Introduction of Time-Frequency Analysis

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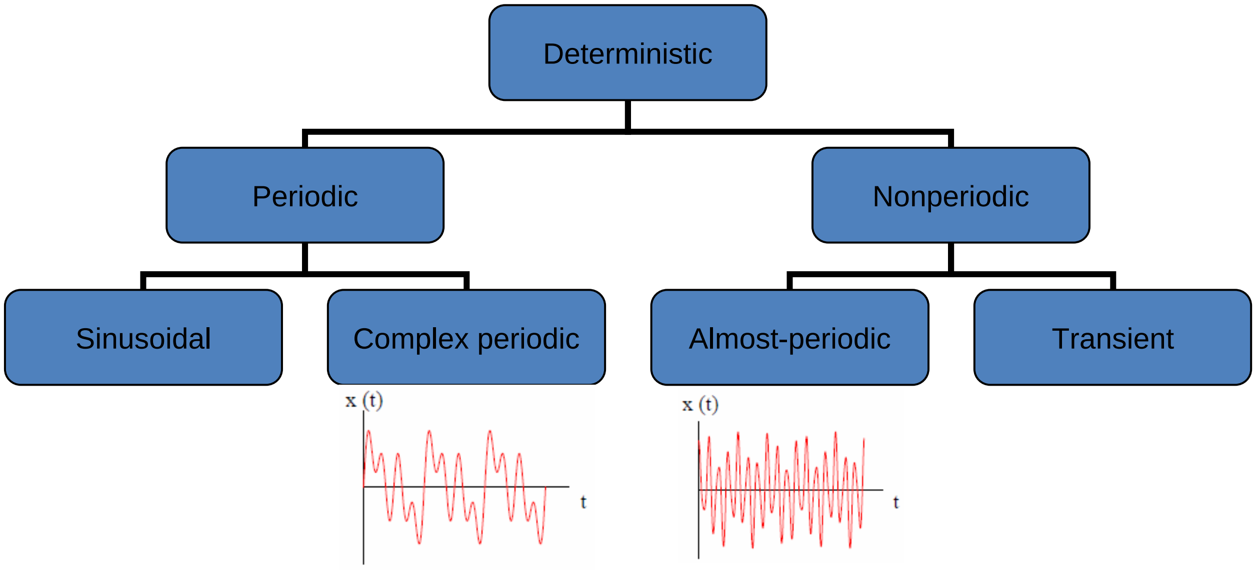
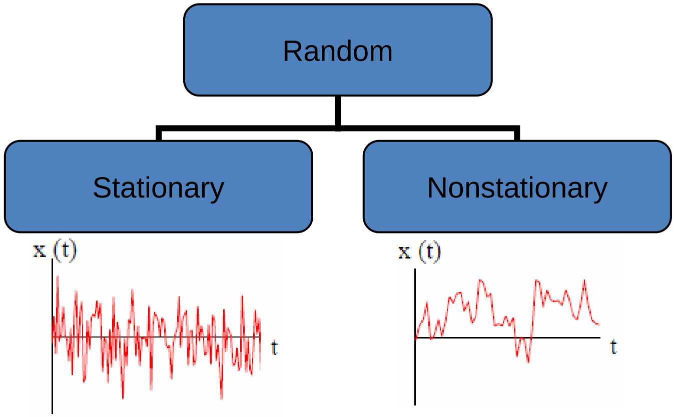
Outline

- **Classifications of signal type**
- **Fourier Transform**
- **Time-Frequency Analysis**
  - Short Time Fourier Transform
  - Wavelet Transform
- **Application**

Classifications of signal type

- deterministic and random signal





Fourier transform

- Fourier series and coefficients
  - – Periodic functions and signals may be expanded into a series of sine and cosine functions

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nt) + \sum_{n=1}^{\infty} b_n \sin(nt)$$

$$= \sum_{n=-\infty}^{\infty} c_n e^{\text{int}} = \sum_{n=-\infty}^{\infty} c_n \left(\cos(nt) + i \sin(nt)\right)$$

