Continuous models of logistics systems

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Unser Lehrstuhlteam

Forschung und Lehre:

- **Verkehrsträgerübergreifender und intermodaler Güterverkehr**
  Dynamik, Organisation und Wandel von Logistiknetzwerken
  Technologien und Umsetzung dezentraler Steuerung
  Integration in Supply-netzwerken
- **Distributions-, Umschlag- und Lagertechnik**
  Lösungen für Transport, Lager, Handhabung
  Prozessgestaltung und Technologie
  Einsatz von ID und Kommunikationstechnik
- **Modellierung, Simulation und Methodenentwicklung für vernetze Transport- und Logistiksysteme**
  Dynamische Modelle und Verfahren der Logistik
  Systematisierung von Lösungswerkzeugen

Entscheidungen rund um das Supply Netz

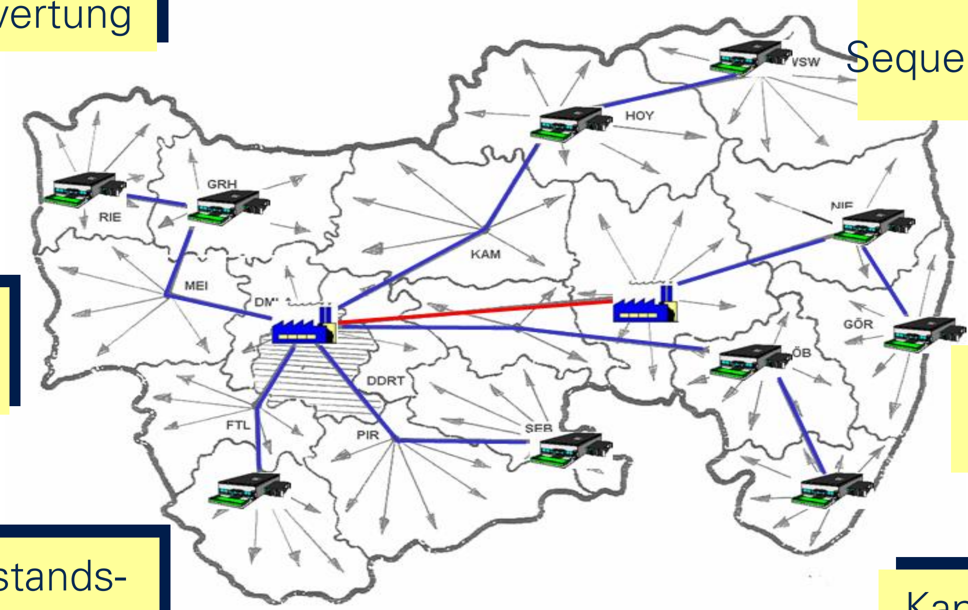
Standort-bewertung

Touren-planung

Sequenzierung

Cluster-analyse

Lagerstufen



Transport-relationen

Bestands-führung

Kapazitäts-problem

Anzahl der Knoten

Projekt: DHL HUB Leipzig

DHL Airhub Leipzig, Modellansicht Sortierhalle

Thema: Aufbau und Anlauf-management Sortierhalle

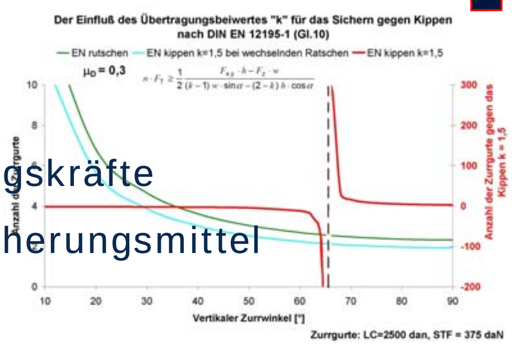
Laufzeit: 23.10.2006 - 22.5.2007

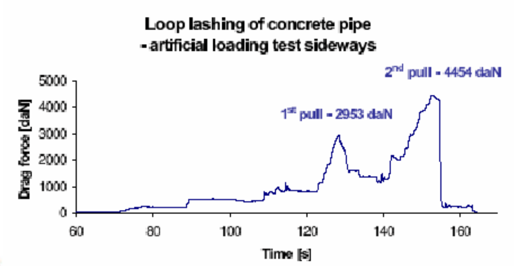
Quelle: DHL

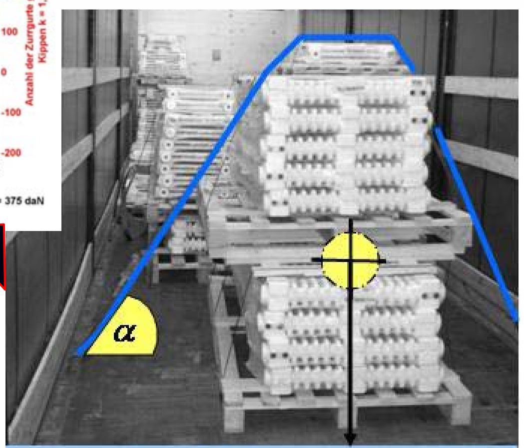
Projekt: Ladungssicherung

Support zur Umsetzung der Standards VDI 2700/DIN 12915:
„Die Ladung muss so gesichert sein, dass unter verkehrsüblichen Fahrzuständen weder einzelne Ladegüter noch die gesamte Ladung unzulässig verrutschen, kippen,…, verrollen darf.“

- Fehleranalyse
- Modellerstellung
- Kalkulation der Sicherungskräfte
- Dimensionierung der Sicherungsmittel







$$F_V = \sin\alpha\left(F_T + (k-1)F_T\right)$$

Stauplanung

+

Sicherungs - verfahren

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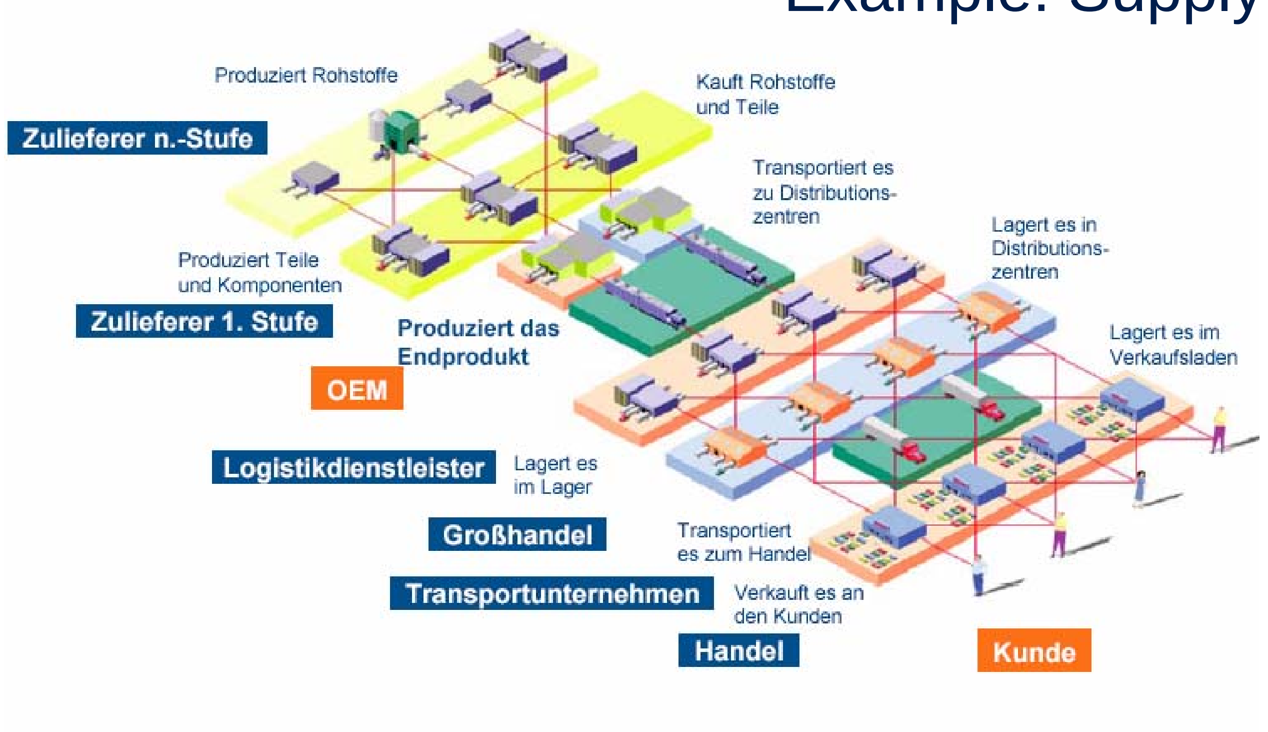
Dynamics in Production and Logistics

