

# MATHEMATIK-ARBEITSPAPIERE

A: MATHEMATISCHE FORSCHUNGSPAPIERE

THE ASSOCIATION BETWEEN TWO RANDOM ELEMENTS:  
A COMPLETE CHARACTERIZATION  
IN TERMS OF ODDS RATIOS

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# The Association Between Two Random Elements: A Complete Characterization In Terms of Odds Ratios

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2. The Odds Ratio Function
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## Summary

For random elements  $X$  and  $Y$  (e.g. vectors or functions) a complete characterization of their association is given by specifying an association function (in terms of odds ratios) which uniquely determines their *joint* distribution within a broad class of distributions with *fixed marginals*. The main result establishes for any association function (satisfying integrability conditions) the existence of a corresponding joint distribution with pre-specified marginals. The proof is constructive in the sense that the joint density is obtained as a limit of an iterative procedure of marginal fittings. Two applications are given, the first deals with *models* for the association. And the second specialises to multivariate *normal marginals* and provides a characterization of multivariate *joint normal* distributions via their association function.

*Keywords:* association, Kullback-Leibler information, logistic regression, marginal fitting, multivariate normal distributions, odds ratio, proportional fitting.

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