- Fourier coefficients on the interval $[-\pi, \pi]$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) \cos(nt) dt$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) \sin(nt) dt$$

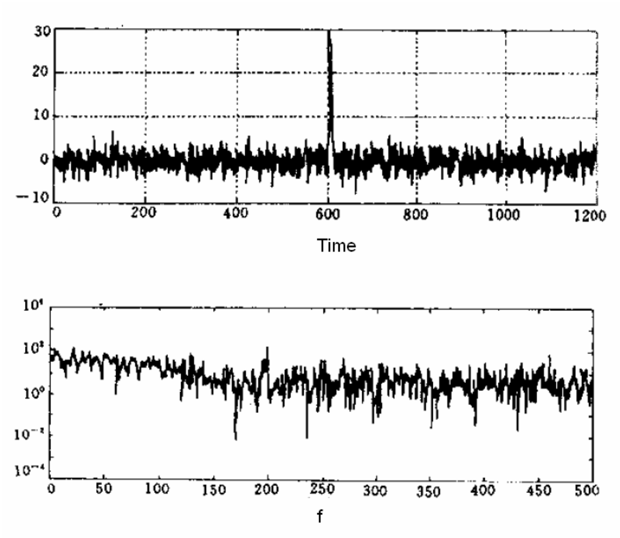
$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} x(t) e^{-\text{int}} dt$$

Fourier transform

- Fourier transform is defined by

$$X(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} x(t) dt \qquad \omega = 2\pi f$$

- Limitations
  - global analysis
  - resolution depends on the time long and the sampling rate
  - no time information in Frequency domain



white noise with impulse

Short Time Fourier Transform

- STFT is defined by

$$STFT\{x(t)\} = X(\tau, w) = \int_{-\infty}^{\infty} x(t) w(t-\tau) e^{-i\omega t} dt$$

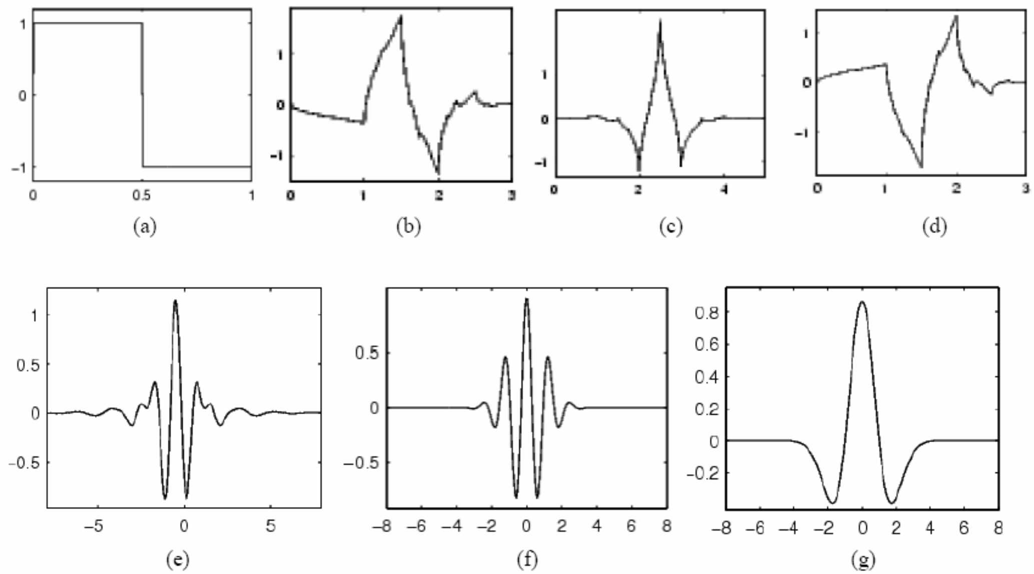
$x(t)$: signal to be transformed $\quad$ $w(t)$: window funktion