Operational level:

Material- and **information** flows

Supply Chain as a network structure

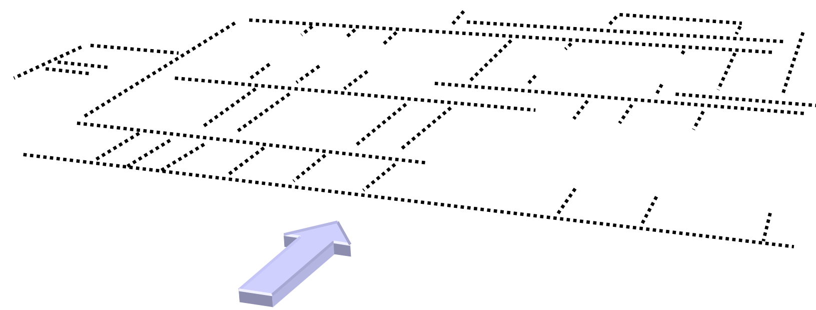
Example: Supply Chain



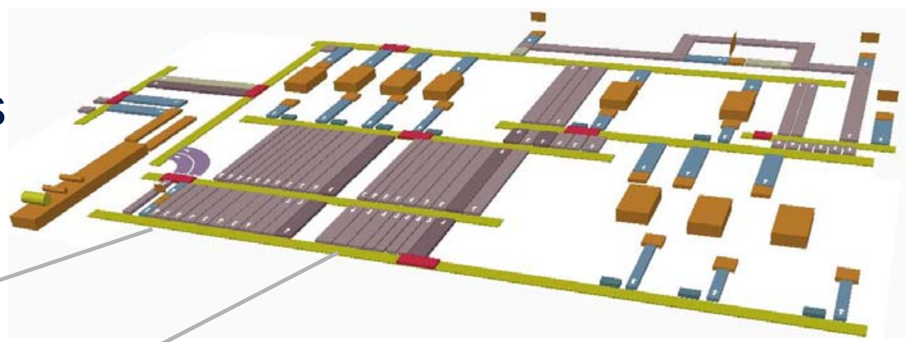
Example: Production System

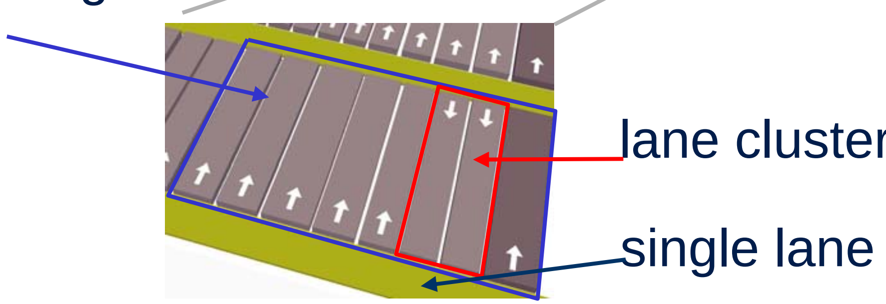
Network Features

➔ System represented as a directed graph

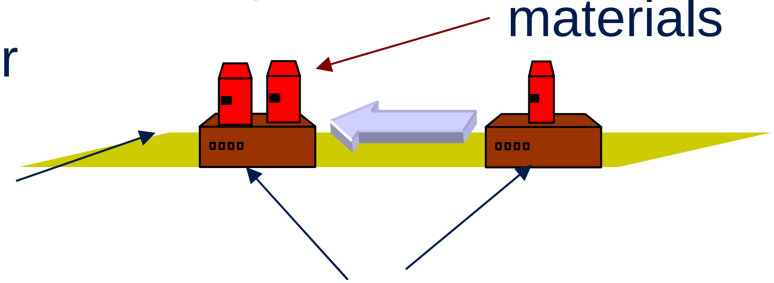


➔ buffer systems





➔ transport



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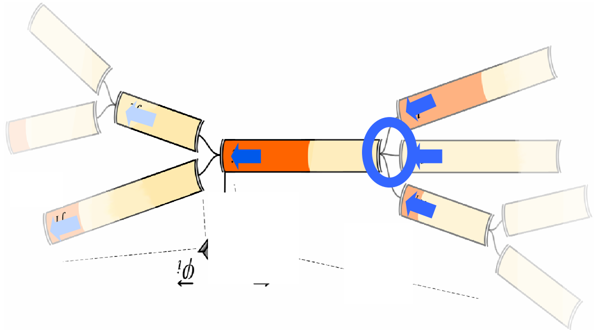
Characteristics of Logistic Systems

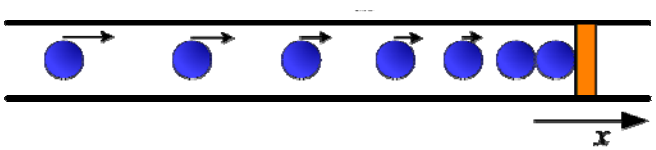
- **Systems of interacting queues**

  Conflicts in usage (e.g.) require priority rules and scheduling strategies which are adaptive to a varying demand.



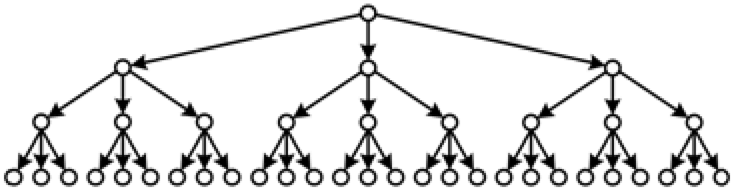
- **Driven many particle systems**

  Nonlinear interactions between intentionally moving units, mutual obstructions, ...



