Some remarks on the stability of manufacturing logistic networks. Stability margins.

Bernd Scholz-Reiter[1], Fabian Wirth[2], Michael Freitag[1], Sergey Dashkovskiy[2], Thomas Jagalski[1], Christoph de Beer[1], Björn Rüffer[2]

[1] Universität Bremen, Department of Planning and Control of Production Systems, Germany {`bsr,fmt,boe,jag`}`@biba.uni-bremen.de`
[2] Universität Bremen, Zentrum für Technomathematik, Germany {`dsn,fabian,rueffer`}`@math.uni-bremen.de`

Summary. The increased complexity of production logistics systems and the requirement of a higher flexibility lead to a change of paradigm in the production logistics: The self controlled systems where decisions are taken autonomously become more attractive. The question of stability is an important issue for the dynamics prediction of such systems. In this paper we are going to touch this question for a special production scenario with self control. The stability region for a corresponding fluid model is found empirically. We point out that further mathematical investigations have to be undertaken to develop some stability criteria for self controlled systems.

1 Introduction

In view of modern technologies and networks complexity the idea to employ self control, i.e., to design a network as an interconnected autonomous units able to make decisions themselves, seems to be a new paradigm in logistics due to its flexibility and robustness [SRFW04], [DWJ04], [SRFdBJ05b], [SRFdBJ05a], [DRW05]. The networks with centralized control commonly used in past decades are well studied in the sense that there are different models like queuing, fluid and discrete models proposed. These models allow to predict the behavior of a system, its efficiency and supply the designer with criteria of stability cf. [Che95], [Dai95]. In this paper we concentrate on the stability properties of production networks with autonomous control. We consider the same production scenario as in [SRFdBJ05b] and [SRFdBJ05a] and state a fluid model for it. With help of simulation we find the stability area of parameters. In Section 2 we briefly describe the model. In the Section 3 we introduce the notion of stability and quote some known results. Section 4 contains simulation results. We collect some conclusions in Section 5.

2 Model description

The considered shop floor scenario is a dynamic flow-line manufacturing system. It consists of n parallel production lines each with m machines M_{ij} and an input buffer B_{ij} in front of each machine (see Figure 1). Every line processes a certain kind of product $A, B, \ldots, X$ by m job steps. The raw materials for each product enter the system via sources; the final products leave the system via drains. The production lines are coupled at every stage and every line is able to process every type of product within a certain stage. The decision about changing the line is made as an autonomous decision by the part itself on the basis of local information about buffer levels and expected waiting times until processing.

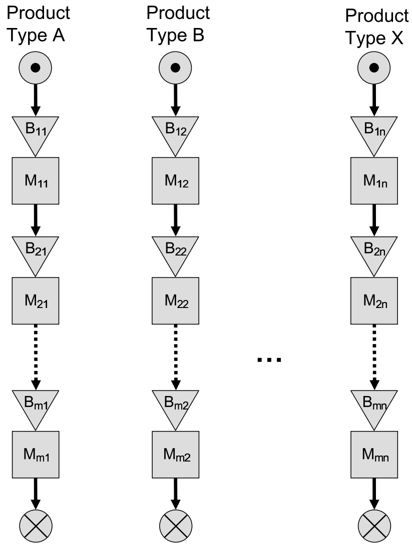

Fig. 1. $m \times n$ machines shop floor scenario.

To handle the complexity of the shop floor the described scenario is reduced to 3×3 machines, i.e., three production lines each with three stages. The parts are autonomous in their decision which machine to choose. They take into account the fact that the processing times are higher on foreign lines than on their associated production line. At each production stage the parts compare the processing times of the parts in the buffers and their own processing time on the respective machine and choose the machine with the minimal time for being processed. Table 1 shows the processing times and the resulting processing rates for the three different product types on the three production lines.