$X(\tau,\omega)$: Fourier Transform of $x(t)w(t-\tau)$ $\quad$ τ: translation parameter

- Resolution in time- and frequency domain
  - one window function has a fixed resolution
  - a wide window gives better frequency resolution but poor time resolution
  - a narrower window gives good time resolution but poor frequency resolution

Wavelet Transform

- Wavelet families



(a) Haar (b) Daubechies4 (c) Coiflet1 (d) Symlet2 (e) Meyer (f) Morlet (g) Mexican Hat

- advantages
  - – study the local features of the signal
  - – good resolution in time- and frequency domain

Wavelet Transform

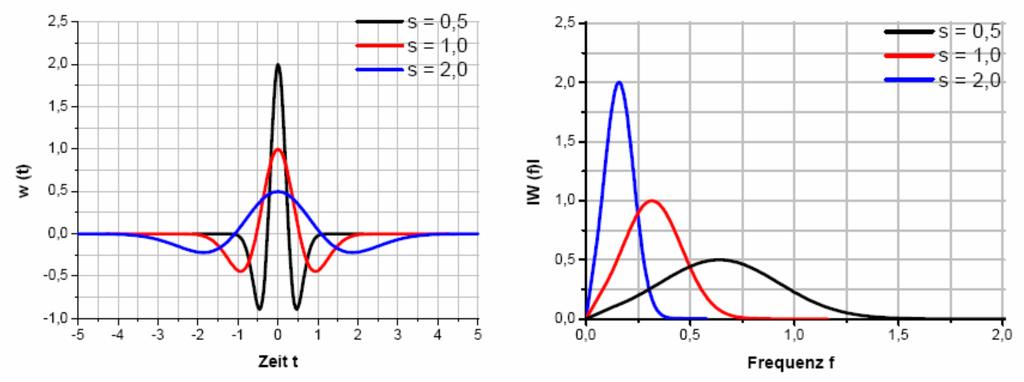
- The wavelet transform is defined by

$$W_{WT}(\tau,s)=\frac{1}{\sqrt{|s|}}\int x(t)\cdot w^*(\frac{t-\tau}{s})dt$$

$x(t)$: signal to be analyzed $\quad w(t)$: mother wavelet or basis function

$$w_{\tau,s}(t)=\frac{1}{\sqrt{|s|}}w(\frac{t-\tau}{s})$$

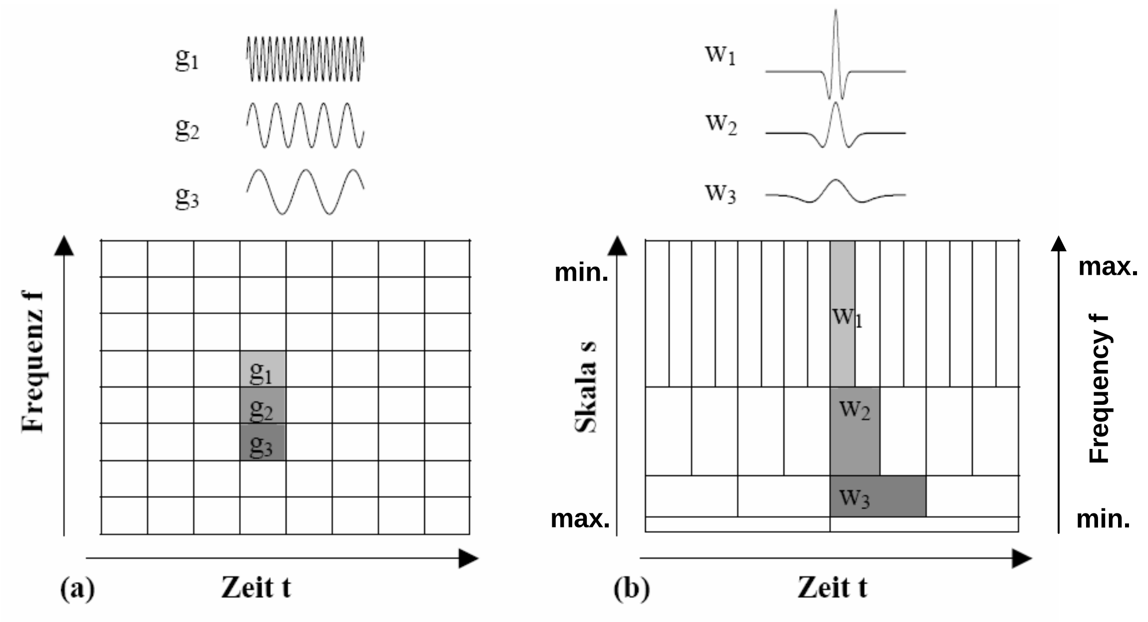
τ : translation parameter $\quad s$: scale parameter (1/frequency)



