Universität Bremen

ITEM

Numerical Methods to Investigating Nanofluids Flow in Curved Tubes

By :

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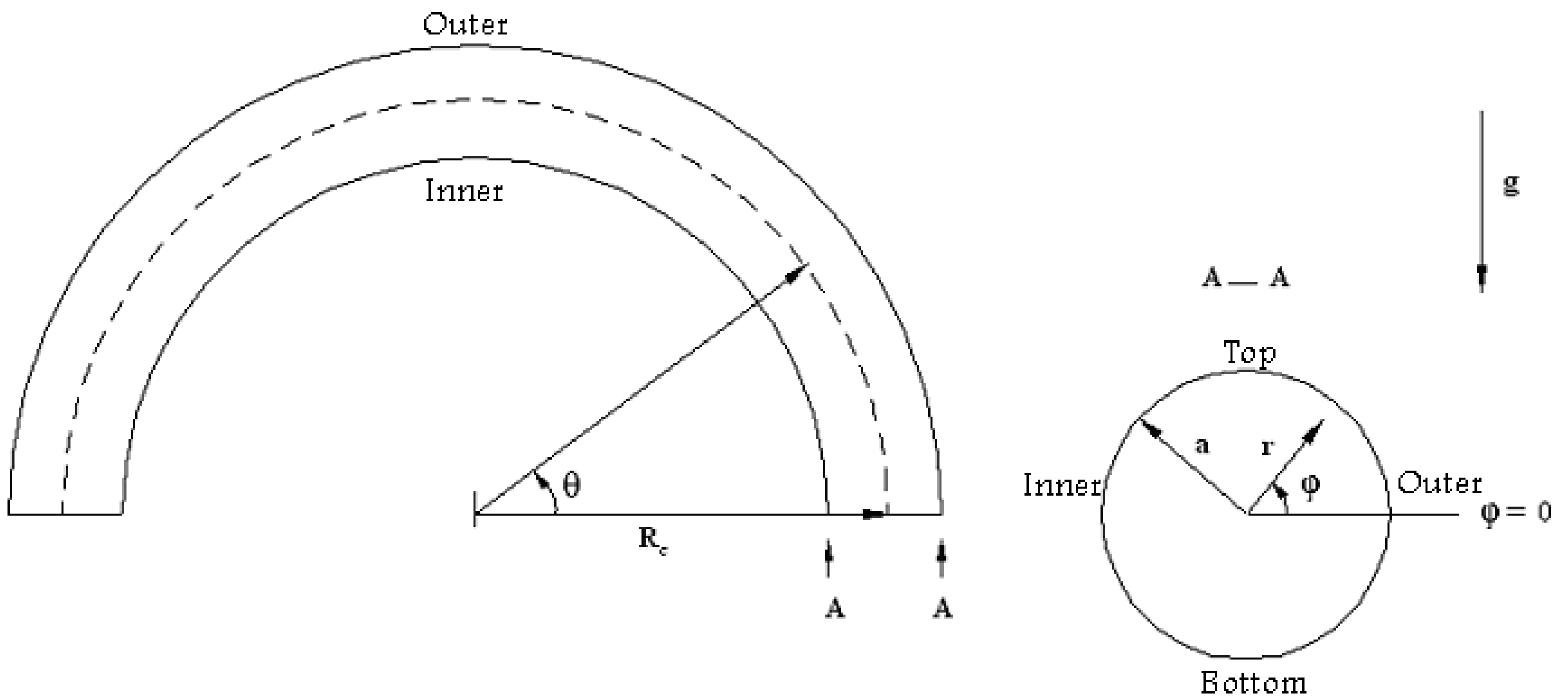
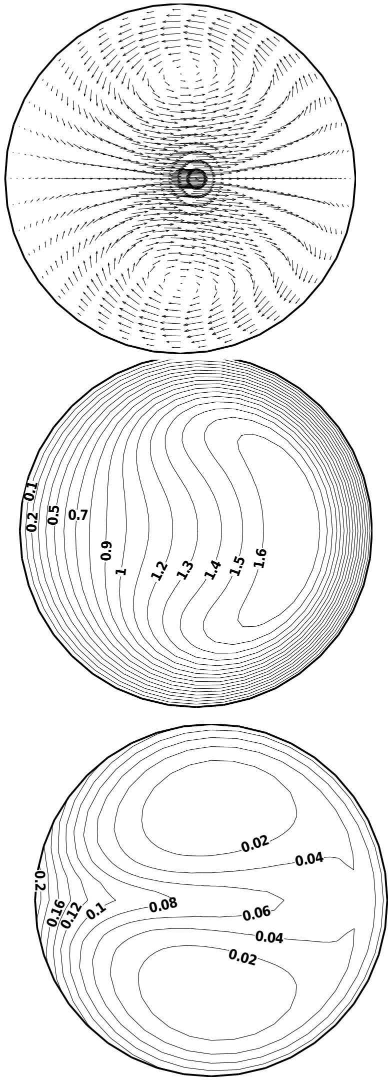
What are nanofluids ?

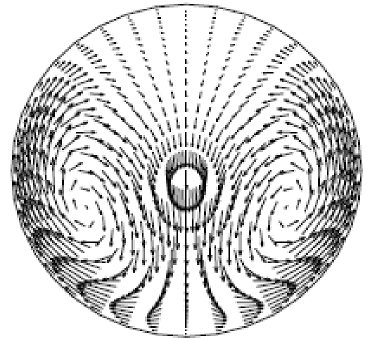
- Nanofluids are a new kind of heat transfer fluid containing a small quantity of nano-sized particles (usually with less than 100 nm diameter) that are uniformly and stably suspended in a base liquid. The dispersion of a small amount of solid nano –particles in convectional fluids changes their thermal conductivity remarkably. In this work base liquid and nano-particles are water and Al_2O_3 , respectively.