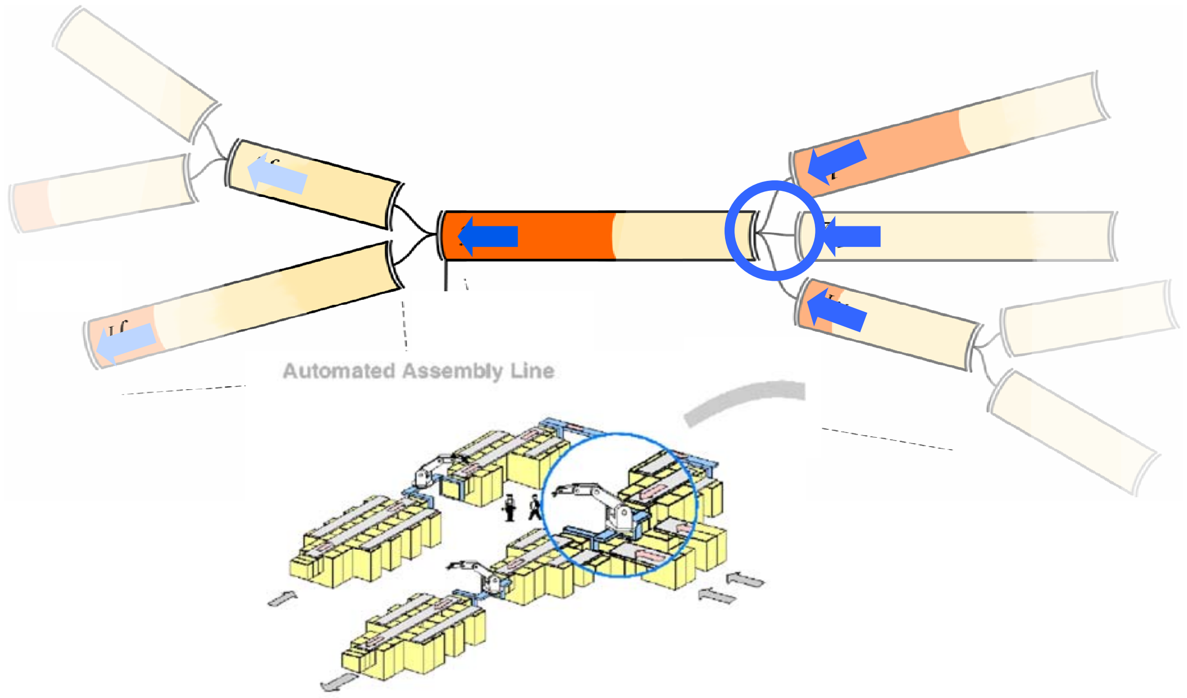
- **Dynamic flows on neworks**

  Dynamical systems as network nodes, coupled through material- and information flows



I
Discontinuities in Processes

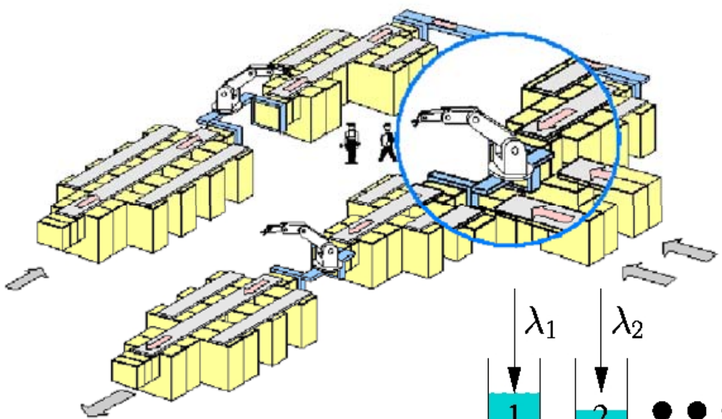
Control (Scheduling, dispatching, policies..) implies often Nonlinearty

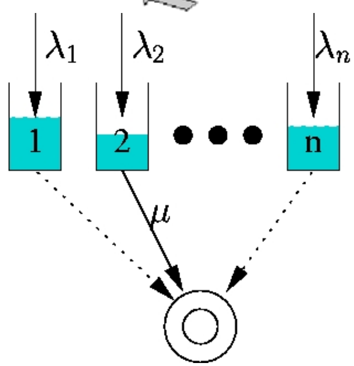


Manufacturing Systems

- Manufacturing systems are examples of a complex systems consisting of inter-acting queues
- Conflicts in usage (e.g. of workstations) require priority rules and scheduling strategies which are adaptive to a varying demand.

Automated Assembly Line





Policy

eg.: CAF Clear a fraction
CONWIP constant work in process

Model:
Continuous material flows
Switching policies

The parallel service of competing tasks is impossible or inefficient. Sequential switching between different opereation modes required

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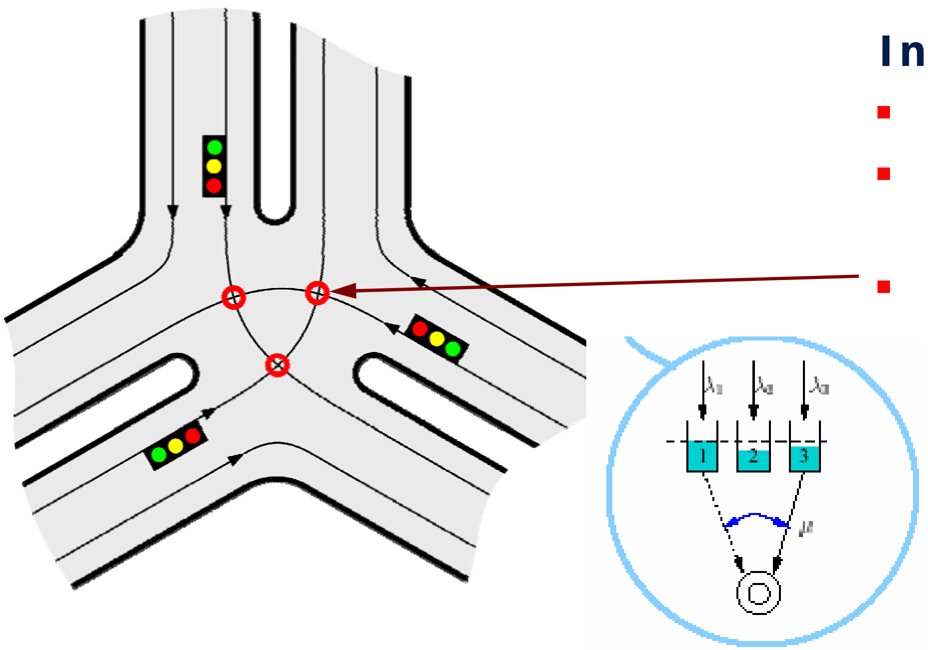
Traffic Flows

- Traffic is a prime example of a complex system consisting of inter-acting queues
- Conflicts in usage (e.g. of intersection areas) require priority rules and scheduling strategies which are adaptive to a varying demand.



