{k=1,\\ k\neq j}}^{m} C_{jk} q_k(t) - Y_j^0 \left| L_j'(P_j^0) \right| p_j(t) - \xi_j(t)$$

$$\frac{dp_j}{dt} = \frac{P_j^0}{N_j^0} \left(-\nu_j n_j(t) - \mu_j \frac{dn_j}{dt} \right)$$

$$\frac{dq_j}{dt} = \frac{Q_j^0}{N_j^0} \left(-\hat{\nu}_j n_j(t) - \hat{\mu}_j \frac{dn_j}{dt} \right)$$

3. Local Stability Analysis

Linearization of this set of equations:

$$N_j(t) = n_j(t) + N_j^0$$

$$P_j(t) = p_j(t) + P_j^0$$

$$Q_j(t) = q_j(t) + Q_j^0$$

$$Y_j(t) = \xi_j(t) + Y_j^0$$

First-order solution (push):

$$\frac{dn_j}{dt} = -q_j(t) + \sum_{\substack{k=1,\\ k\neq j}}^{m} T_{jk} q_k(t)$$

Sign changes

$$\frac{dp_j}{dt} = \frac{P_j^0}{N_j^0}\left(-\nu_j n_j(t) - \mu_j \frac{dn_j}{dt}\right)$$

$$\frac{dq_j}{dt} = \frac{Q_j^0}{N_j^0}\left(+\hat{\nu}_j n_j(t) + \hat{\mu}_j \frac{dn_j}{dt}\right)$$

3. Local Stability Analysis

Solution for linear supply chain:

Pull: $C_{jk}=\delta_{j+1,k}$

Push: $T_{jk}=\delta_{j-1,k}$

$$\frac{dn_j}{dt} = \pm q_j(t) \mp q_{j\pm 1}(t)$$

$$\frac{dq_j}{dt} = \frac{Q_j^0}{N_j^0}\left(\mp \hat{\nu}_j n_j(t) \mp \hat{\mu}_j \frac{dn_j}{dt} \right)$$

$$\Rightarrow \quad \frac{d^2 q_j}{dt^2} + 2\gamma_j \frac{dq_j}{dt} + \omega_j^2 q_j(t) = 2\gamma_j \frac{dq_{j\pm 1}}{dt} + \omega_j^2 q_{j\pm 1}(t) = f_j(t)$$

with $$\gamma_j = \frac{\hat{\mu}_j Q_j^0}{2N_j^0} > 0$$

$$\omega_j^2 = \frac{\hat{\nu}_j Q_j^0}{N_j^0} > 0$$

Damped and driven harmonic oscillator!

3. Local Stability Analysis

Solution for linear supply chain:

$$\frac{d^2 q_j}{dt^2} + 2\gamma_j \frac{dq_j}{dt} + \omega_j^2 q_j(t) = 2\gamma_j \frac{dq_{j\pm 1}}{dt} + \omega_j^2 q_{j\pm 1}(t) = f_j(t)$$

Same stability properties for pull and push systems, in particular:

Bullwhip effect (convective instability due to resonance)
if final demand (original supply) oscillates with frequency β as

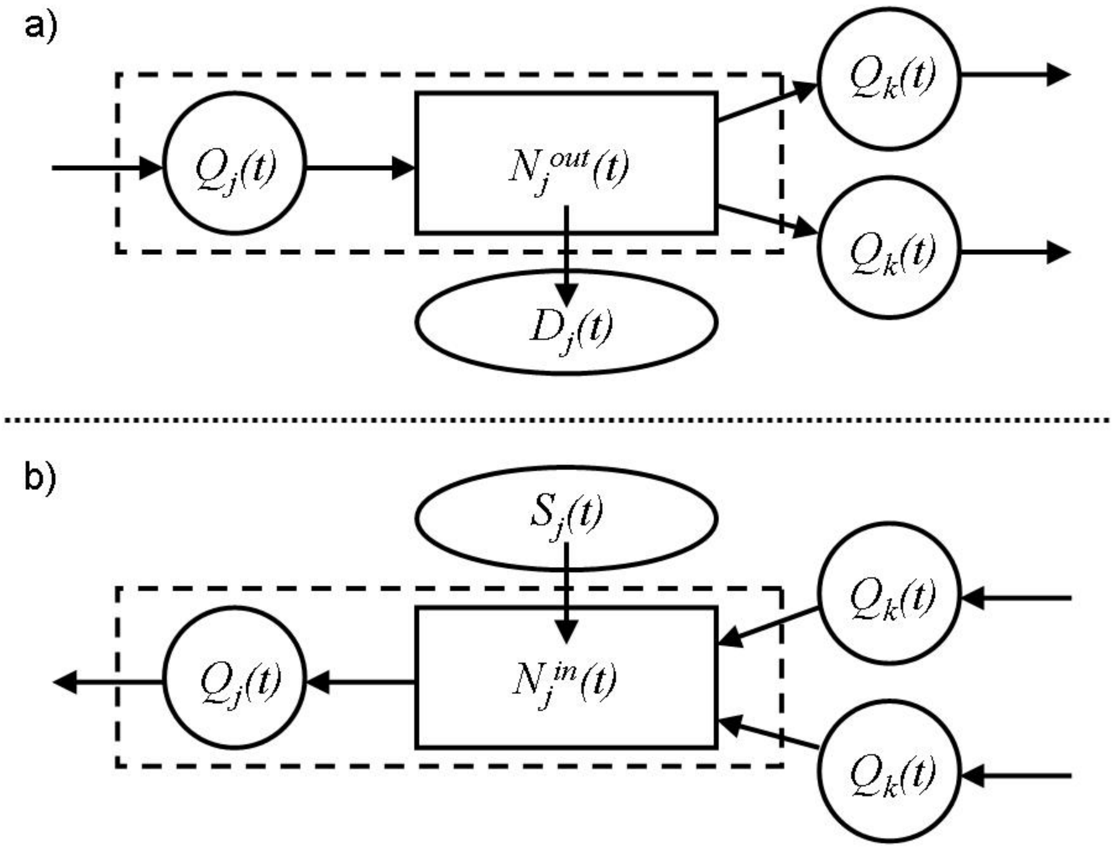
$$0 < \beta^2 < 2\omega_j^2$$

Difference: direction of amplifying propagation of perturbations –
Starting from the final customer (pull) or the initial supplier (push)

3. Local Stability Analysis

Idea: Mixed production strategies (pull/push/CONWIP)

⇒ Power for local optimization of production processes?



