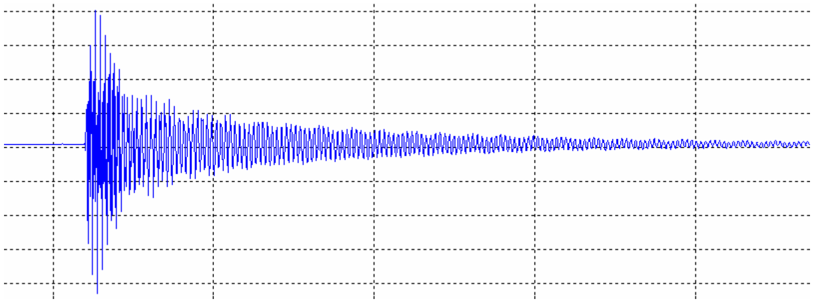
Signal Processing

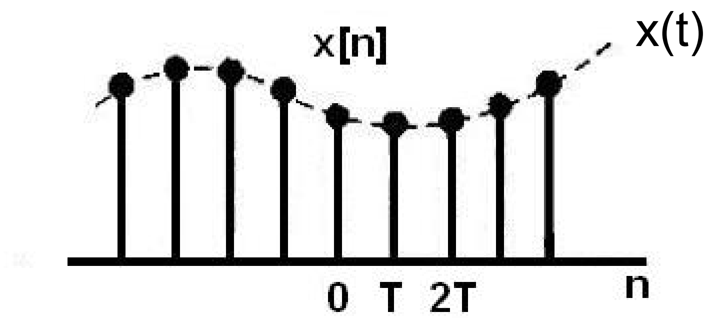


outline

- Sampling
- Aliasing
- Filters
- Example with MatLab

sampling

Sampling is the reduction of a continuous to a discrete signal.

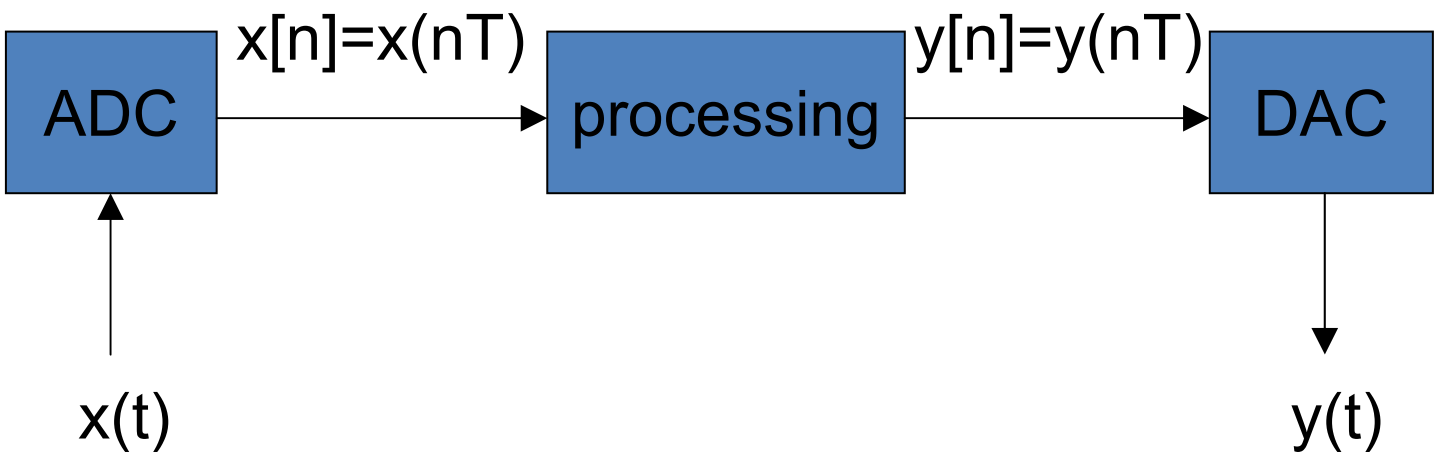


Sampled signal: x[n] = $x(nT)$, with n = 0, 1, 2, 3, ...