Computational domain

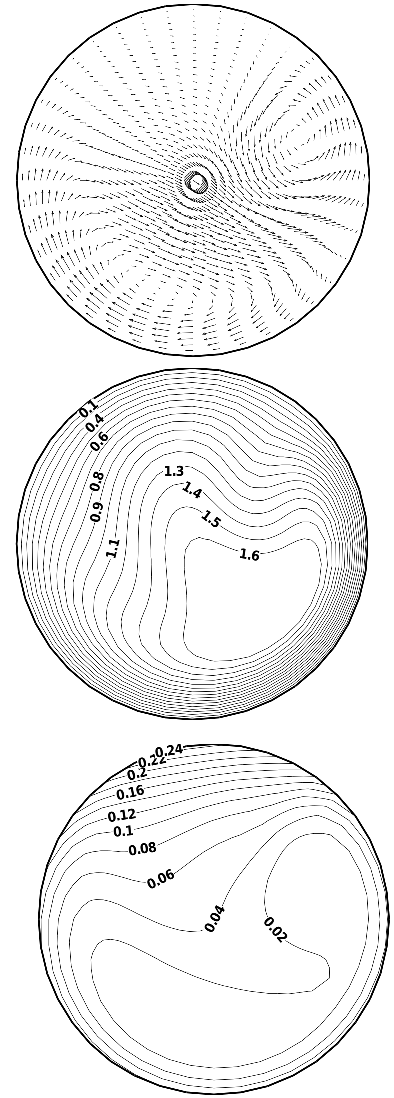




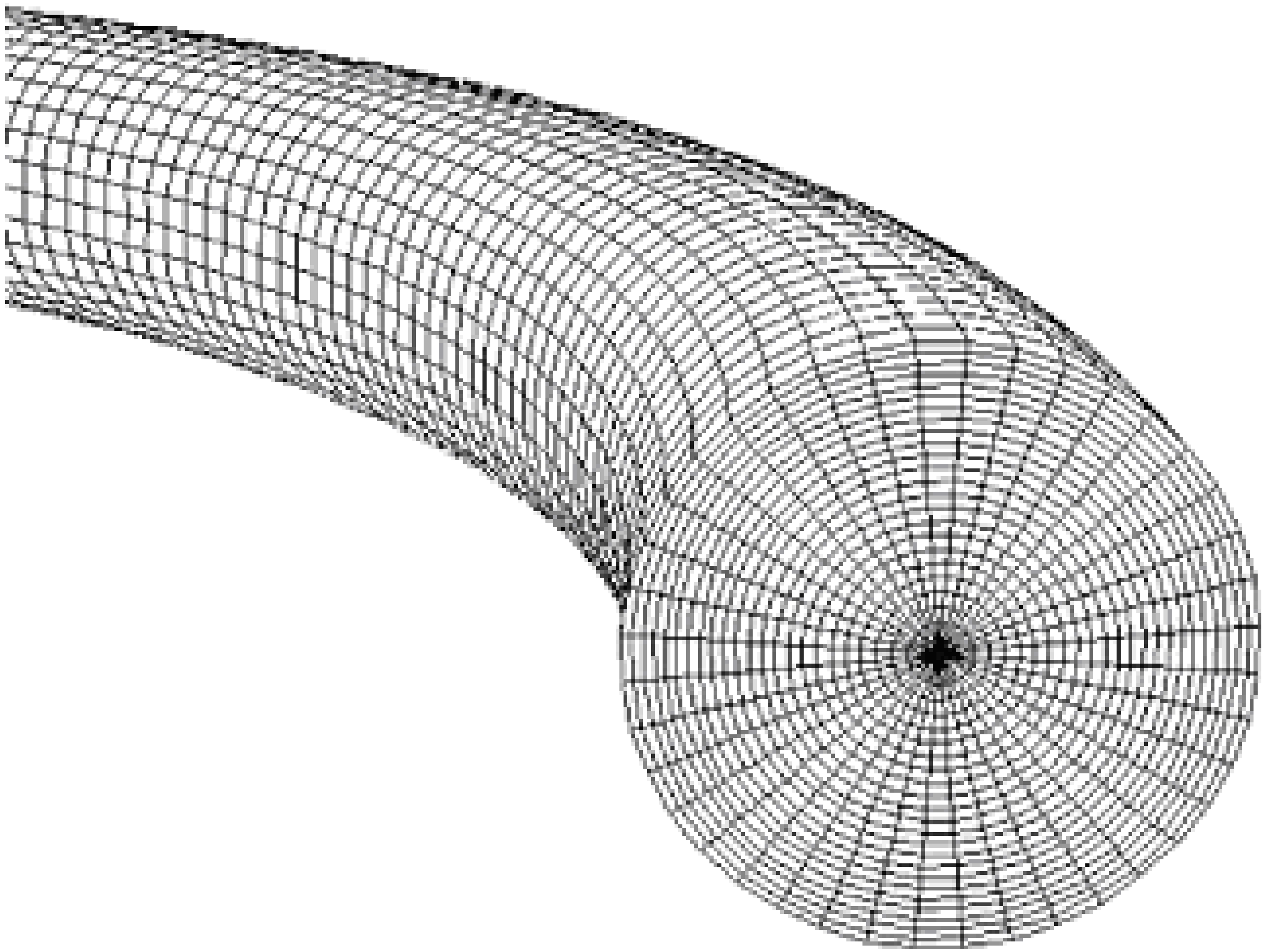
Centrifugal force



Buoyancy force



Mixed convection in curved tube



Governing equations

Continuity:

$$\frac{\partial u_j}{\partial x_j} = 0$$

Momentum:

$$\frac{\partial}{\partial x_j}(\rho_{eff} u_i u_j) = \frac{\partial}{\partial x_j}\left(\mu_{eff}\left(\frac{\partial u_i}{\partial x_j}\right)\right) - \frac{\partial p}{\partial x_i} - \rho_{eff,0} g_i \beta_{eff}(T - T_0)$$