3. Local Stability Analysis

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⇒ Power for local optimization of production processes?

$$\frac{1}{Q_j(t)}\frac{dQ_j}{dt} = \hat{\xi}_j \left\{ \hat{\nu}_j^{out} \left(\frac{\hat{N}_j^{out}}{N_j^{out}(t)} - 1 \right) - \hat{\mu}_j^{out} \frac{1}{N_j^{out}(t)} \frac{dN_j^{out}}{dt} \right\}$$

$$+ (1-\hat{\xi}_j) \left\{ -\hat{\nu}_j^{in} \left(\frac{\hat{N}_j^{in}}{N_j^{in}(t)} - 1 \right) + \hat{\mu}_j^{in} \frac{1}{N_j^{in}(t)} \frac{dN_j^{in}}{dt} \right\} + \hat{\alpha}_j \left(\frac{\hat{Q}_j}{Q_j(t)} - 1 \right)$$

3. Local Stability Analysis

Idea: Mixed production strategies (pull/push/CONWIP)

$\Rightarrow$ Power for local optimization of production processes?

$$\frac{1}{Q_j(t)}\frac{dQ_j}{dt} = \boxed{\hat{\xi}_j \left\{ \hat{\nu}_j^{out} \left(\frac{\hat{N}_j^{out}}{N_j^{out}(t)} - 1 \right) - \hat{\mu}_j^{out} \frac{1}{N_j^{out}(t)} \frac{dN_j^{out}}{dt} \right\}} \quad \textbf{Pull}$$

$$+ (1-\hat{\xi}_j) \left\{ -\hat{\nu}_j^{in} \left(\frac{\hat{N}_j^{in}}{N_j^{in}(t)} - 1 \right) + \hat{\mu}_j^{in} \frac{1}{N_j^{in}(t)} \frac{dN_j^{in}}{dt} \right\} + \hat{\alpha}_j \left(\frac{\hat{Q}_j}{Q_j(t)} - 1 \right)$$

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$$\boxed{+ (1-\hat{\xi}_j) \left\{ -\hat{\nu}_j^{in} \left(\frac{\hat{N}_j^{in}}{N_j^{in}(t)} - 1 \right) + \hat{\mu}_j^{in} \frac{1}{N_j^{in}(t)} \frac{dN_j^{in}}{dt} \right\}} + \hat{\alpha}_j \left(\frac{\hat{Q}_j}{Q_j(t)} - 1 \right)$$

Push

3. Local Stability Analysis

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⇒ Power for local optimization of production processes?

$$\frac{1}{Q_j(t)}\frac{dQ_j}{dt} = \hat{\xi}_j \left\{ \hat{\nu}_j^{out} \left(\frac{\hat{N}_j^{out}}{N_j^{out}(t)} - 1 \right) - \hat{\mu}_j^{out} \frac{1}{N_j^{out}(t)} \frac{dN_j^{out}}{dt} \right\}$$
$$+ (1-\hat{\xi}_j) \left\{ -\hat{\nu}_j^{in} \left(\frac{\hat{N}_j^{in}}{N_j^{in}(t)} - 1 \right) + \hat{\mu}_j^{in} \frac{1}{N_j^{in}(t)} \frac{dN_j^{in}}{dt} \right\} \boxed{+ \hat{\alpha}_j \left(\frac{\hat{Q}_j}{Q_j(t)} - 1 \right)}$$

CONWIP

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$$\frac{1}{Q_j(t)}\frac{dQ_j}{dt} = \hat{\xi}_j \left\{ \hat{\nu}_j^{out} \left(\frac{\hat{N}_j^{out}}{N_j^{out}(t)} - 1 \right) - \hat{\mu}_j^{out} \frac{1}{N_j^{out}(t)} \frac{dN_j^{out}}{dt} \right\}$$
$$+ (1 - \hat{\xi}_j) \left\{ -\hat{\nu}_j^{in} \left(\frac{\hat{N}_j^{in}}{N_j^{in}(t)} - 1 \right) + \hat{\mu}_j^{in} \frac{1}{N_j^{in}(t)} \frac{dN_j^{in}}{dt} \right\} + \hat{\alpha}_j \left(\frac{\hat{Q}_j}{Q_j(t)} - 1 \right)$$

or short:

$$\Delta^{(\hat{\alpha},\hat{Q})} Q(t) = \hat{\xi}^{in} \Delta^{(\hat{\alpha}^{in},\hat{N}^{in})} N^{in}(t) - \hat{\xi}^{out} \Delta^{(\hat{\alpha}^{out},\hat{N}^{out})} N^{out}(t)$$

$$\hat{\alpha}^{in,out} = \hat{\nu}^{in,out} / \hat{\mu}^{in,out}$$
$$\hat{\xi}^{out} = \hat{\xi}\hat{\mu}^{out}, \quad \hat{\xi}^{in} = (1 - \hat{\xi})\hat{\mu}^{in}$$
$$\Delta^{(a,b)} X := \frac{1}{X}\frac{dX}{dt} - a\left(\frac{b}{X} - 1\right) \quad \text{with} \quad b \equiv \hat{X}$$

For a single production unit:

$$\frac{d^2q}{dt^2} + 2\gamma\frac{dq}{dt} + \omega^2 q(t) = f(t)$$

$$\gamma = \frac{1}{2}\left[\hat{\alpha} + \hat{Q}\left(\frac{\hat{\xi}^{in}}{\hat{N}^{in}} + \frac{\hat{\xi}^{out}}{\hat{N}^{out}}\right)\right]$$

$$\omega^2 = \hat{Q}\left(\frac{\hat{\alpha}^{in}\hat{\xi}^{in}}{\hat{N}^{in}} + \frac{\hat{\alpha}^{out}\hat{\xi}^{out}}{\hat{N}^{out}}\right)$$

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$$\omega^2 = \hat{Q}\left(\frac{\hat{\alpha}^{in}\hat{\xi}^{in}}{\hat{N}^{in}} + \frac{\hat{\alpha}^{out}\hat{\xi}^{out}}{\hat{N}^{out}}\right)$$

Periodic supply or demand at unit j with frequency β:

Amplification factor:

$$\left\{1 + \frac{\beta^4 - 2\beta^2\omega^2}{\omega^4 + 4\beta^2\gamma^2}\right\}^{-1/2}$$

Maximum amplification (resonance):

$$\beta_{\max}^2 = \frac{\omega^4}{4\gamma^2}\left(\sqrt{1 + \frac{8\gamma^2}{\omega^2}} - 1\right)$$