Intersection

- Mutually excluding flows
- Controlled by switching traffic ligths
- Each intersection point gives one more side condition

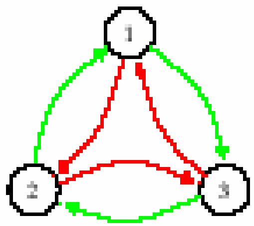
Hybrid Dynamical Systems

Continuous

Dynamical System

$$\dot{x} = f(x)$$

Discrete Automaton



Hybrid Dynamical System

continuous + discrete dynamics

set of (simple) dynamics + switching rules

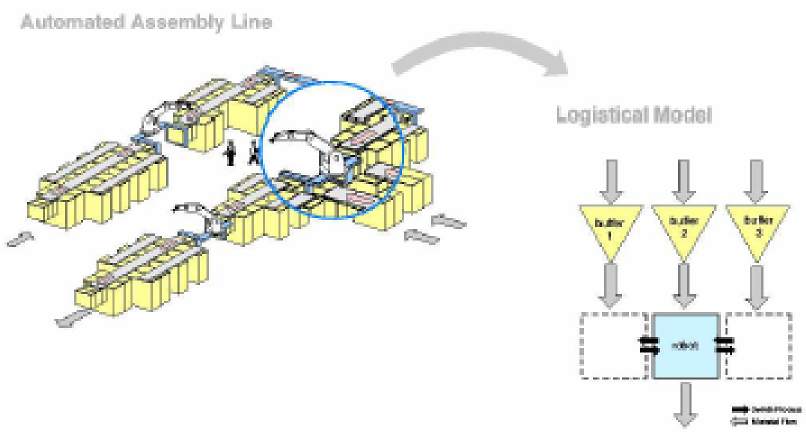
$\dot{x} = f(x, q)$ $x \in \mathbb{R}^n$, $q \in Q$, Q countable , finite

trajectory:

$(x(t), q(t))$ where $q(t) = q_i$ for $t_{i-1} < t \leq t_i$

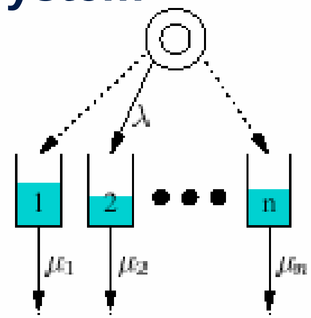
t_i's: switching times (point events)

Examples



Ein Drei Puffer Switched Server System als Teil einer automatisierten Produktionslinie

Switched arrival system



balanced $\sum_i \mu_i = \lambda$

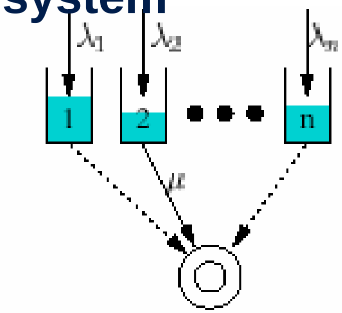
normalised
$\lambda = 1, \sum_i x_i = 1$

velocities
$\dot{x}_q = (e_q - \mu), q = 1,2,3$

switching
$(x_q = 0) \to$ fill q

Chaotic dynamics

Switched server system



balanced $\sum_i \lambda_i = \mu$

normalised
$\mu = 1, \sum_i x_i = 1$

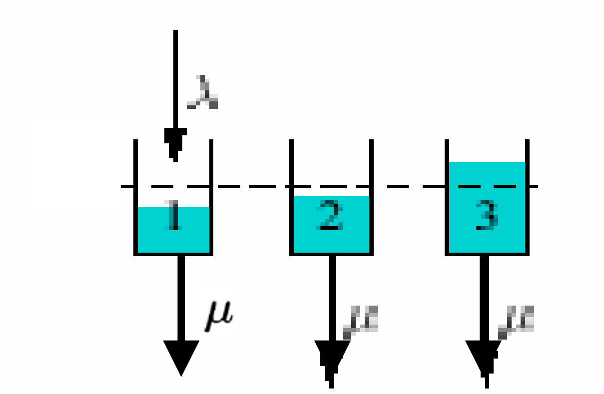
velocities
$\dot{x}_n = (\lambda - e_n), q = 1,2,3$

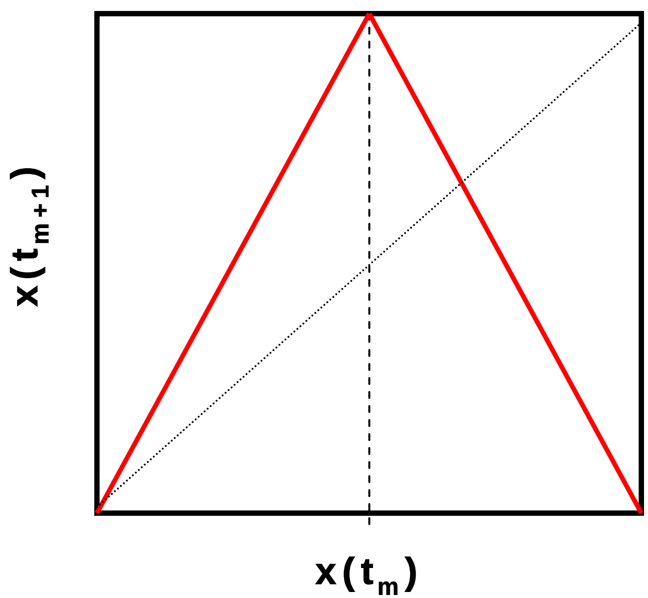
switching
$(x_q = 0) \to$
serve $[q+1]_{modN}$

Periodic dynamic

Switched Arrival is Chaotic

(a simple picture)





Consider N=3, and $\lambda=3$, $\mu=1$

We are interested in sucessive „initial“ filling levels :
→ sample at detachment times t_m

$$x(t_{m+1}) = T_m \lambda$$

$$x_i(t_m) - T_m \mu = 0$$

or:

$$1 - x_i(t_m) - T_m \mu = 0$$

$$T_m = t_{m+1} - t_m$$

$$\rightarrow x(t_{m+1}) = 1 - 2|x(t_m) - 0.5|$$

The dynamics of sucessive filling levels is given by the **tent map**

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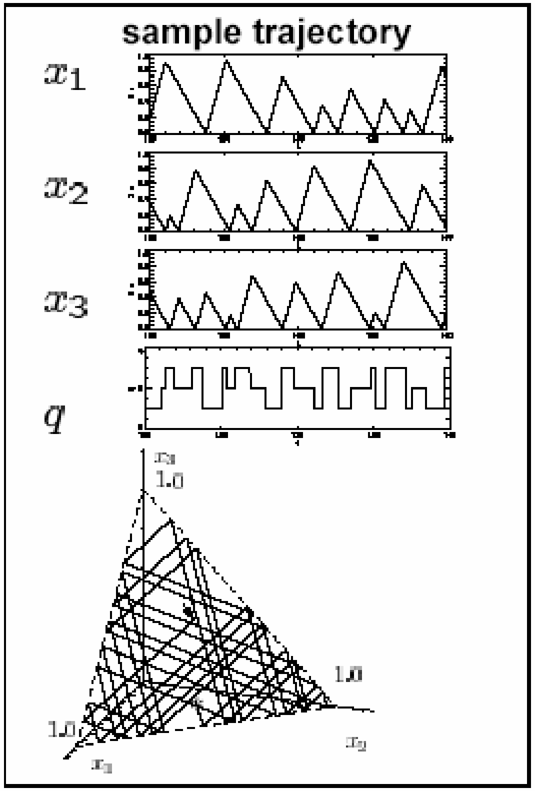
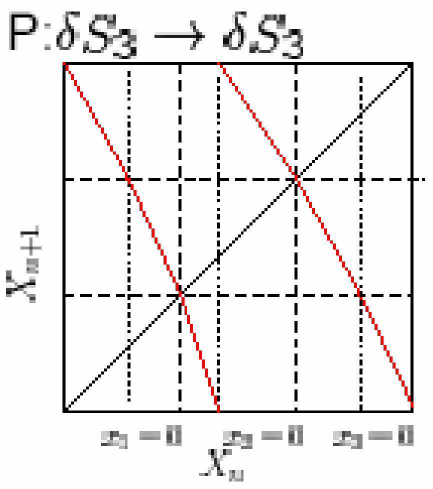
Switched Arrival Systems: Poincaré Maps