	Processing times [h:min]/ Processing rates [1/h] at production line n 1	2	3
Part Type A	2:00 / 0.5	2:30 / 0.4	3:00 / 0.33
Part Type B	3:00 / 0.33	2:00 / 0.5	2:30 / 0.4
Part Type C	2:30 / 0.4	3:00 / 0.33	2:00 / 0.5

Table 1. Processing times and resulting processing rates of the 3x3 machine model.

To analyse the system's behaviour at varying demand and workload fluctuations, an arrival function $\lambda(t)$ is defined and set as a sine function:

$$\lambda(t) = \lambda_m + \alpha \cdot \sin(t + \varphi) \tag{1}$$

Here, λ_m is the mean arrival rate, α is the amplitude of the sine function, and φ indicates a phase shift. The arrival functions for the three product types A, B and C are identical except for the phase shift of 1/3 period. This phase shift is chosen to simulate a seasonal varying demand for the three different products.

3 The notion of stability

There are several notions of stability for different models in the literature, see [Dai95] for Harris recurrence of queuing networks, [Che95] for week and strong stability of fluid models, [DRW05] for Input-to-State stability in control systems. Roughly speaking this properties mean that the states of the system (or length of the queues) remains bounded if the external input to the system is bounded.

Let us consider a queuing network with K classes of customers processed with service times $m_k,\ k = 1, \dots, K$; routing matrix P, external arrival rates $\alpha = (\alpha_1, \dots, \alpha_K)$ and let C_i be the set of classes processed on the server $i, \quad i = 1, \dots, I$. The effective arrival rate λ_k to the class k is given by $\lambda = (I - P')^{-1}\alpha'$. Then for some special networks and some special service disciplines the condition

$$\rho_i := \sum_{k \in C_i} \lambda_k m_k < 1 \tag{2}$$

was found to be a sufficient for stability of the corresponding network. However changing the discipline may cause instability, see [Bra94].

It is known that stability of fluid limit guarantees the stability of the corresponding queuing network. However if the fluid limit is unstable one can conclude nothing about the stability of queuing network. In the following we describe some simulation results concerning stability of the described model. The known criteria are not applicable for our model because of self controlled routing.

4 Simulation results

4.1 Stability region using fluid models

The fluid model for the 3×3 machines model is given by the following set of equations:

$$Q_{ijk}(t) = Q_{ijk}(0) + \int_0^t \alpha_{ijk}(s) - \mu_{ijk}\dot{T}_{ijk}(s)ds, \tag{3}$$

$$\alpha_{ik} \equiv \sum_{j=1}^{3} \alpha_{ijk}, \tag{4}$$

$$\alpha_{ijk}(t) = \alpha_{ik}(t) \cdot \mathbb{1}_{\left\{W_{ij}(t)+\frac{\alpha_{ik}(t)\cdot dt}{\mu_{ijk}} \leq \min_{l\neq j}\left\{W_{il}(t)+\frac{\alpha_{ik}(t)\cdot dt}{\mu_{ilk}}\right\}\right\}}, \tag{5}$$

$$\alpha_{i+1,k}(t) = \sum_{j=1}^{3} \mu_{ijk}\dot{T}_{ijk}(t), \text{ for } i = 1, 2, \tag{6}$$

$$\dot{T}_{ijk}(t) = \begin{cases} 0, & \text{if } Q_{ijk}(t) = 0, \\ \frac{Q_{ijk}(t)}{\sum_{l=1}^{3} Q_{ijl}(t)}, & \text{else,} \end{cases} \tag{7}$$

$$W_{ij}(t) = \sum_{k=1}^{k} \frac{Q_{ijk}(t)}{\mu_{ijk}}. \tag{8}$$

Here i and j denote the row and column of each machine, k refers to the product type, Q is the queue length in fractions of products, W the work load, μ is constant and describes the maximal possible service rates, T is the cumulative allocation time per machine and product type, it increases whenever the respective server spends time serving the corresponding type of part. α_{ik} denotes the arrival rate of part k in row i, which then is directed to exactly one machine.