For a single production unit:

if supply and demand vary periodically with common frequency β:

$$s(t) = q^{0|in} \cos(\beta t + \theta^{0|in})$$
$$d(t) = q^{0|out} \cos(\beta t + \theta^{0|out})$$

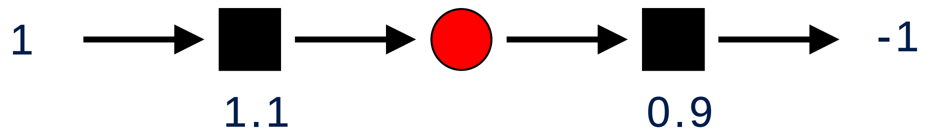
minimization of Bullwhip effect by mixed strategies that fulfill

$$\arctan(\hat{\alpha}^{in}\beta) - \arctan(\hat{\alpha}^{out}\beta) + \theta^{0|in} - \theta^{0|out} = \pi$$

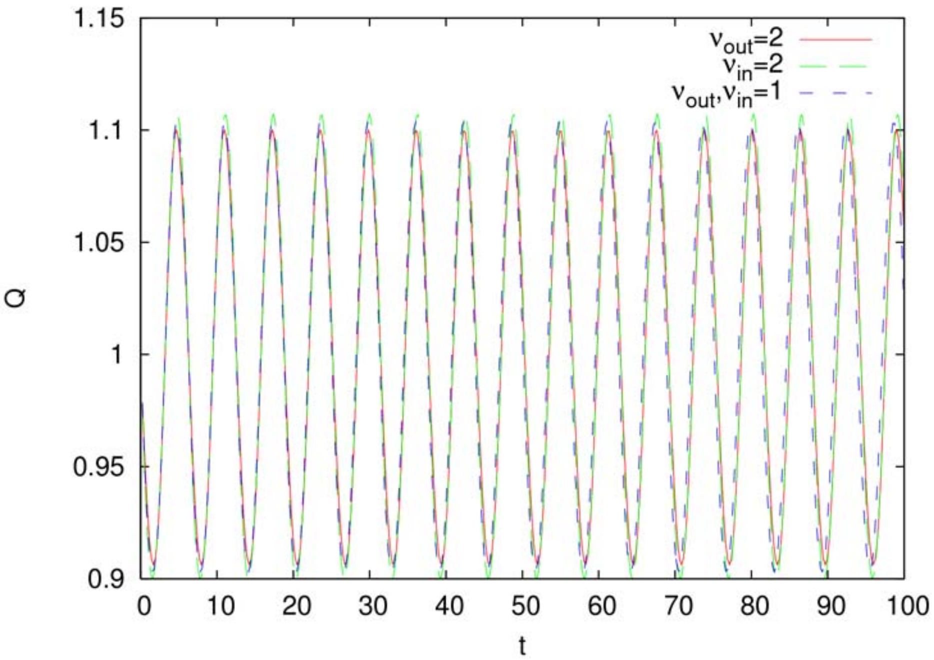
$$\frac{q^{0|in}}{\hat{N}^{in}} \hat{\xi}^{in} \sqrt{1 + \left(\hat{\alpha}^{in}\right)^2 \beta^2} = \frac{q^{0|out}}{\hat{N}^{out}} \hat{\xi}^{out} \sqrt{1 + \left(\hat{\alpha}^{out}\right)^2 \beta^2}$$

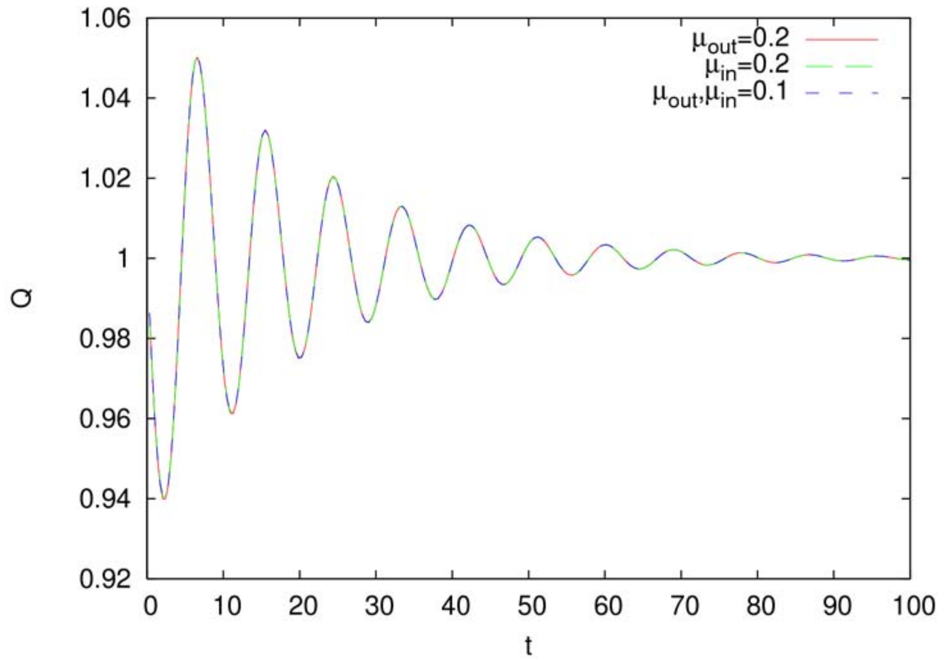
$\Rightarrow$ Possible approach for local control?