hyperplane: $\sum_i x_i = 1$

velocities: $\dot{x}_q = (e_q - \mu)$

q^+ unique at boundary

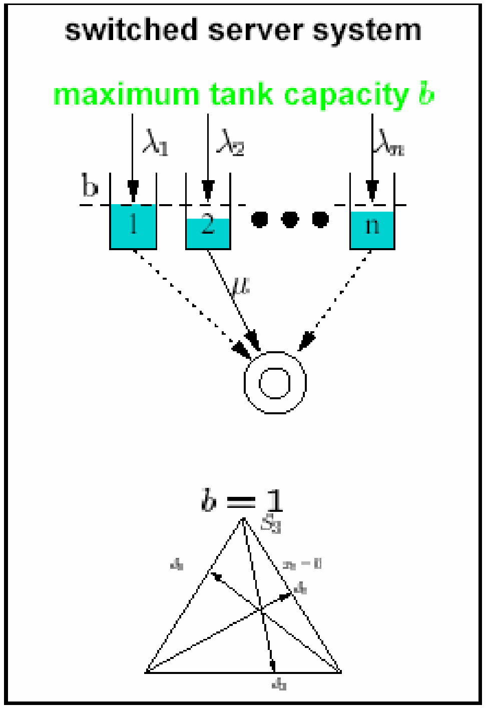


Poincaré map: sample the system at switching times t_i

$\delta S_n \to \delta S_n$

$n = 3 : (0,1) \to (0,1) : X_n \mapsto X_{n+1}$

Switched Server – restricted Buffer



balanced: $\sum_i \lambda_i = \mu$

normalized:

$\mu = 1, \sum_i x_i = 1$

velocities: $\dot{x}_q = (\lambda - e_q)$

switching rule 1: [free]

$(x_q = 0) \rightarrow$ serve $[q+1]_{modN}$

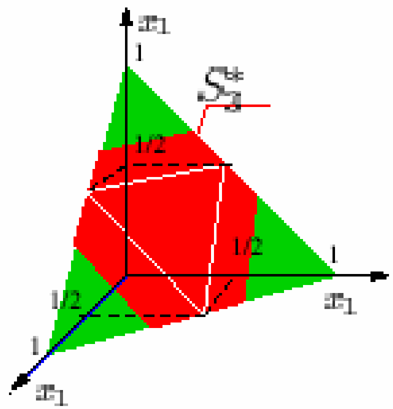
switching rule 2:

$(x_q = b) \rightarrow$ serve q

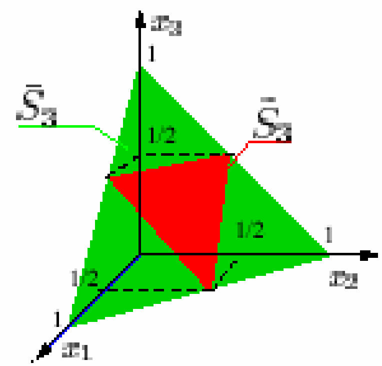
The continuous state evolves linear inside S^*:

$$S_n^*(b) = \{x \in \mathbb{R}^n | \sum_i x_i = 1, 0 \leq x_i \leq b; \text{ for } i = 1, ..., n\}$$

$1 > b > 1/2$



$b = 1/2 \quad b = 1$



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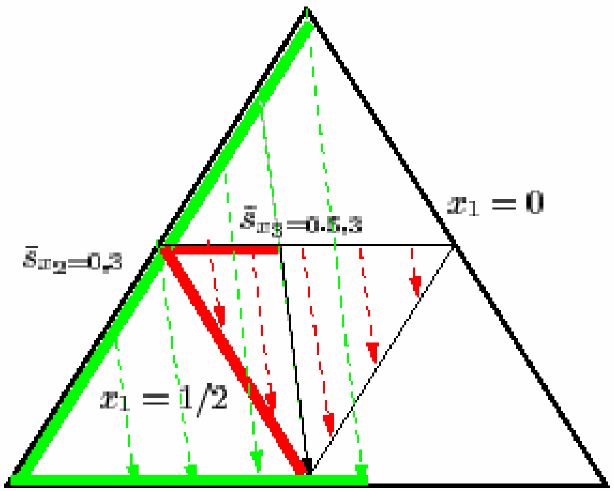
Limiting Cases



The dynamics of a **switched server systems with $b = 1/(n-1)$** is (with mirorred handedness and a scaling factor) the dynamics of a **switched arrival system with $b = 1$** and vice versa.

For $n > 2$ two limiting cases: (**n-Simplexes**):

$b = 1$

$\bar{S}_n = \{x \in \mathbb{R}^n | \sum_i x_i = 1, 0 \leq x_i \leq 1;\}$.

$b = 1/(n-1)$

$\bar{S}_n = \{x \in \mathbb{R}^n | \sum_i x_i = 1, 0 \leq x_i \leq 1/(n-1);\}$

Restricted buffer sizes can cause chaos !

Bifurcations in Switched Server Systems

changing switching threshold b:

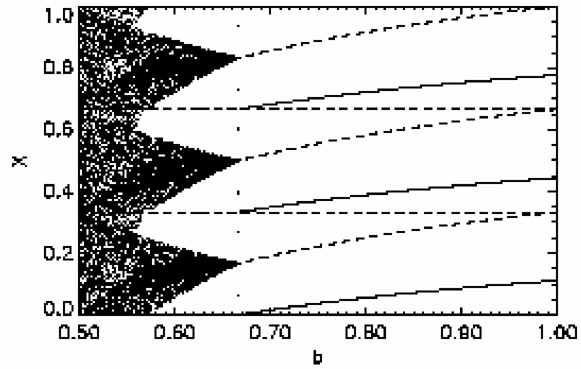
shifted branches

bifurcations

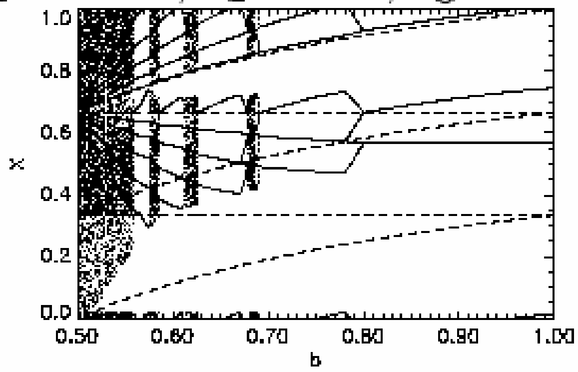
appeareance of new reachable branches

appearence of new orbits

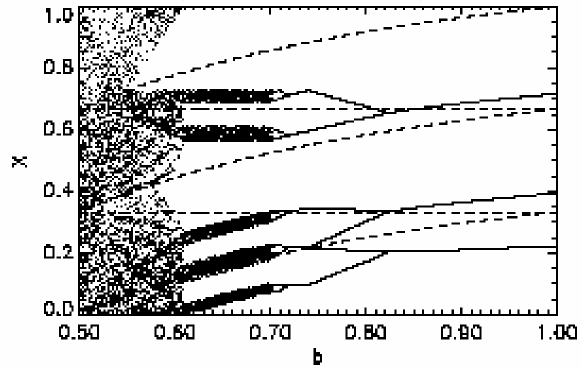
$\lambda_1 = \lambda_2 = \lambda_3 = 1/3$