Fluid limits do not capture individual parts, they can be seen as a macroscopic view of the process, such that all external arrival rates become their averages. The autonomous routing is captured in (5) under the restriction of (4), that is, the input rate can only turn to one machine at the time. By the macroscopic perspective we may assume (7), since in the FIFO queues of each server and by the intelligent routing algorithm the parts are assumed to be equally distributed, so that each server spends service time for each class as it relates to the total queue length in front of the machine.

Using standard discrete time numerical methods, we calculate estimates on the stability region of the fluid limit, which in turn is a subset of the stability region of the discrete event system, see Figure 2. Here we used an exemplary interarrival time of 144 minutes (2:24h as in [SRFdBJ05b, SRFdBJ05a]) for product type C and varied the interarrival times for types A and B.

4.2 Instability region using discrete event simulation

To analyse the stable parameter region using the discrete event simulation analogue to the continuous method the arrival rate for part type C is set as constant, in this case $\lambda_C = 0.4/h$. The other two arrival rates are still sine functions with an amplitude of $\alpha = 0.15$. The mean of the sine curves are independently varied, i.e., one of the mean arrival rates is held constant while

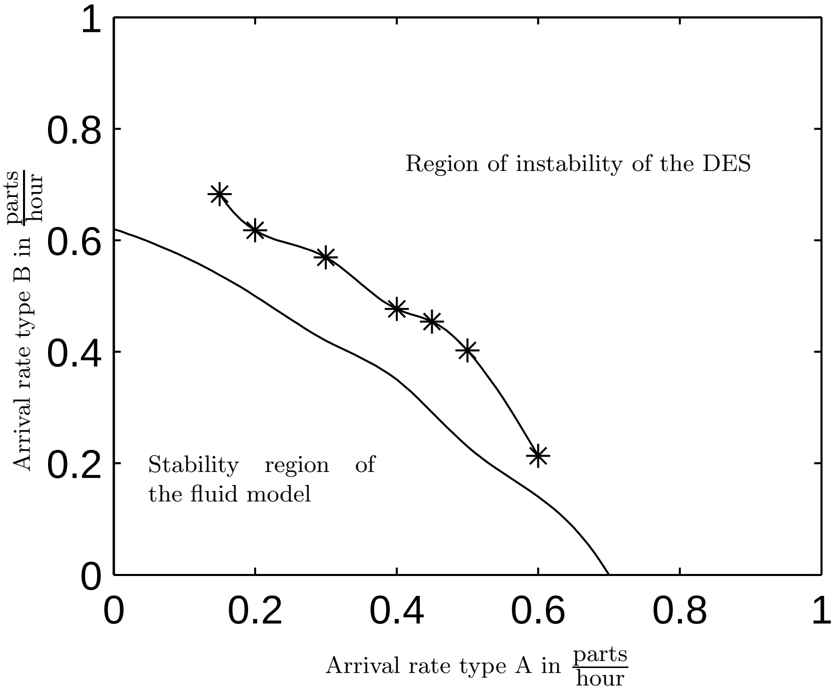

Fig. 2. Subset of the stability region of the 3×3 machine model with given interarrival rate of 0.4 parts of type C per hour.

the other is increased unless the buffer levels begin to rise to infinity. The maximum mean arrival rate before the buffer levels begin to rise to infinity is called the critical rate. The result of this stability analysis is shown in Figure 2.

The minimal mean arrival rate of type A is $\lambda = 0.15/h$ because the amplitude is $\alpha = 0.15/h$ and no negative arrival rates are allowed.

5 Conclusions

Mu nochmal umgeschrieben werden:

Arrival rates corresponding to the points above the curve in Figure **??** yield an unstable behavior. Comparing Figure 2 and Figure **??** we conclude that the set of parameters, λ_A, λ_B is into three parts: For input rates below the line in Figure 2 the system is stable, if the input rates are above the line in Figure **??** the system is not stable. Finally, between these two regions nothing is known about the stability of the 3×3 machine model.

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