Energy:

$$\frac{\partial}{\partial x_i}\left[(\rho c_p)_{eff} u_i T\right] = \frac{\partial}{\partial x_i}\left(k_{eff}\frac{\partial T}{\partial x_i}\right)$$

The effective properties of the nanofluid

$$\rho_{eff,0}=\left(1-\varphi_s\right)\rho_{f,0}+\varphi_s\rho_{s,0}$$

$$\mu_{eff}=\left(123\varphi_{s^2}+7.3\varphi_s+1\right)\mu_f$$

$$(c_p)_{eff}=\left[\frac{\left(1-\varphi_s\right)(\rho c_p)_f+\varphi_s(\rho c_p)_s}{\left(1-\varphi_s\right)\rho_f+\varphi_s\rho_s}\right]$$

$$k_{eff}=\left(\frac{k_s+2k_f-2\varphi_s\left(k_f-k_s\right)}{k_s+2k_f+\varphi_s\left(k_f-k_s\right)}\right)k_f$$

$$\beta_{eff}=\left[\frac{1}{1+\frac{\left(1-\varphi_s\right)\rho_f}{\varphi_s\rho_s}}\frac{\beta_s}{\beta_f}+\frac{1}{1+\frac{\varphi_s}{1-\varphi_s}\frac{\rho_s}{\rho_f}}\right]\cdot\beta_f$$

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Boundary conditions

- At the *tube inlet* $v_r=0$; $v_\varphi=0$, $v_z= v_0$; $T=T_0$

- At the *fluid-solid interface* (r=a):

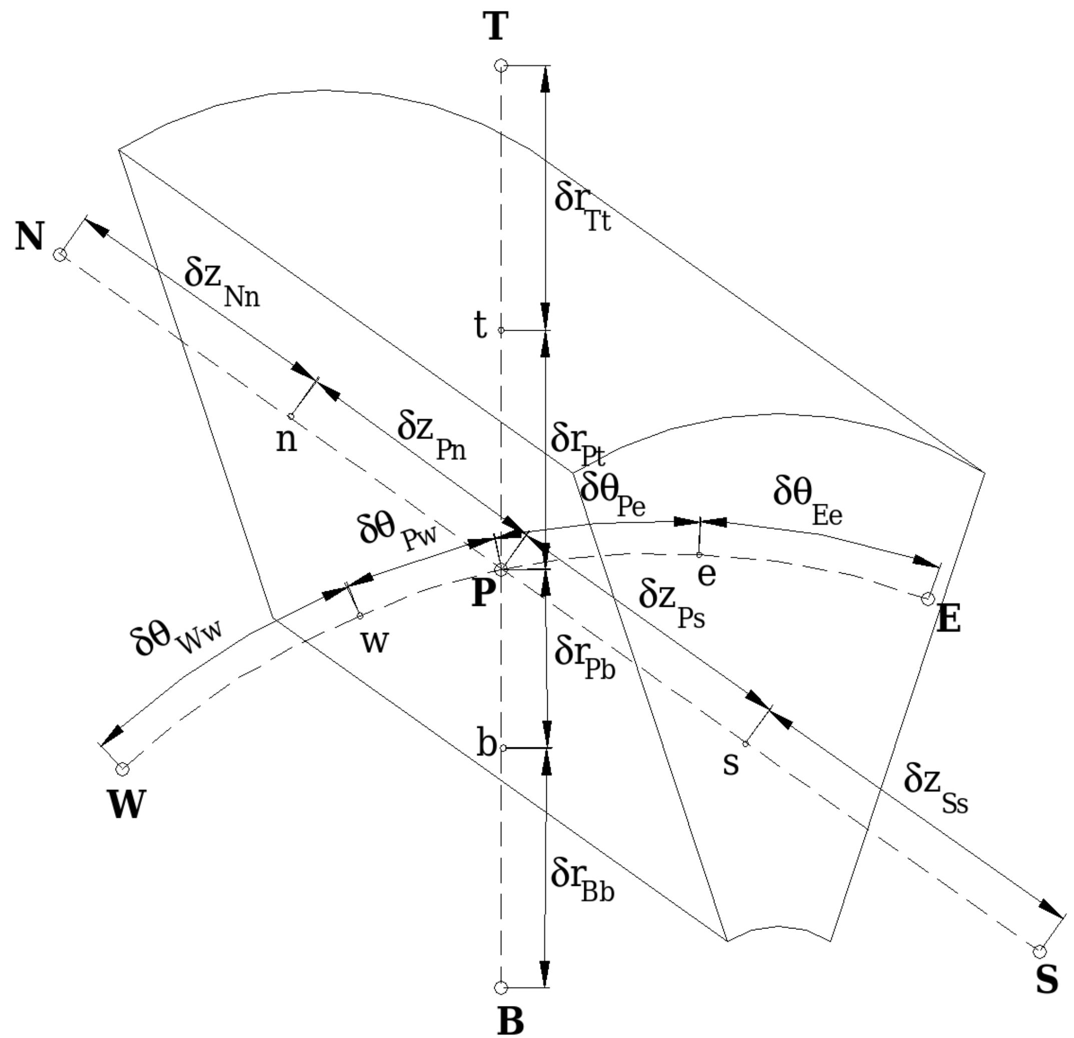
$$v_r= v_\varphi= v_z=0 \;; \qquad q_w=-k\frac{\partial T}{\partial r}$$

- At the *tube outlet* ($\theta=180\,^0$) the diffusion flux in the direction normal to the exit plane is assumed to be zero and an overall mass balance correction is applied.