Sampling frequency: fs=1/T in Hz

sampling

In practice, the continuous signal is sampled using an analog-to-digital converter(ADC)



aliasing

Sinusoidal function

x(t)=Asin(2πft+φ), f>0

f = 400 Hz

375 Hz

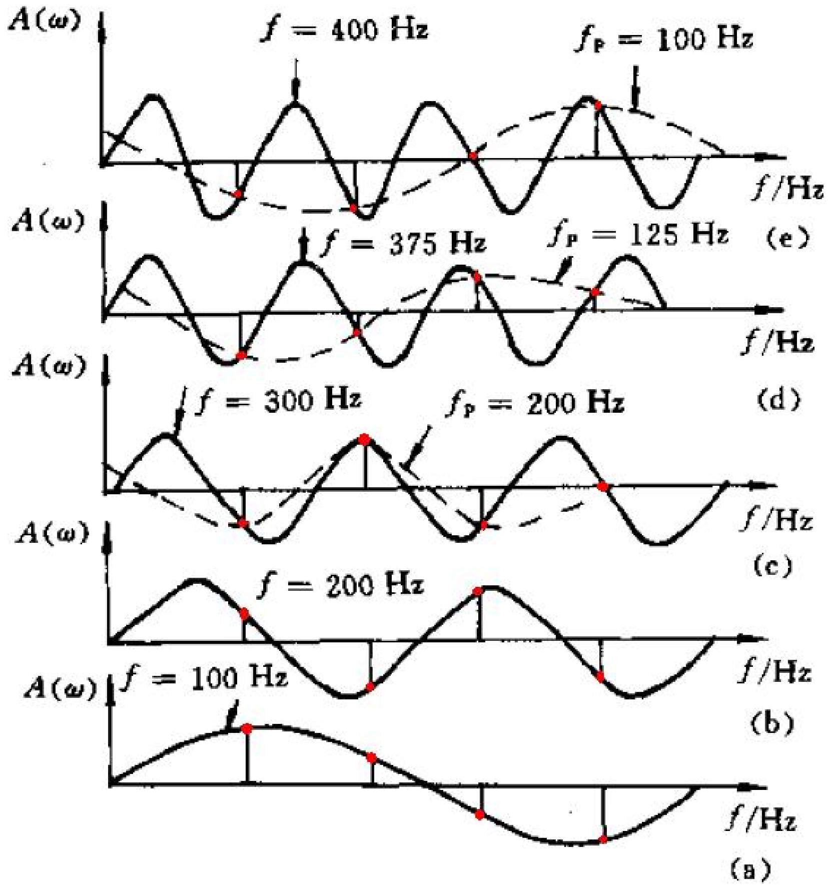
300 Hz

200 Hz

100 Hz

f_s = 500Hz

$\boxed{f_s \geqslant 2f}$



- With aliasing, the higher frequency signal has taken on the identity of the lower frequency.

Nyquist-Shannon sampling theorem

Let x(t) be a bandlimited signal with
$X(j2\pi f)=0$ for $|f_{max}|=f_N$

Then x(t) is uniquely determined by samples
$x[n]=x(nT)$, $n=0,1,2,......$, if $f_s \geq 2f_N$