Simulation Results: In accordance with linear theory,

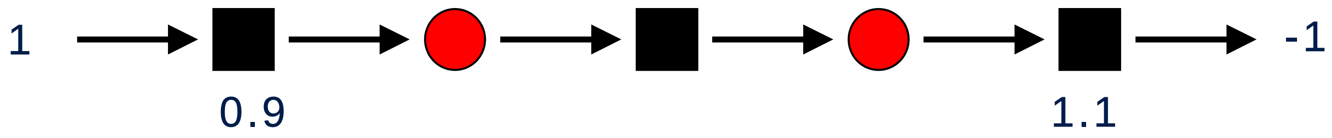


frequency depends exclusively on $\nu_{in,out}$
damping depends exclusively on $\mu_{in,out}$
results don't change as long as $\mu_{in}+\mu_{out}$ and $\nu_{in}+\nu_{out}$ are fixed



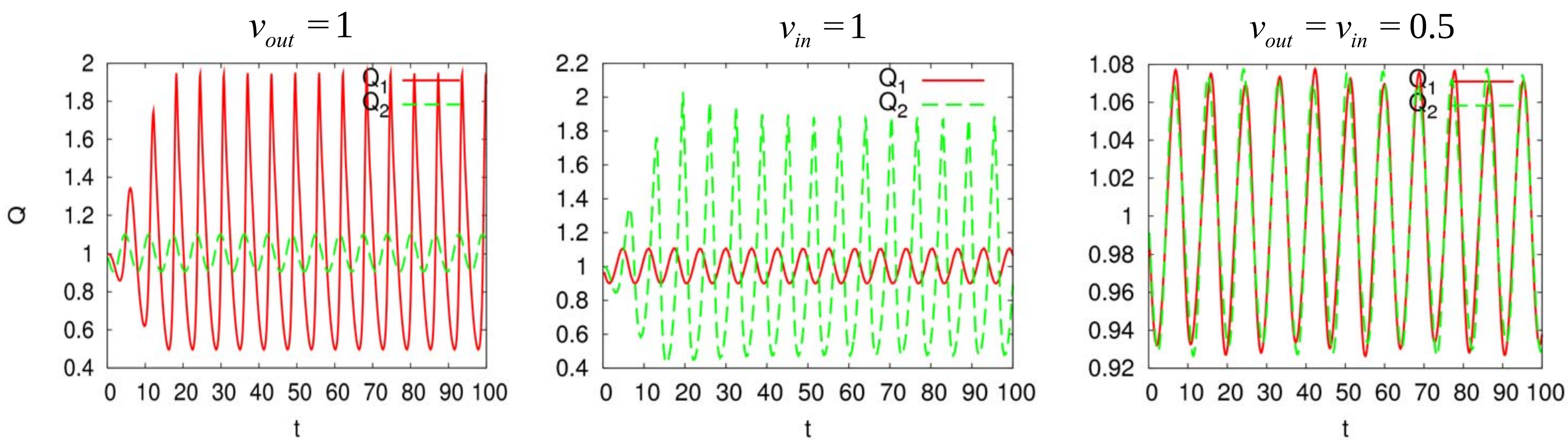


Simulation Results: Linear chain with two manufacturers

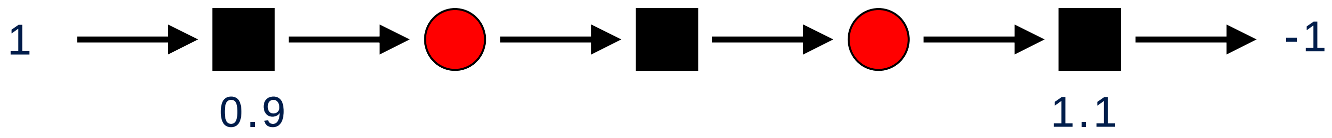


- **Pull:** Enhanced fluctuations at first producer
- **Push:** Enhanced fluctuations at second producer
- **Mixed:** Smaller fluctuations with nearly no phase shift

⇒ Improvement of “network” performance

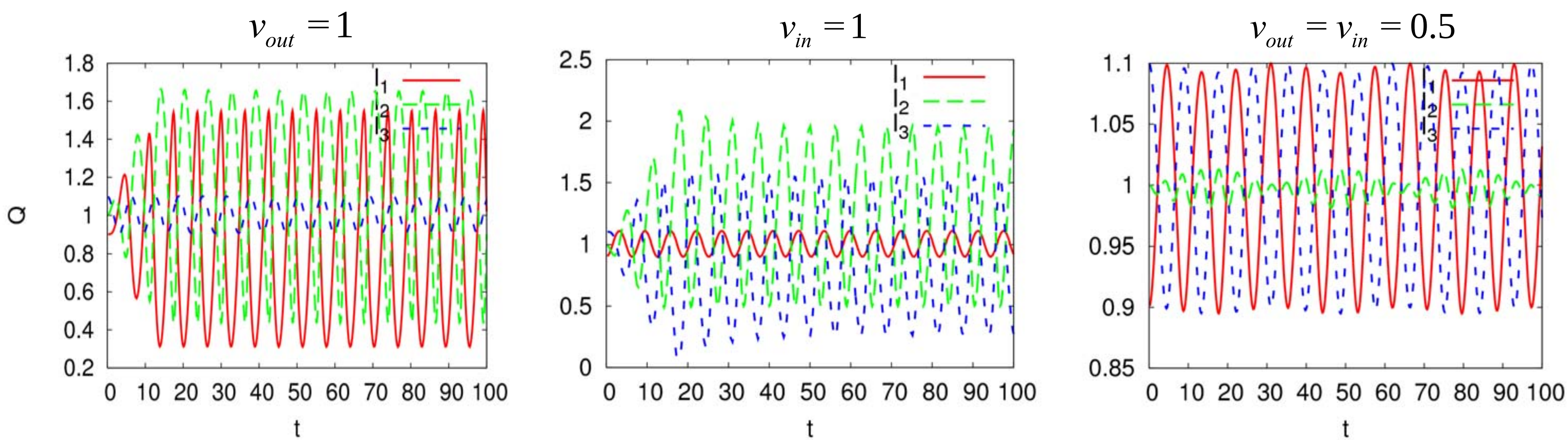


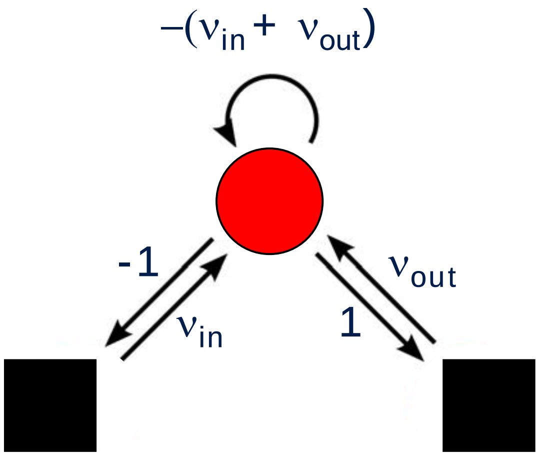
Simulation Results: Linear chain with two manufacturers - Inventories



- **Pull / Pull:** same frequencies, common buffer fluctuates strongest
- **Mixed:** double frequency, but only small fluctuations at common buffer

$\Rightarrow$ Reduction of required storage capacity





Single producer:

Jacobian:

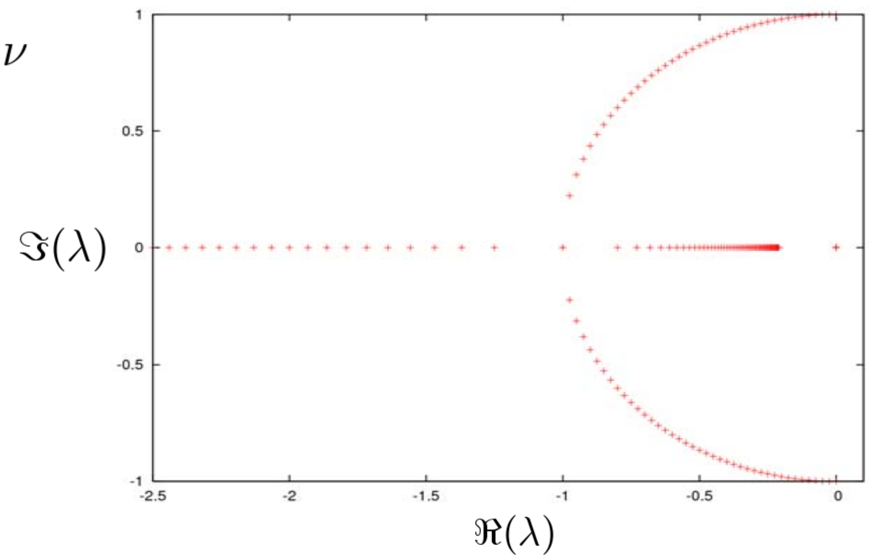
$$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ \nu_{in} & \nu_{out} & -(\mu_{out}+\mu_{in}) \end{pmatrix}$$

Char. polynom:

$$P(\lambda) = \lambda^3 + (\mu_{out}+\mu_{in})\lambda^2 + (\nu_{out}+\nu_{in})\lambda \overset{!}{=} 0$$

$$\lambda = -\frac{\mu_{out}+\mu_{in}}{2} \pm \sqrt{\frac{\mu_{out}+\mu_{in}}{2} - \nu}$$

$$\boxed{\ddot{q} + 2\Gamma\dot{q} + \omega_0^2 = 0 \Rightarrow q = Ae^{\lambda t}}$$



4. Network Effects

Two producers with equal strategy:

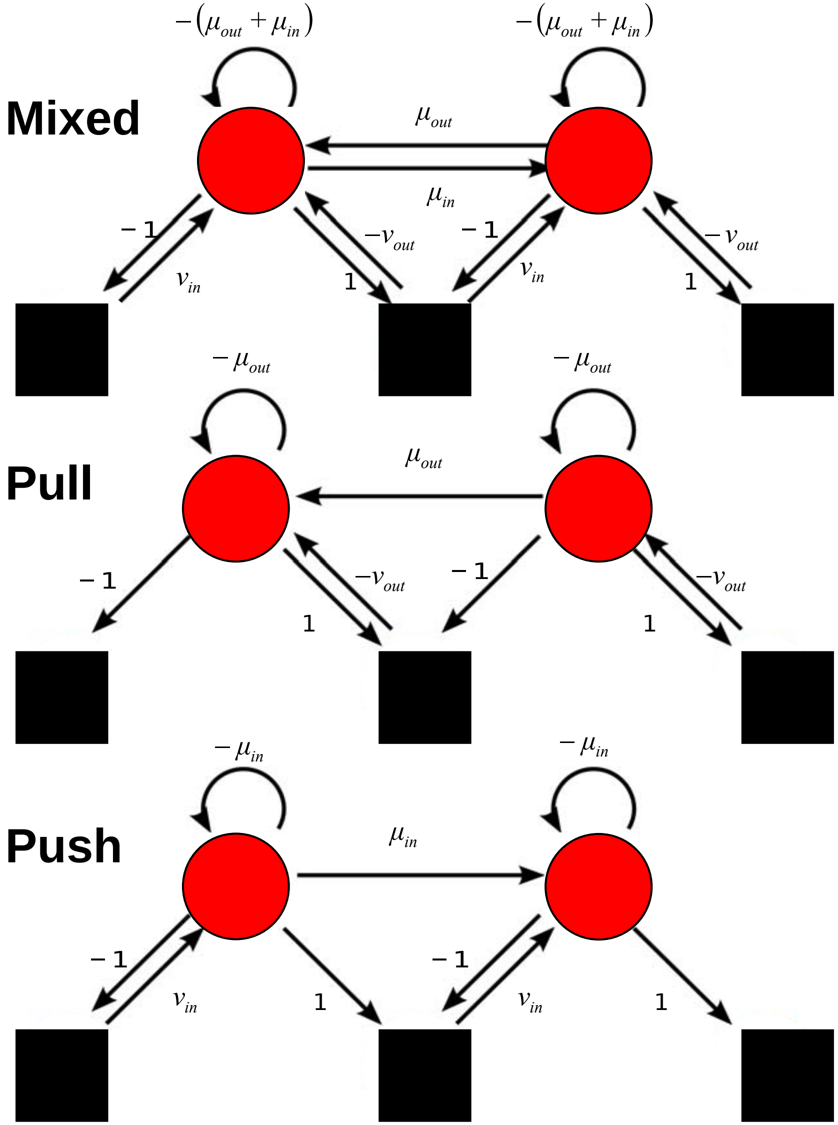
$$\begin{pmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ \nu_{in} & -\nu_{out} & 0 & -\mu_{out} - \mu_{in} & \mu_{out} \\ 0 & \nu_{in} & -\nu_{out} & \mu_{in} & -\mu_{out} - \mu_{in} \end{pmatrix}$$

Pull / Push:

- Edges connecting production units do not contribute to characteristic polynom
- both units share characteristic polynom, which is the square of the polynom for single producer
- degenerated eigenvalues

Mixed strategy:

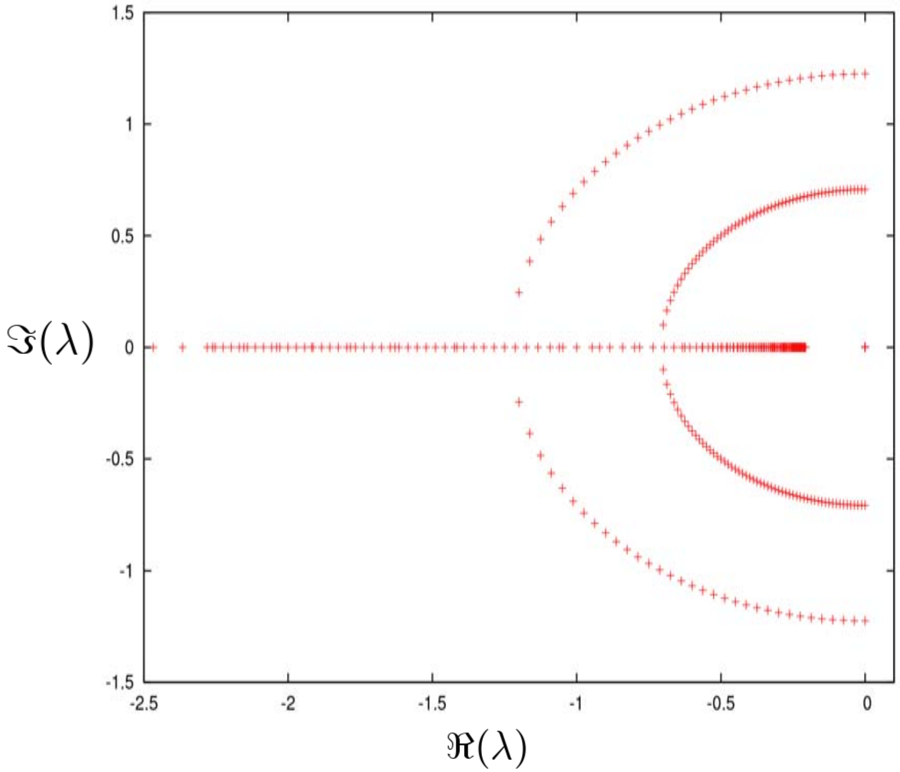
- no degeneration
- cycles contribute to characteristic polynom, i.e., source of instability

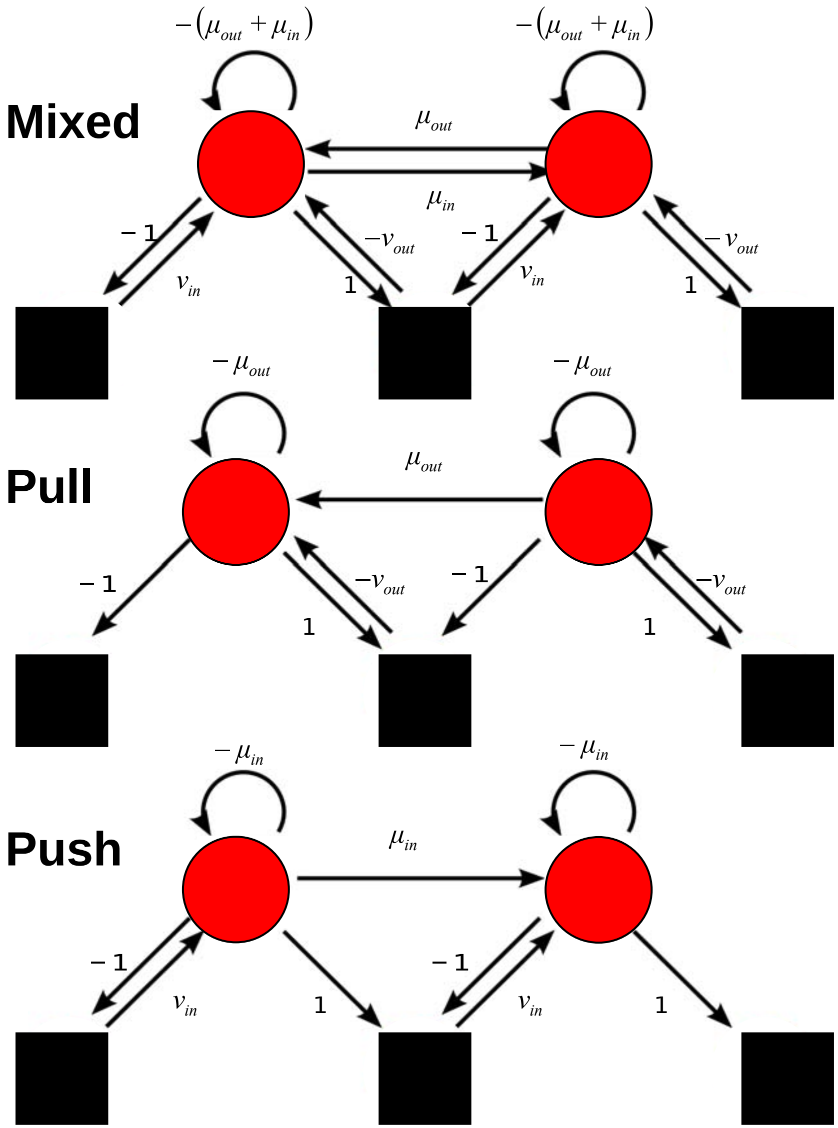


4. Network Effects

Two producers with equal strategy:

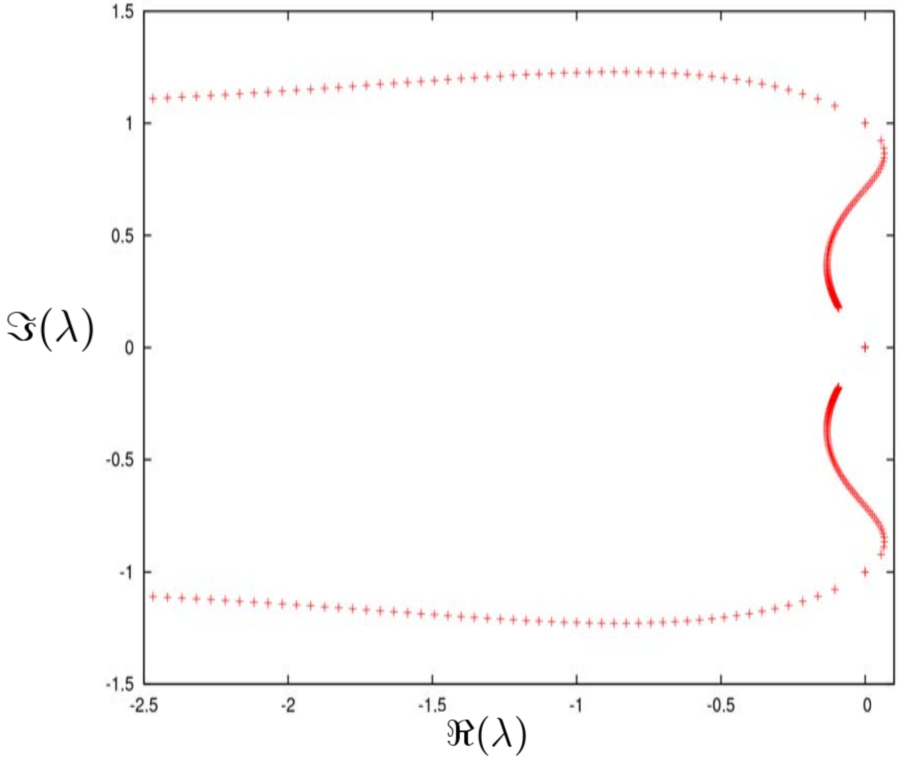
$$\nu_{in} = \nu_{out} = 0.5, \mu_{in} = \mu_{out}$$