$\lambda_1 = 0.15, \lambda_2 = 0.8, \lambda_3 = 0.05$



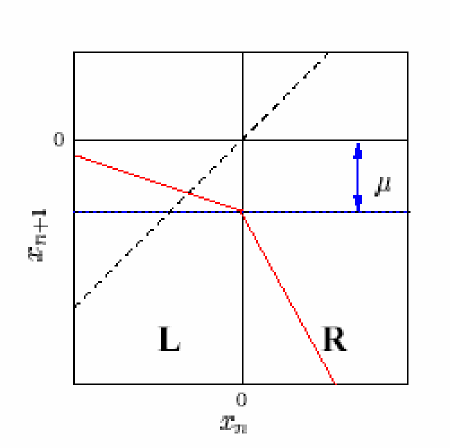
$\lambda_1 = 0.49, \lambda_2 = 0.11, \lambda_3 = 0.4$



The dynamics depends on λ and b.

Border Collision Bifurcations

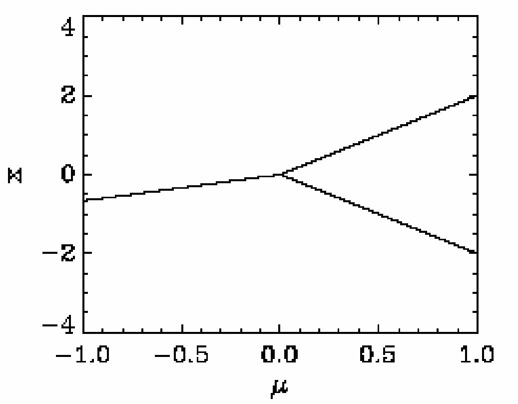
Period doubling Bifurcation

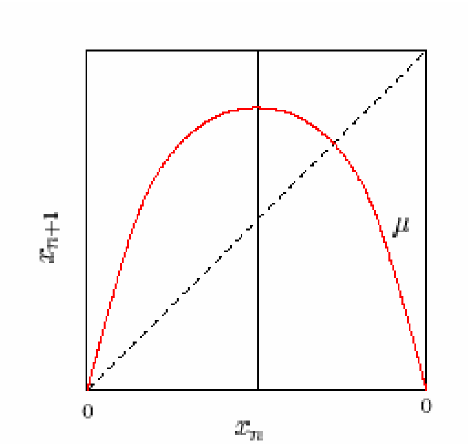


Continous, piecewise linear map:
Border Collision Bifurcation

$$f(x) = \begin{cases} f_L(x) = & a_L x + \mu \quad \text{für} \quad x \le 0 \\ f_R(x) = & a_R x + \mu \quad \text{für} \quad x > 0. \end{cases}$$

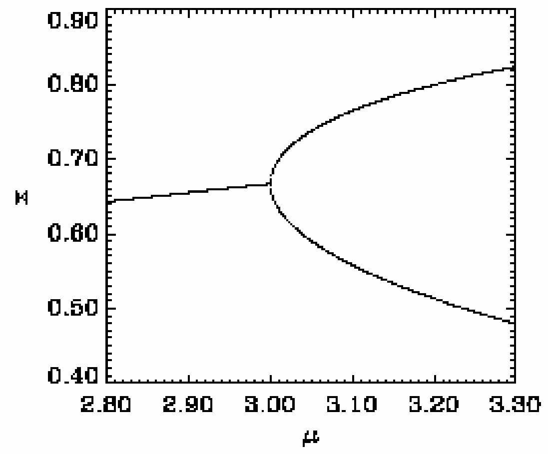
$-1 < a_L < 0$, $a_R < -1$ und $a_L a_R < 1$





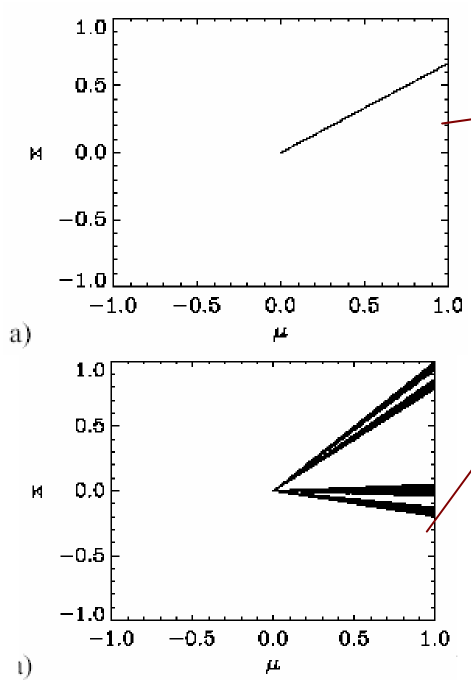
Continuous map:
period doubling cascade

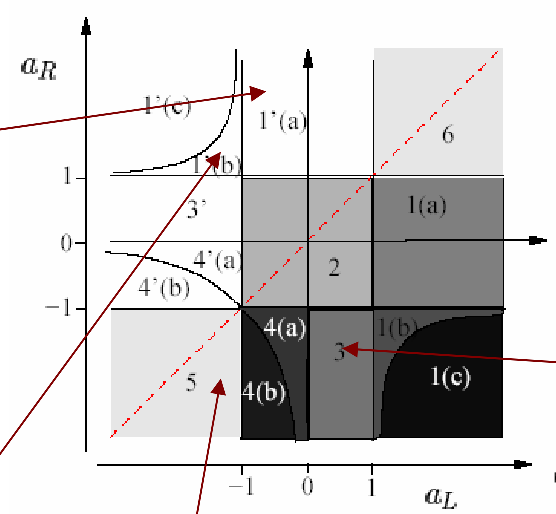
$$f(x) = \mu x(1 - x)$$



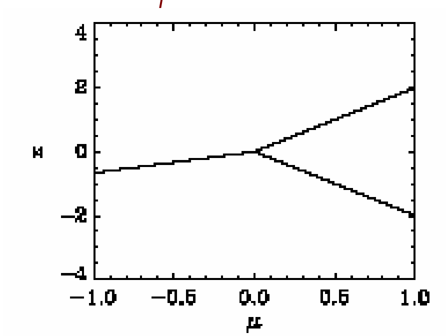
Border Collision Bifurcations

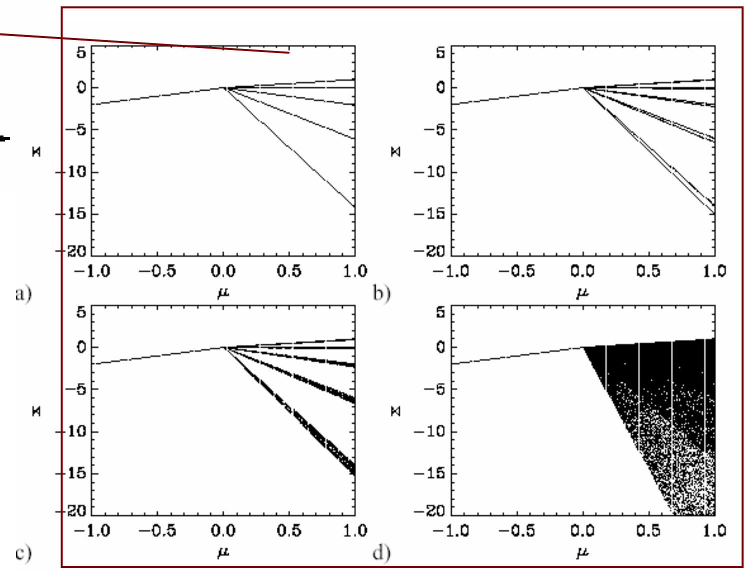
Continuous, non-smooth 1 dimensional maps, two branches





$$f(x) = \begin{cases} f_L(x) = a_L x + \mu & \text{for } x \leq 0 \\ f_R(x) = a_R x + \mu & \text{for } x > 0. \end{cases}$$