'Mexican Hat' Wavelet

Wavelet Transform

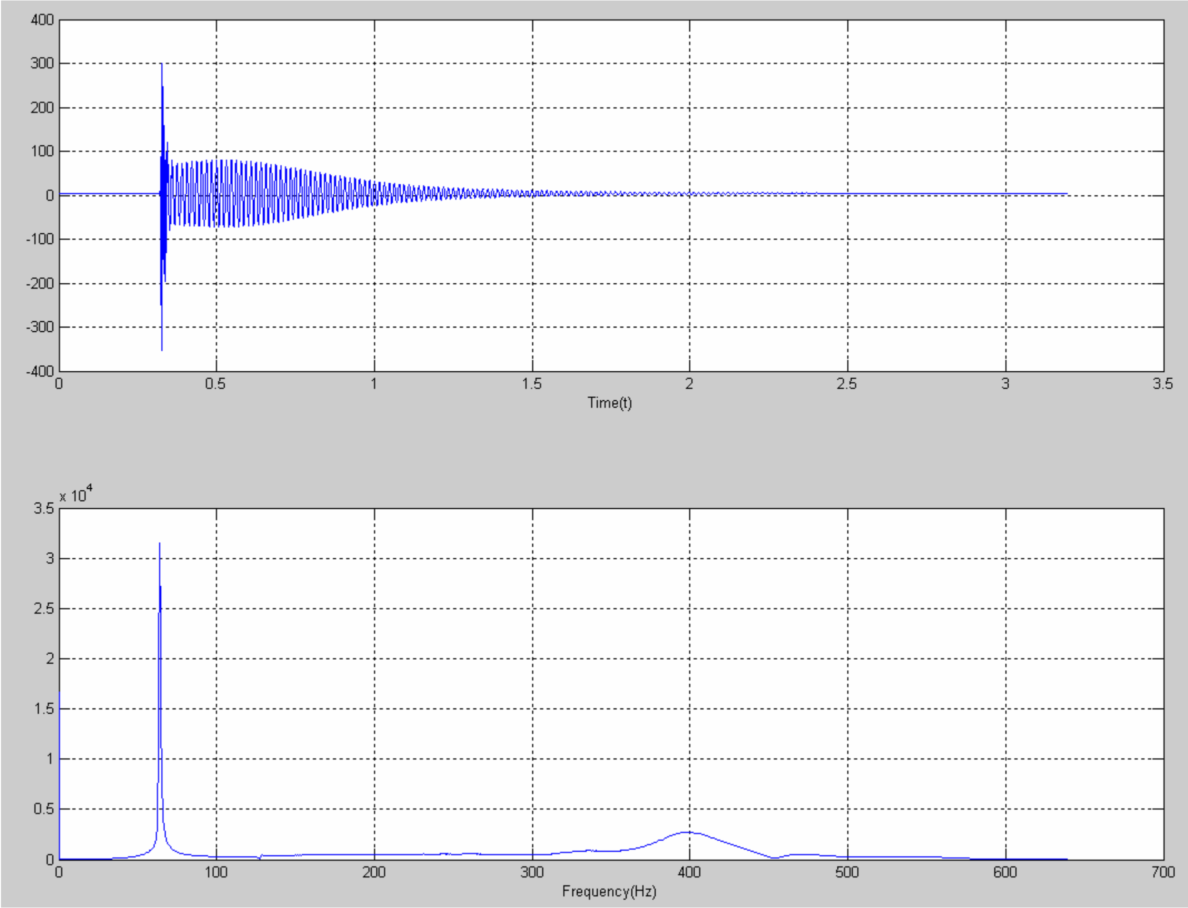
- comparison STFT and Wavelet Transform



- features of wavelet transform
  - resolution in time- and frequency domain is also conflicting
  - automatic adjustable resolution fit to most practical applications