Numerical procedure

- *Control volume technique*
- *Second order upwind*
- *SIMPLEC*
- The discretization grid is uniform in circumferential direction and non-uniform in the other two directions. It is finer near the tube entrance and near the wall where the velocity and temperature gradient are significant.

Discretization Method



$$\rho \frac{\partial}{\partial X_j}\left(U_j \Phi\right) = \frac{\partial}{\partial X_j}\left(\Gamma \frac{\partial \Phi}{\partial X_j}\right) + S$$

.Diffusion and Source coefficient for variable

Φ	Γ	S
U_j	μ_{eff}	$-(\partial P/\partial X_i)+\rho_{nf,0}\beta_{nf}.(T- T_0).$
T	$k_{eff} / (c_p)_{nf}$	g_i 0

$$J_j = \rho U_j \Phi - \Gamma \frac{\partial \Phi}{\partial X_j}$$

$$\frac{\partial J_j}{\partial X_j} = S$$

:Thus

$$J_t A_t - J_b A_b + J_e A_e - J_w A_w + J_n A_n - J_s A_s = \overline{S}\, \Delta V$$

:Where

$$\overline{S} = S_c + S_p \Phi_p$$

$$J_j A_j = \rho_e U_e A_e \Phi_e^{\text{¿}} + \frac{\Gamma(\Phi_E - \Phi_p)}{\delta x_e}$$

$$\Phi_e^{\text{¿}} = \frac{3}{2}\Phi_p - \frac{1}{2}\Phi_W \qquad if \quad (\rho U_e A_e) > 0$$

$$\Phi_e^{\text{¿}} = \frac{3}{2}\Phi_E - \frac{1}{2}\Phi_{EE} \qquad if \quad (\rho U_e A_e) < 0$$

$$a_p \Phi_p = \sum_{nb} a_{nb}\Phi_{nb} + b$$

$$a_e = [[-F_e,\ 0]] + D_e A_e \quad , \quad a_w = [[F_w,\ 0]] + D_w A_w$$

$$a_t = [[-F_t,\ 0]] + D_t A_t \quad , \quad a_b = [[F_b,\ 0]] + D_b A_b$$

$$a_n = [[-F_n,\ 0]] + D_n A_n \quad , \quad a_s = [[F_s,\ 0]] + D_s A_s$$

$$b = S_c \Delta V + b^{\text{¿}}$$

$$[[F_e,\ 0]] = [[-F_e,\ 0]] + F_e \quad , \quad [[-F_w,\ 0]] = [[F_w,\ 0]] + F_w$$

$$[[F_t,\ 0]] = [[-F_t,\ 0]] + F_t \quad , \quad [[-F_b,\ 0]] = [[F_b,\ 0]] + F_b$$

$$[[F_n,\ 0]] = [[-F_n,\ 0]] + F_n \quad , \quad [[F_s,\ 0]] = [[F_s,\ 0]] + F_s$$

:From the continuity equation

$$F_e - F_w + F_t - F_b + F_n - F_s = 0$$

:Thus

$$a_p \Phi_p = \sum_{nb} a_{nb} \Phi_{nb} + b^{¿} + S_c \Delta V$$

$$a_p = \sum_{nb} a_{nb} + S_p = a_e + a_w + a_t + a_b + a_s + a_n + S_p$$

$$b^{¿} = \left\{ \frac{1}{2}(\Phi_p - \Phi_W)[\![F_e, 0]\!] - \frac{1}{2}(\Phi_E - \Phi_{EE})[\![-F_e, 0]\!] \right\} + \left\{ \frac{1}{2}(\Phi_p - \Phi_E)[\![-F_w, 0]\!] - \frac{1}{2}(\Phi_W - \Phi_{WW})[\![F_w, 0]\!] \right\} +$$

$$\left\{ \frac{1}{2}(\Phi_p - \Phi_b)[\![F_t, 0]\!] - \frac{1}{2}(\Phi_T - \Phi_{TT})[\![-F_t, 0]\!] \right\} + \left\{ \frac{1}{2}(\Phi_p - \Phi_T)[\![-F_b, 0]\!] - \frac{1}{2}(\Phi_B - \Phi_{BB})[\![F_b, 0]\!] \right\} +$$

$$\left\{ \frac{1}{2}(\Phi_p - \Phi_S)[\![F_n, 0]\!] - \frac{1}{2}(\Phi_N - \Phi_{NN})[\![-F_n, 0]\!] \right\} + \left\{ \frac{1}{2}(\Phi_p - \Phi_N)[\![-F_s, 0]\!] - \frac{1}{2}(\Phi_S - \Phi_{SS})[\![F_s, 0]\!] \right\}$$