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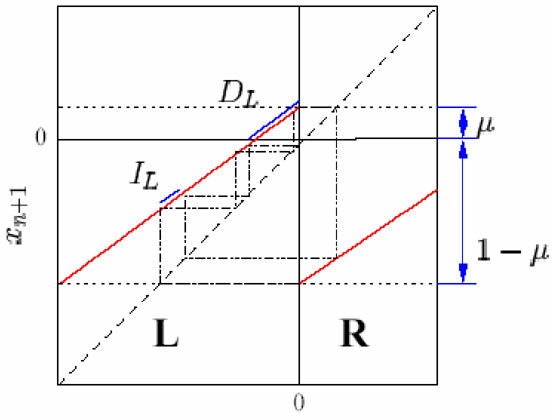
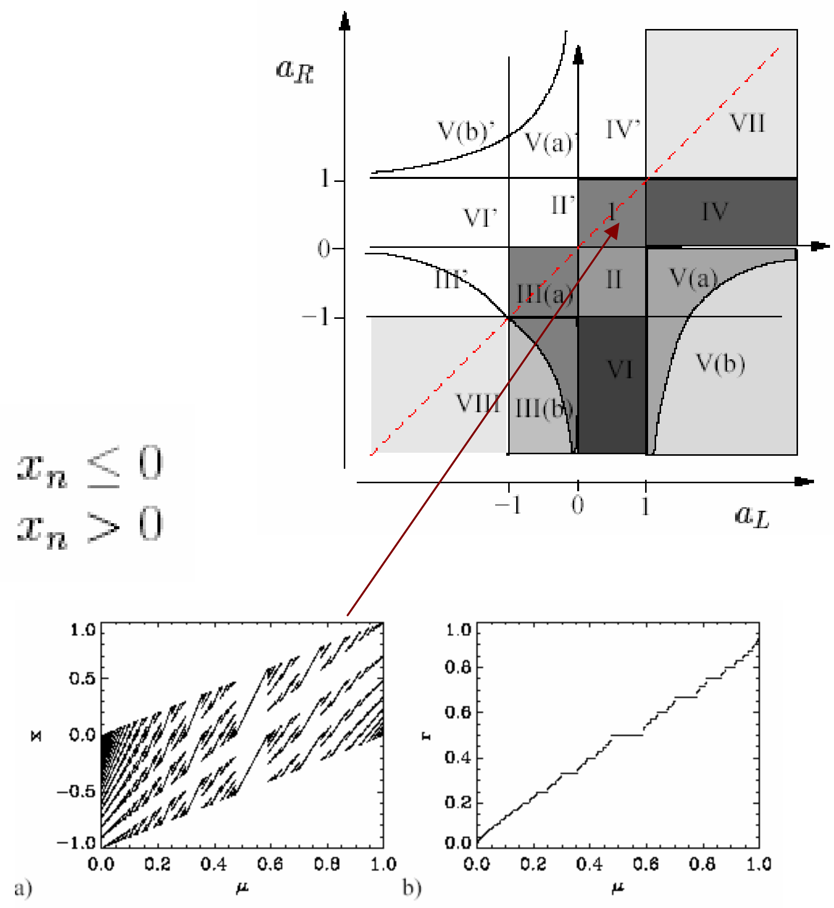
Border Collision Bifurcations

discontinuous, non-smooth 1 dimensional maps, two branches



$$x_{n+1} = \begin{cases} a_L x_n + \mu & \text{for} \quad x_n \leq 0 \\ a_R x_n - (l - \mu) & \text{for} \quad x_n > 0 \end{cases}$$



Perfomance measures

Utilization Factors

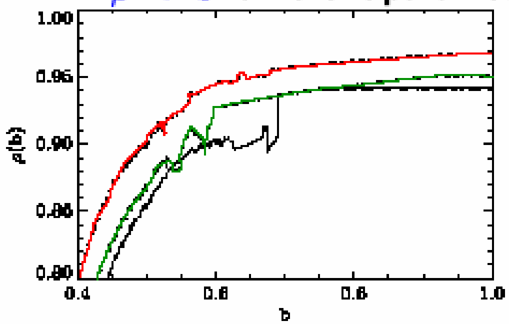
For systems with set-up times

utilization factor: $\rho = \lim_{T\to\infty} \frac{\int_0^T dt\,\mu(t)}{\int_0^T dt\,\mu_0}$

- small $b \to$ decreased utilization
- different attractors $\to$ jumps in utilization factors
- increased $\tau \to$ decreased utilization

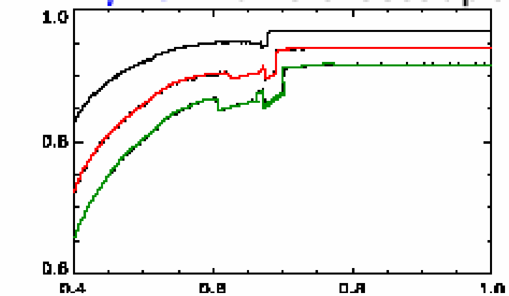
Small buffers cause production losses.

ρ vs. b: different parameters λ_i



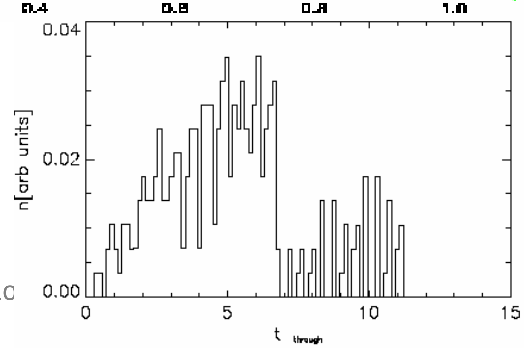
λ_1	λ_2	λ_3
0.3	0.4	0.3
0.1	0.6	0.3
0.1	0.8	0.1

ρ vs. b: different set-up times τ



λ_1	λ_2	λ_3
0.3	0.4	0.3

$\tau = 0.1$
$\tau = 0.2$
$\tau = 0.3$



Chaos? - not really!