Application

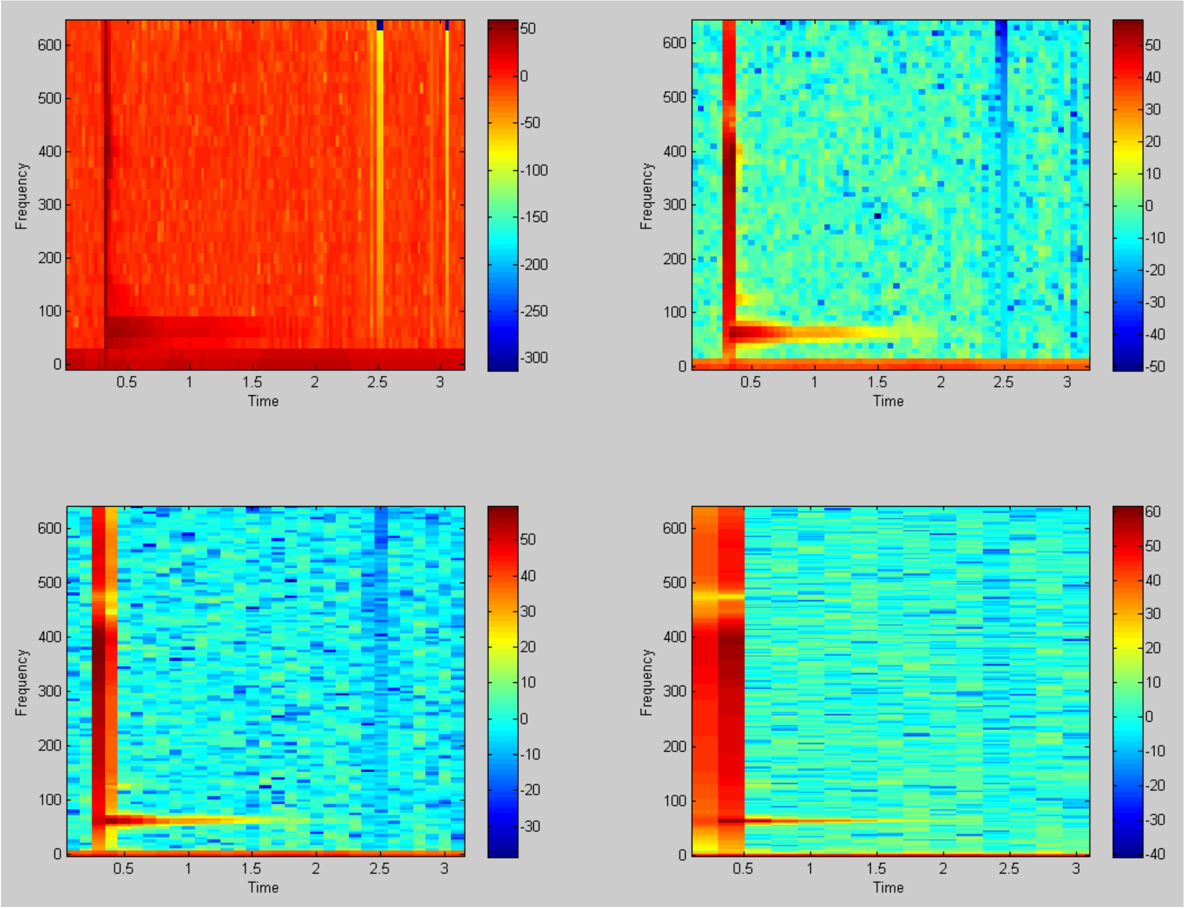
FFT Analysis



fs=1280Hz, T=3.2s

Application

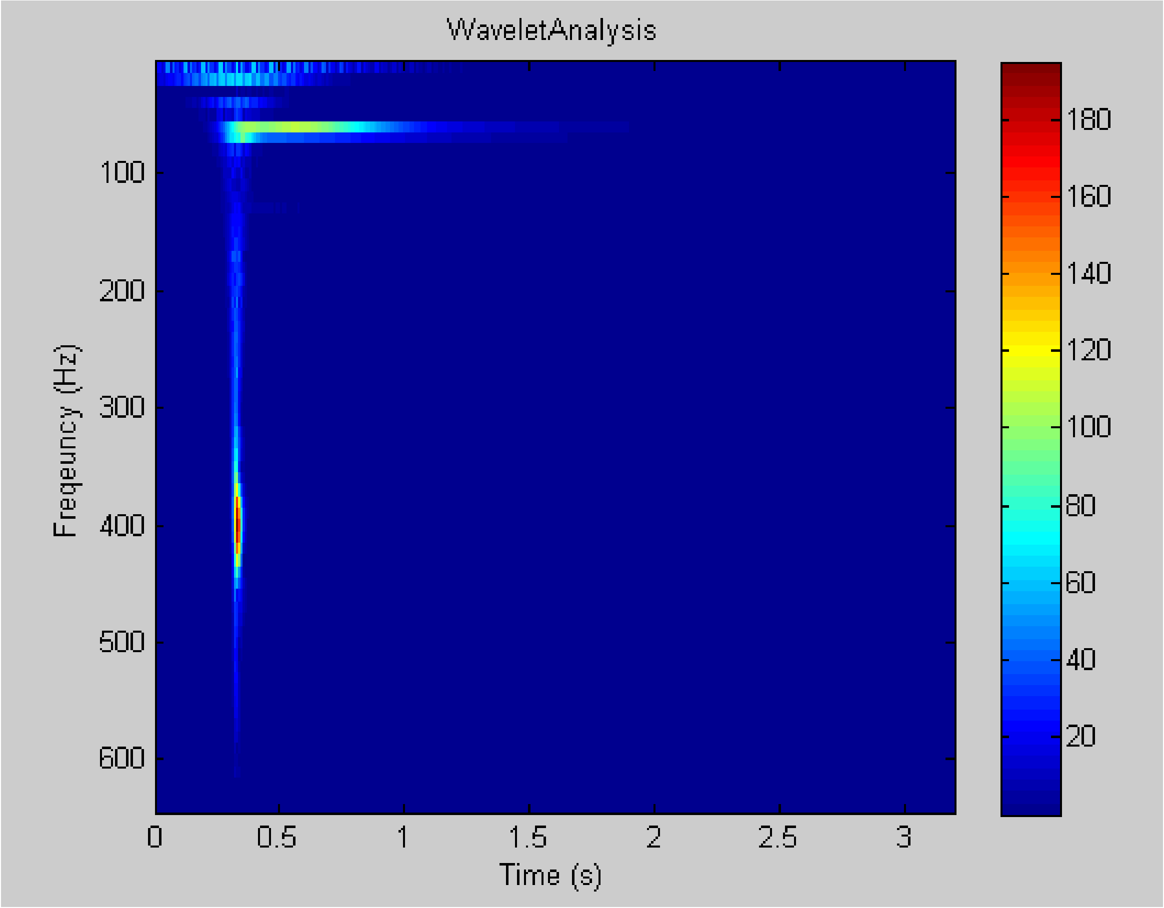
STFT Analysis



width of window function = 64, 128, 256 and 512

Application

Wavelet Analysis



Thank you for your Atention