Two phase approach .2

Continuity: $$\nabla\cdot\left(\rho_m V_m\right)=0$$

Momentum: $$\nabla\cdot\left(\rho_m V_m V_m\right)=-\nabla p+\nabla\cdot\left(\mu_m \nabla V_m\right)+\rho_{m,0} g\beta_m\left(T-T_b\right)+\nabla\cdot\left(\sum_{k=1}^{n}\varphi_k\rho_k V_{dr,k}V_{dr,k}\right)$$

Energy: $$\nabla\cdot\sum_{k=1}^{n}\left(\varphi_k V_k\rho_k T\right)=\nabla\cdot\left(\lambda_{eff}\nabla T\right)$$

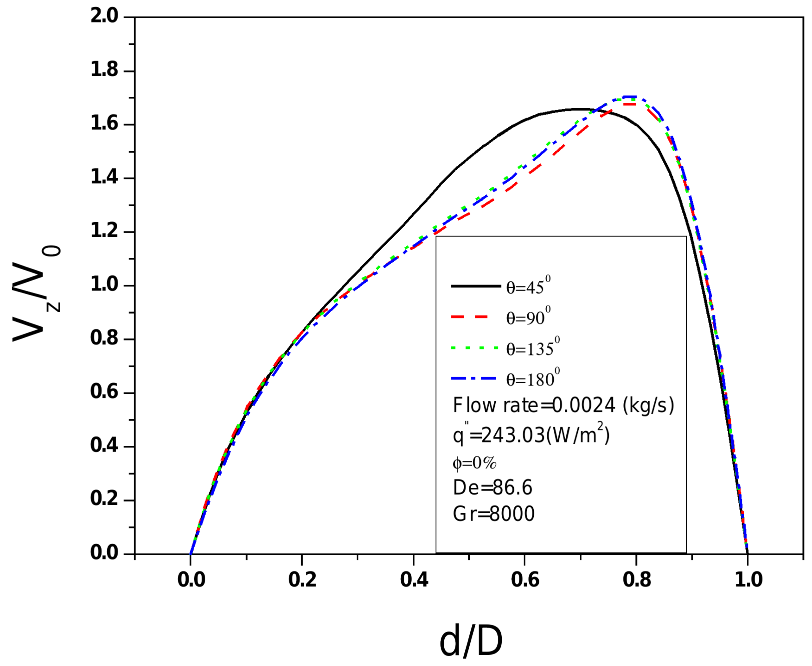
Volume fraction: $$\nabla\cdot\left(\varphi_p\rho_p V_m\right)=-\nabla\cdot\left(\varphi_p\rho_p V_{dr,p}\right)$$

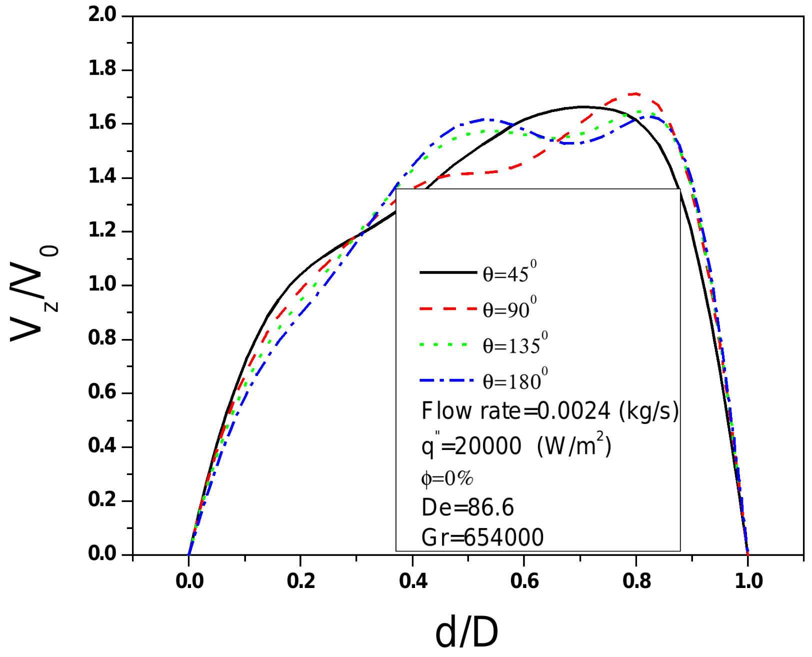
$$\rho_m = \sum_{k=1}^{n} \varphi_k \rho_k$$

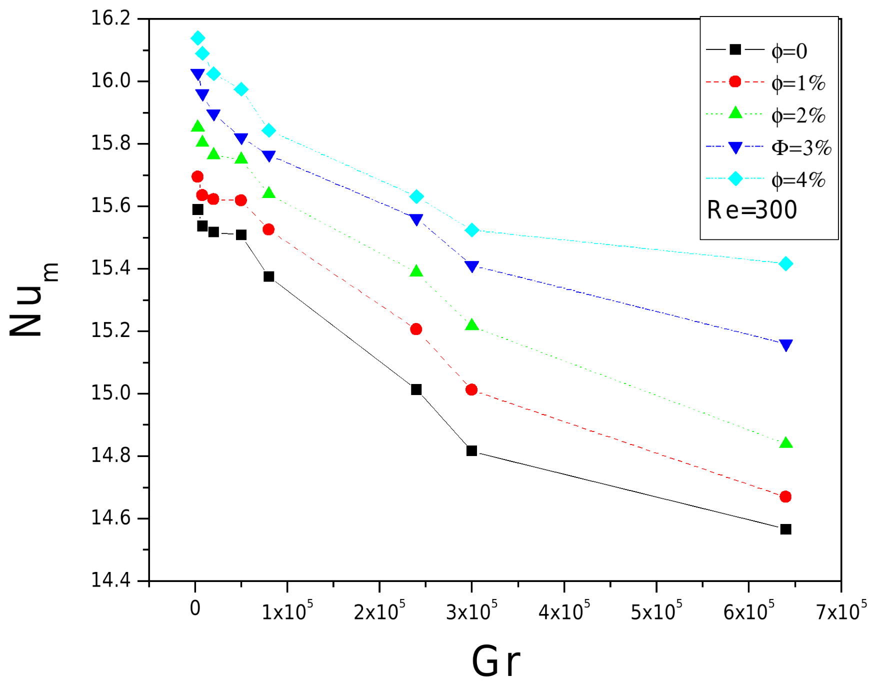
$$\mu_m = \sum_{k=1}^{n} \varphi_k \mu_k$$