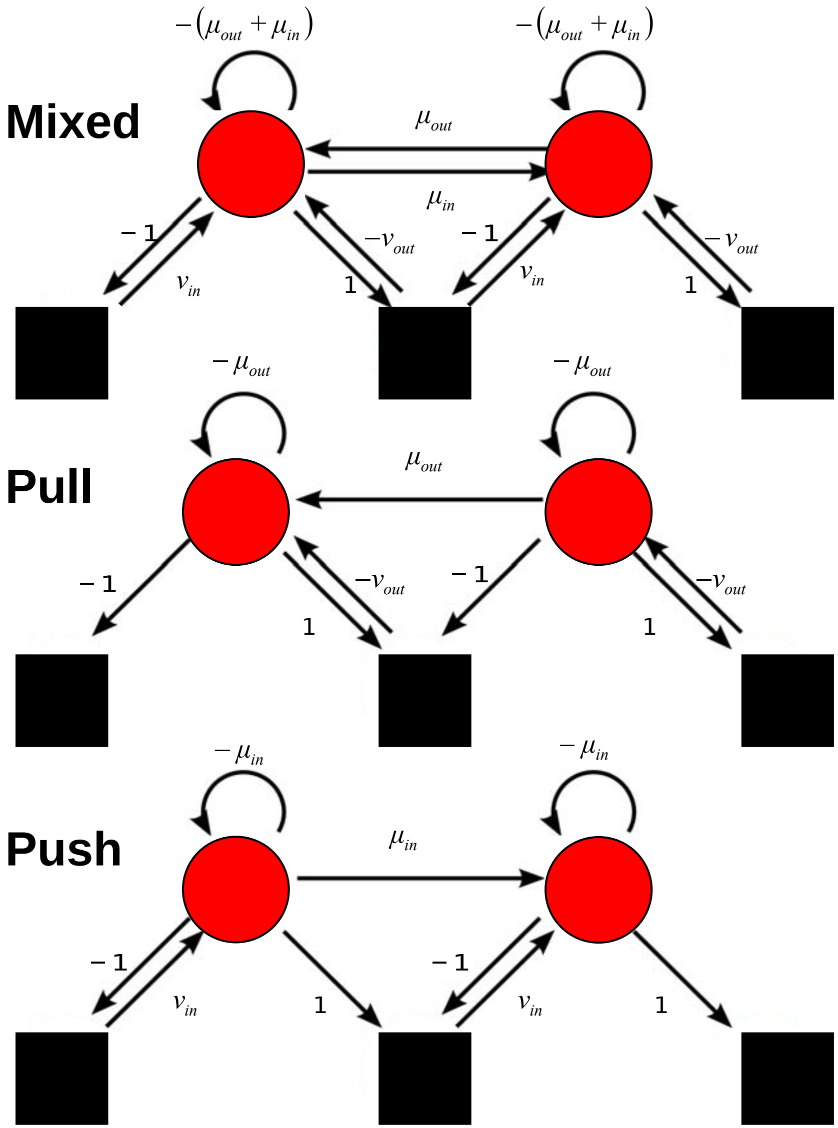




Two producers with equal strategy:

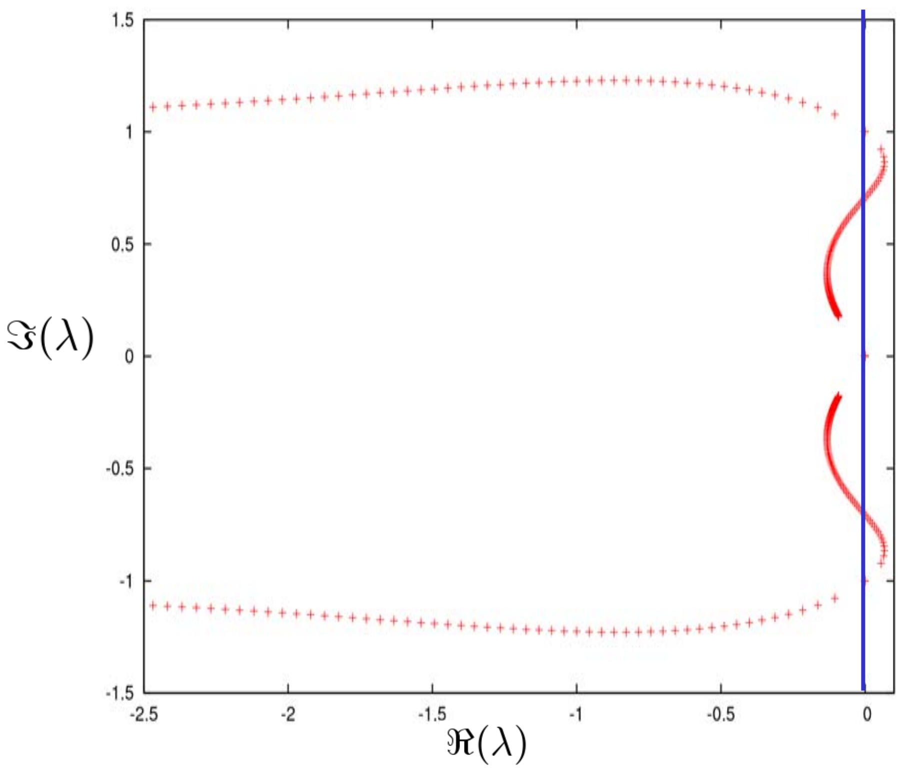
$$\nu_{in} = 0, \nu_{out} = 1, \mu_{out} = 0$$