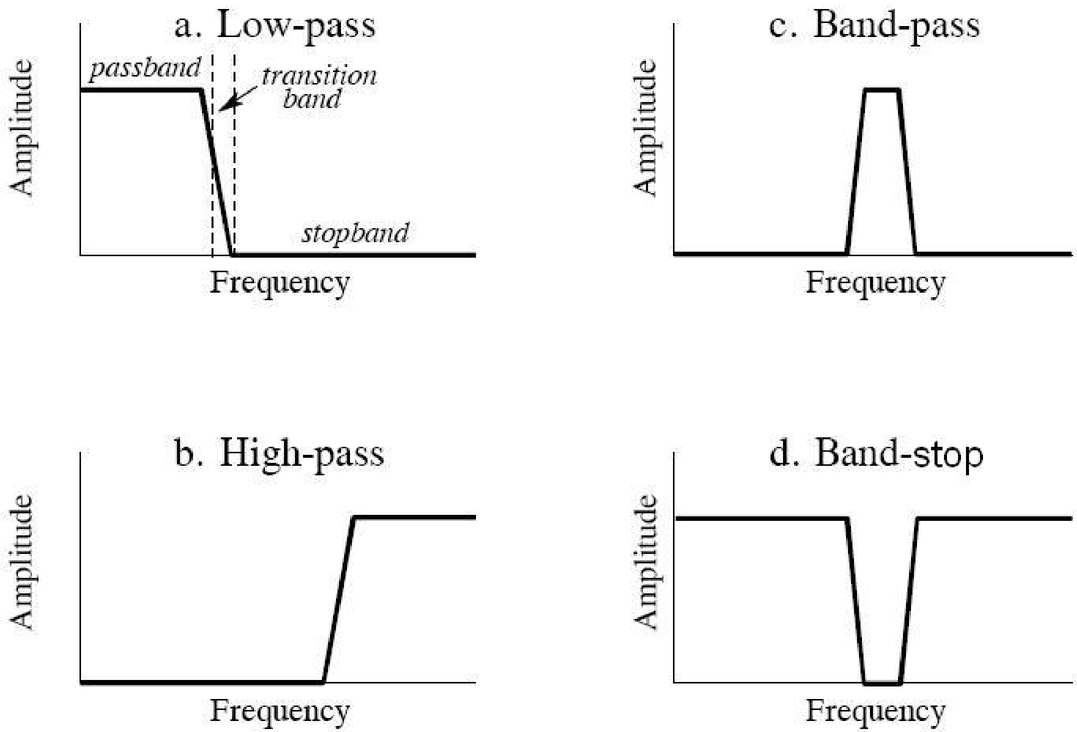
The frequency f_N is commonly referred to as the Nyquist frequency, and the frequency $2f_N$ that must be exceeded by the sampling frequency is called the Nyquist rate.

to prevent or reduce aliasing

- Increase the sampling rate, to above twice the highest frequency.
- Make the anti-aliasing filter (analog filter) before sampling the analog signal.
- There is no way to separate the original signal from the sampled signal, after aliasing.

digital filters

Filters are most often used to enhance signals by removing unwanted components from them.



software: http://www.falstad.com/dfilter/

Filters in Matlab

The operation performed by filter is described in the time domain by the difference equation:

$$y(n) = \begin{matrix} b(1)x(n) + b(2)x(n-1) + \ldots + b(nb+1)x(n-nb) \\ -a(2)y(n-1) - \ldots - a(na+1)y(n-na) \end{matrix}$$

An equivalent representation is the z-transform or frequency-domain description:

$$Y(z) = \frac{b(1) + b(2)z^{-1} + \ldots + b(nb+1)z^{-nb}}{1 + a(2)z^{-1} + \ldots + a(na+1)z^{-na}} X(z)$$

Butterworth low-pass filter

Butterworth filter:

```
[b,a]=butter(n,fc/(fs/2))
```

The function:

```
y=filter(b,a,x)
```

For example:

```
clear
t=(0:0.001:0.5);
x=sin(50*pi*t)+randn(size(t));
[b,a]=butter(10,30/500);
y=filter(b,a,x);
plot(t,x);
```

Thank you for your attention

Reference:

1. „Signals & Systems“, Alan V. Oppenheim, Alan S. Willsky, ISBN 0-13-651175-9
2. „Discrete-Time Signal Processing“, Alan V. Oppenheim, Ronald W. Schafer, ISBN 0-13-083443-2
3. „The Student Edition of MATLAB“, Prentice Hall, Englewood Cliffs, ISBN 0-13-855982-1