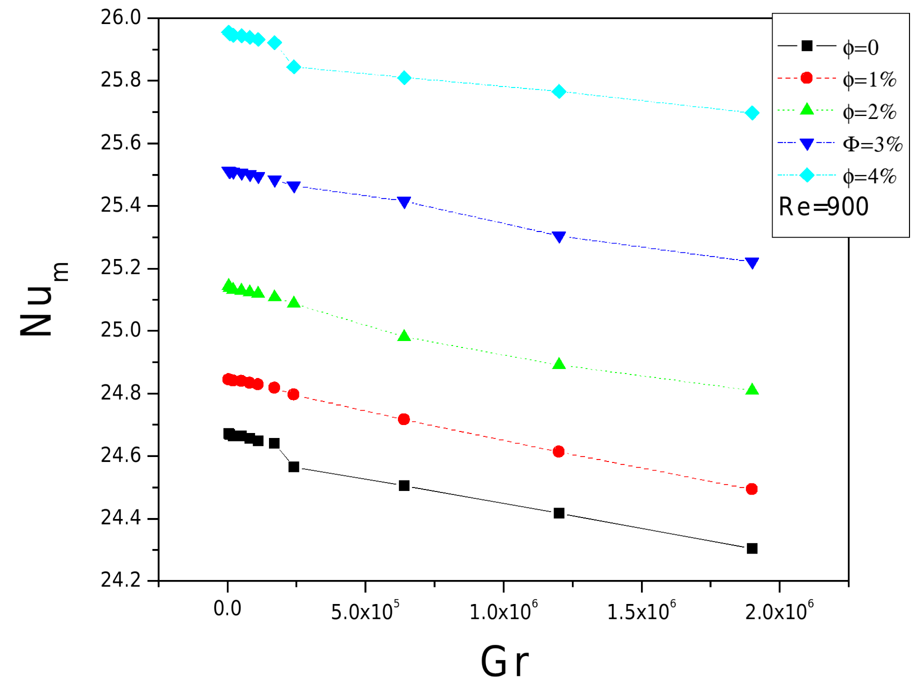
$$V_m = \frac{\sum_{k=1}^{n} \varphi_k \rho_k V_k}{\rho_m}$$

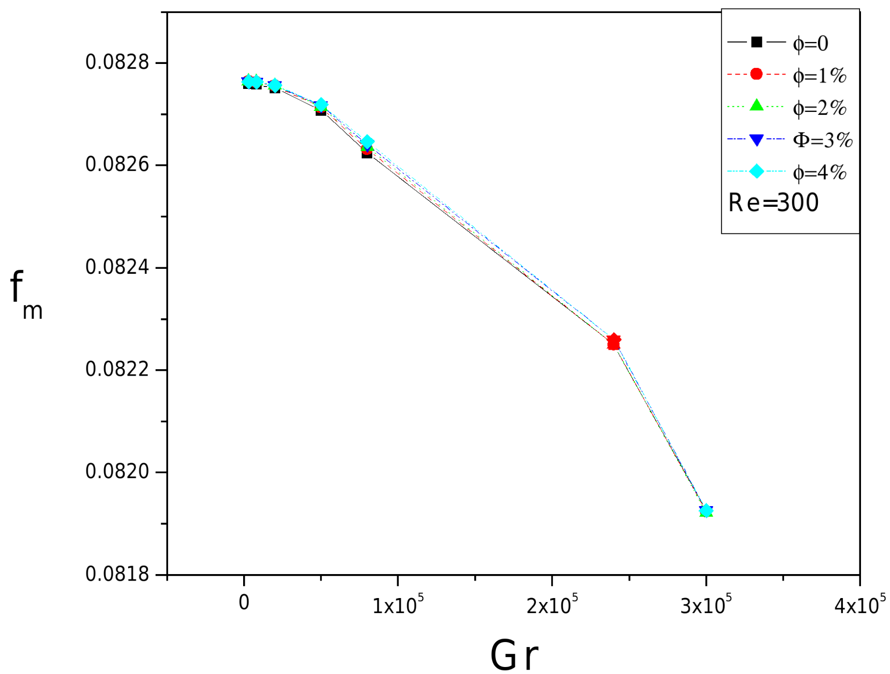
$$\lambda_{eff} = \sum_{k=1}^{n} \varphi_k \lambda_k$$