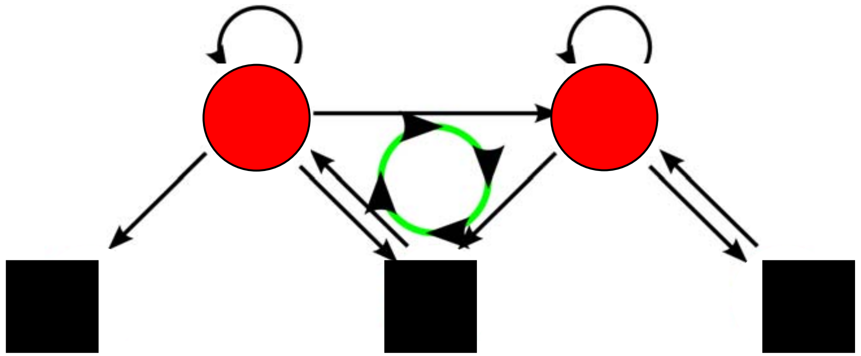
Two producers with equal strategy:

$$\nu_{in} = 0, \nu_{out} = 1, \mu_{out} = 0$$



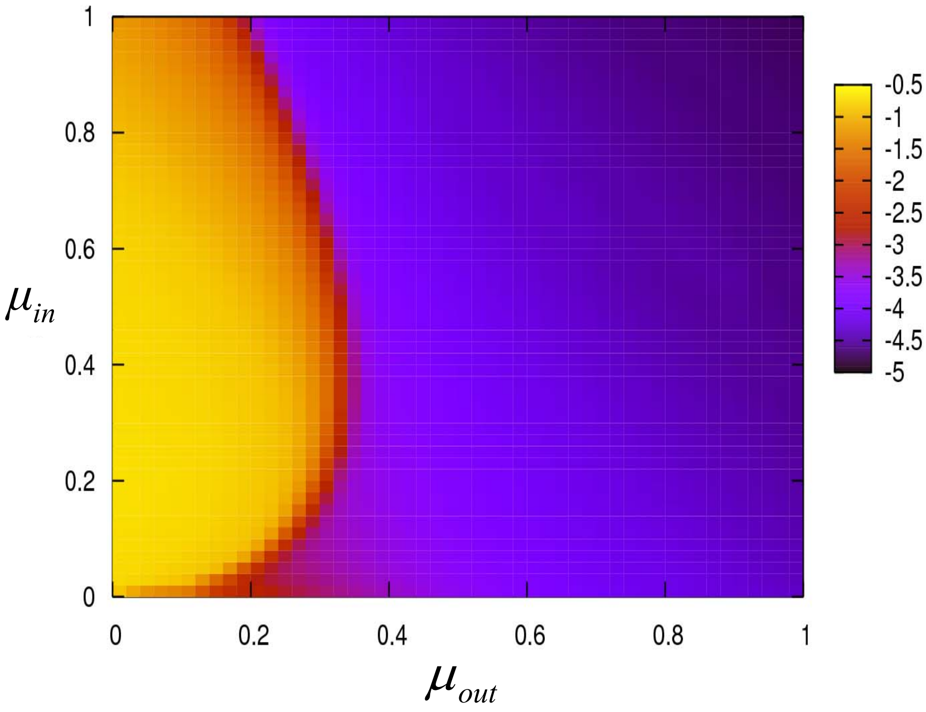
if only (ν_{out},μ_{in}) or (μ_{out},ν_{in}) are non-zero:

Positive real parts for small μ_{in} (μ_{out})
$\Rightarrow$ Destabilization due to feedback loop
$\Rightarrow$ can be suppressed by non-zero μ_{out} (μ_{in})

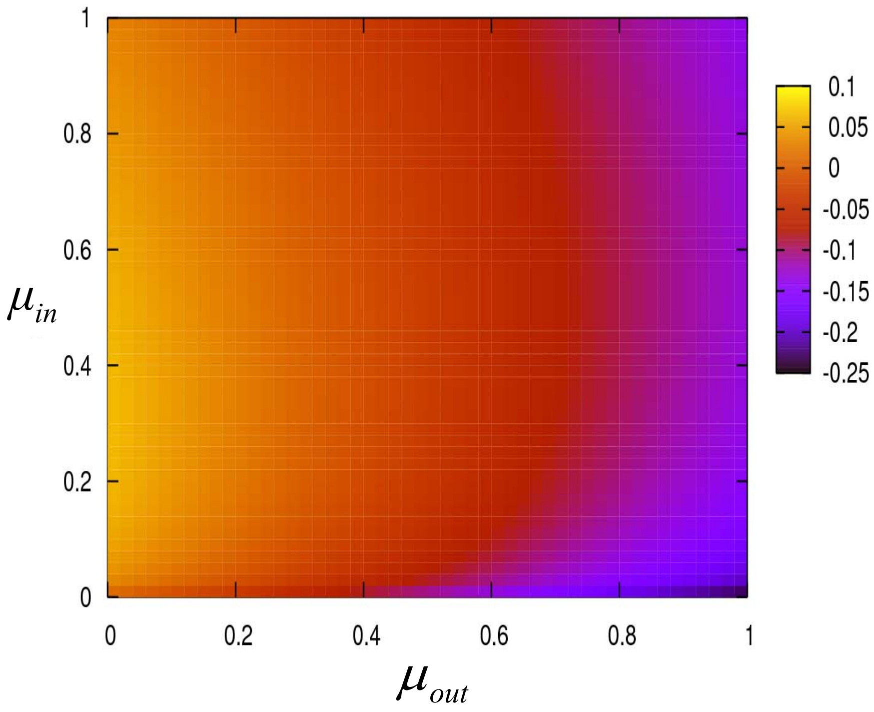


Two producers with equal strategy:

$\log(std(Q(t)))$



$\Re(\lambda)$



5. Summary

- Generalised input-output model of commodity flows
- Local stability:
  - damped and driven harmonic oscillator
  - formal equivalence of pull and push strategies
  - generic instability for long-periodic oscillations: Bullwhip effect
  - stabilizing effect of mixed strategies: local adjustment of phases and amplitudes of driving forces (demand, supply)
- Global stability: additional instabilities due to dynamic feedback loops in the presence of mixed strategies

6. Outlook

- Analytical treatment of network effects (Laplace transforms?)
- Self-organized global control by local, adaptive, mixed production strategies?
- Study of price-production feedback and its implications
- Macroeconomic model for commodity availability
- Application for optimization of real-world production system


Thank you for your attention!