

MATHEMATIK-ARBEITSPAPIERE

A: MATHEMATISCHE FORSCHUNGSPAPIERE

**SOME RESULTS ON CONVERGENCE OF MOMENTS
AND CONVERGENCE IN DISTRIBUTION WITH
APPLICATIONS IN STATISTICS**

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Some Results on Convergence of Moments and Convergence in Distribution with Applications in Statistics

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Summary

Many asymptotical results in statistics state only convergence in distribution although convergence of certain moments (like expectation and variance) are also of interest and sometimes no more difficult to derive. We consider notions of convergence which require convergence in distribution together with convergence of certain moments or even convergence of the moment-generating function of the norm. The common tools - like the central limit theorem, Slutsky's rules, Cramèr-Wold's device and the delta-method - for establishing convergence in distribution are extended to the stronger notions. For illustration, selected statistical applications are given concerning transformations of binomial rates, the log-odds-ratio, the linear model and sample quantiles (e.g. the median).

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