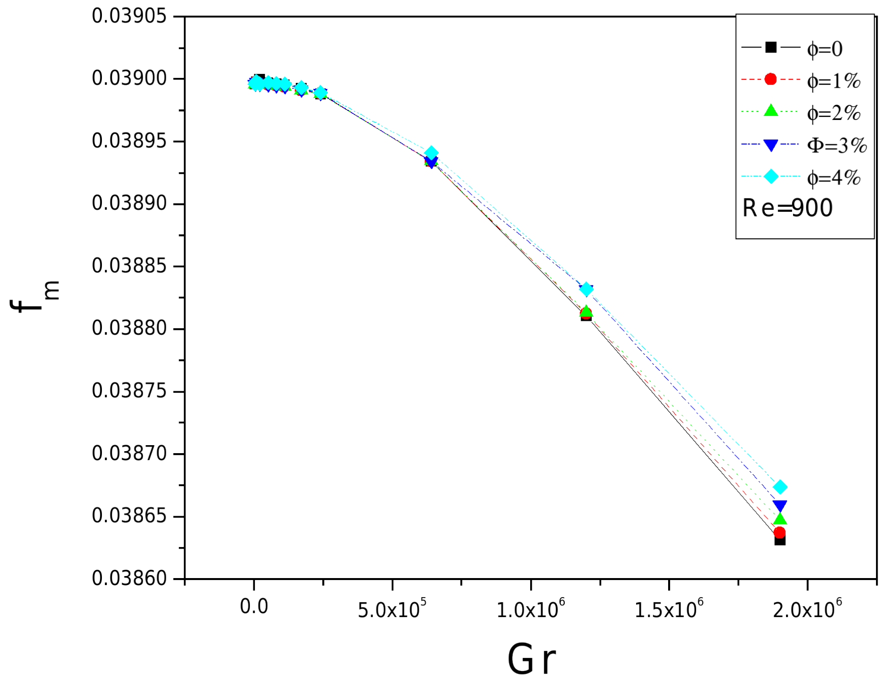

V_z/V_0
d/D
θ=45°
θ=90°
θ=135°
θ=180°
Flow rate=0.0024 (kg/s)
q''=243.03(W/m²)
ϕ=0%
De=86.6
Gr=8000

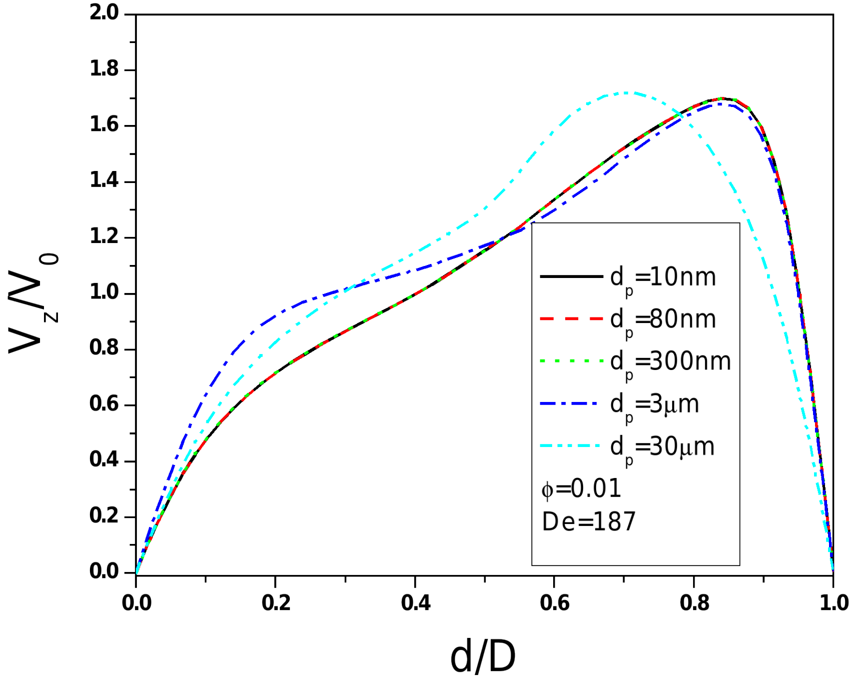

V_z/V_0
d/D
θ=45°
θ=90°
θ=135°
θ=180°
Flow rate=0.0024 (kg/s)
q''=20000 (W/m²)
ϕ=0%
De=86.6
Gr=654000

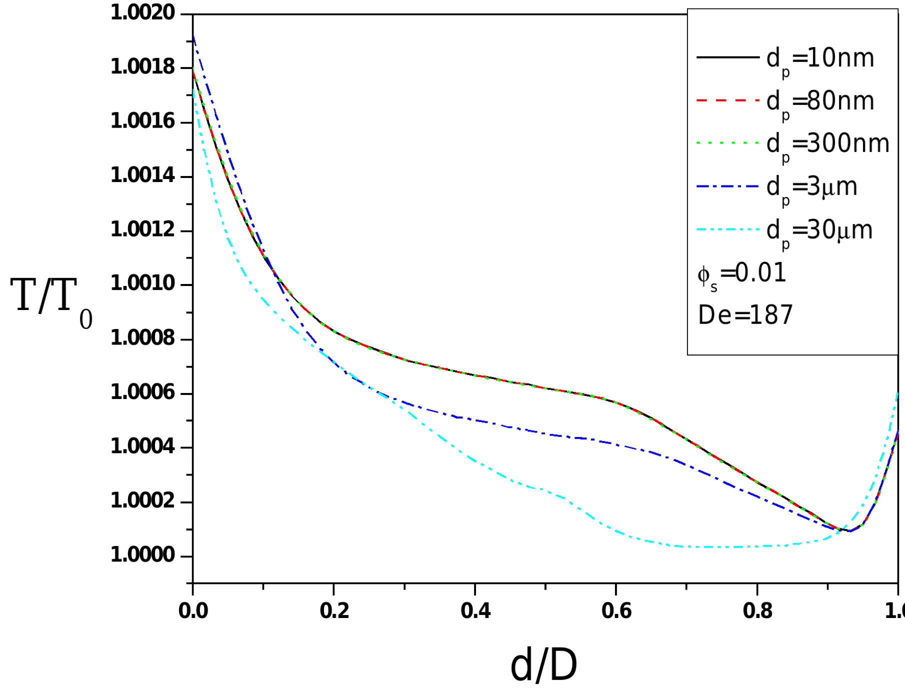

16.2
16.0
15.8
15.6
15.4
15.2
15.0
14.8
14.6
14.4
Nu_m
0
1x10^5
2x10^5
3x10^5
4x10^5
5x10^5
6x10^5
7x10^5
Gr
ϕ=0
ϕ=1%
ϕ=2%
Φ=3%
ϕ=4%
Re=300

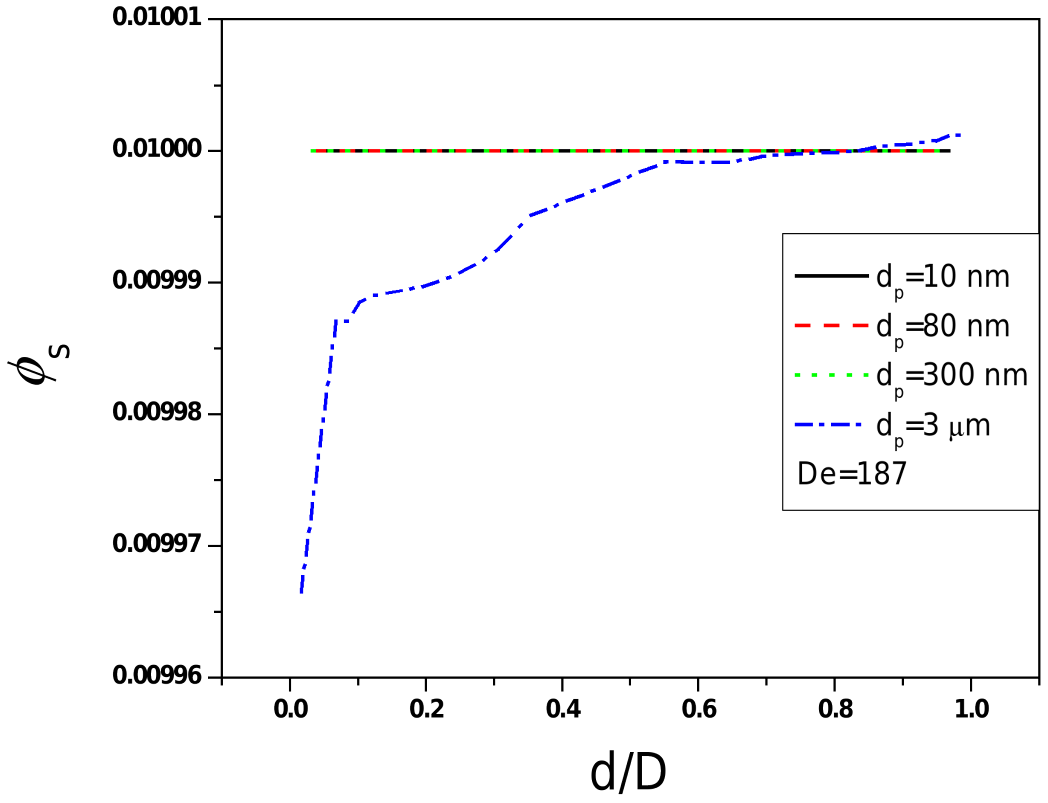

26.0
25.8
25.6
25.4
25.2
25.0
24.8
24.6
24.4
24.2
Nu_m
0.0
5.0x10^5
1.0x10^6
1.5x10^6
2.0x10^6
Gr
ϕ=0
ϕ=1%
ϕ=2%
Φ=3%
ϕ=4%
Re=900


0.0828
0.0826
0.0824
0.0822
0.0820
0.0818
f
m
0
1x10^5
2x10^5
3x10^5
4x10^5
Gr
ϕ=0
ϕ=1%
ϕ=2%
Φ=3%
ϕ=4%
Re=300


0.03905
0.03900
0.03895
0.03890
0.03885
0.03880
0.03875
0.03870
0.03865
0.03860
f
m
0.0
5.0x10^5
1.0x10^6
1.5x10^6
2.0x10^6
Gr
ϕ=0
ϕ=1%
ϕ=2%
Φ=3%
ϕ=4%
Re=900


2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
V_z/V_0
0.0
0.2
0.4
0.6
0.8
1.0
d/D
d_p=10nm
d_p=80nm
d_p=300nm
d_p=3μm
d_p=30μm
φ=0.01
De=187


1.0020
1.0018
1.0016
1.0014
1.0012
1.0010
1.0008
1.0006
1.0004
1.0002
1.0000
T/T_0
0.0
0.2
0.4
0.6
0.8
1.0
d/D
d_p=10nm
d_p=80nm
d_p=300nm
d_p=3μm
d_p=30μm
ϕ_s=0.01
De=187


0.01001
0.01000
0.00999
0.00998
0.00997
0.00996
ϕ_s
0.0
0.2
0.4
0.6
0.8
1.0
d/D
d_p=10 nm
d_p=80 nm
d_p=300 nm
d_p=3 μm
De=187

Thanks

? Questions