Discrete Material Flow

Consider discrete parts of material

Example

model:

inter arrival times: ϑ_i (fixed)

inter departure times: Θ, (fixed)

switching rules

balanced

Deterministic Queuing Model

- A fundamentally other type of dynamical system
- no deterministic chaos possible
- no normalization feasible

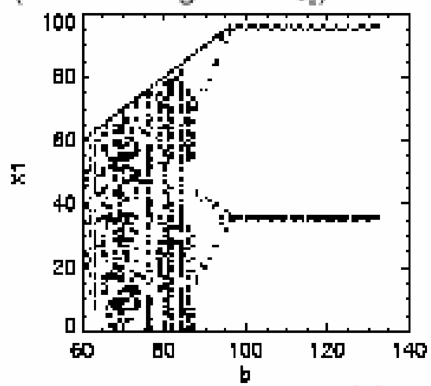
ϑ_1	ϑ_2	ϑ_3
0.3	0.5	0.3

$$\Theta = 0.1153$$

total content: 132 ± 1

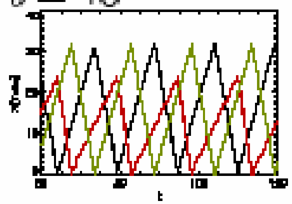
Content of buffer 1

(at switching times t_i)

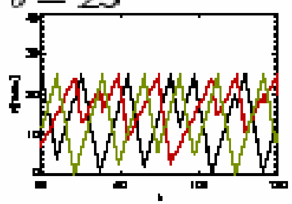


total content: 42 ± 1

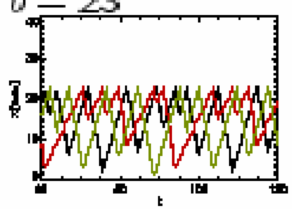
$b = 40$



$b = 25$



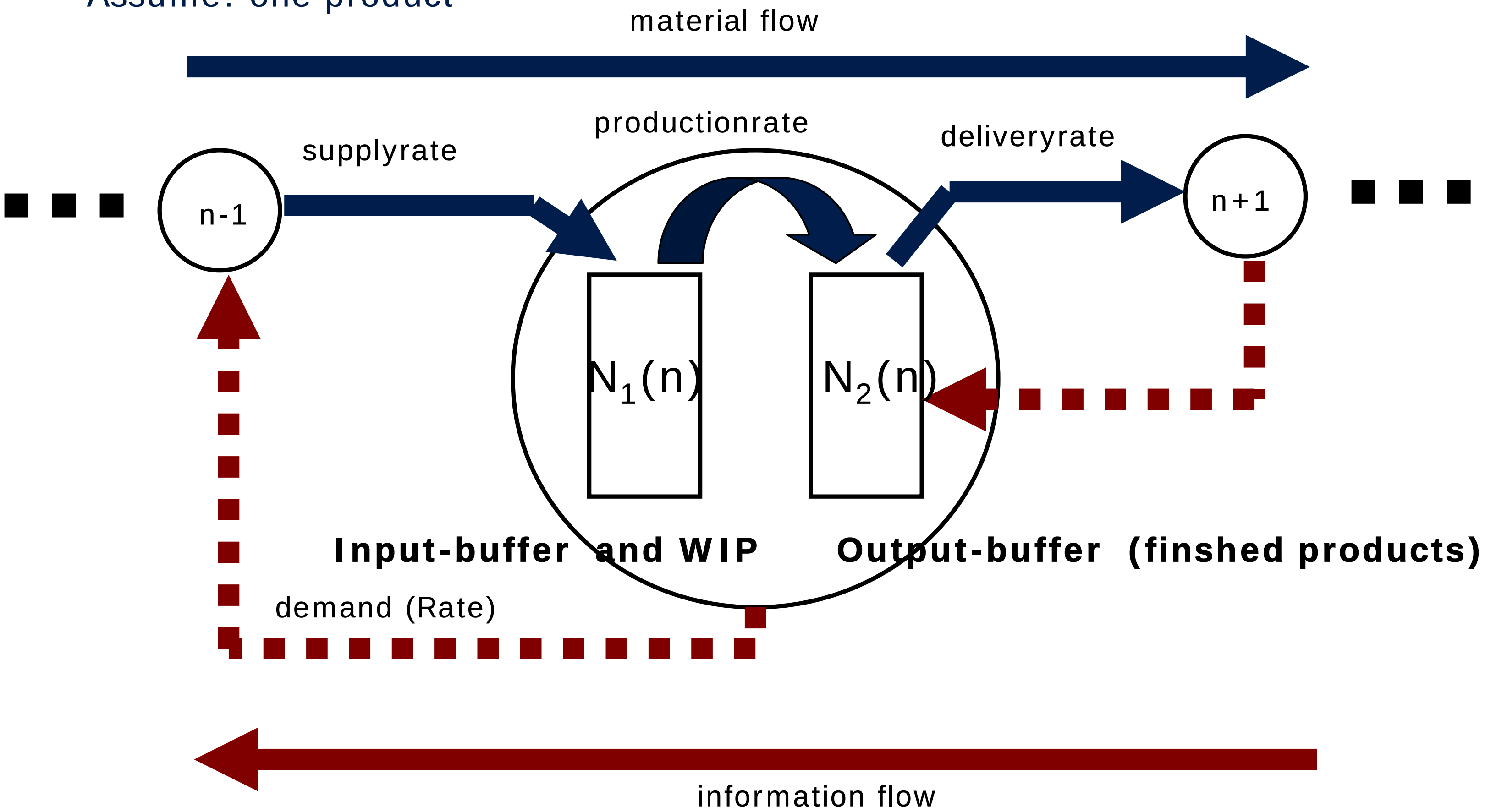
$b = 23$



Basic dynamical features are conserved !

Supply Chain:Modelling of Nodes

Assume: one product



Modelling of Nodes: Production

The **Input buffer** N_1 at producer b changes in time t according to

$$\frac{dN_1(b)}{dt} = Q_b^{in}(t) - Q_b^{prod}(N_1, t)$$

$Q_b^{in}(t)$… rate at which producer b receives ordered products from supplier $b-1$

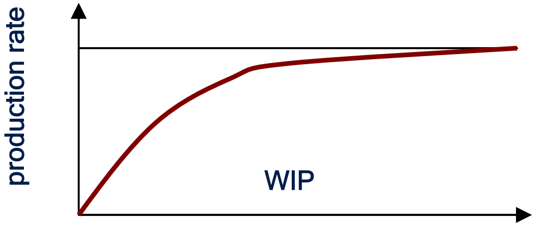
which is $\frac{1}{\tau} Q_b^{dem}(t)$ … if the inventory of supplier $b-1$ is not 0 !

If the supplier has no finished products in stock, it is: $\frac{1}{\tau} Q_{b-1}^{prod}(t)$

$Q^{dem}{}_b$ is the desired delivery rate (the **order rate**). Its **adaptation** takes on average some **time** interval τ.

$Q_b^{prod}(N_1, t) = P(N_1(b,t))$ … is the actual production rate of producer b,

$P(N_1(b,t))$ … is the production function of suited form



Modelling of Nodes: Demand Prediction

The **Inventory buffer** N_2 at **producer** b changes in **time** t according to

$$\frac{dN_2(b)}{dt} = Q_b^{prod}(N_1, t) - Q_b^{out}(t)$$

The **temporal change of the demand rate** is proportional to the deviation of the actual **delivery** from the desired one W_b (the **order rate**) and the demand of the upstream producer.

$$\frac{dQ_b^{dem}}{dt} = c_1(W_b(t) - Q_b^{in}(t)) - c_2(Q_b^{prod}(t) - Q_{b+1}^{dem})$$

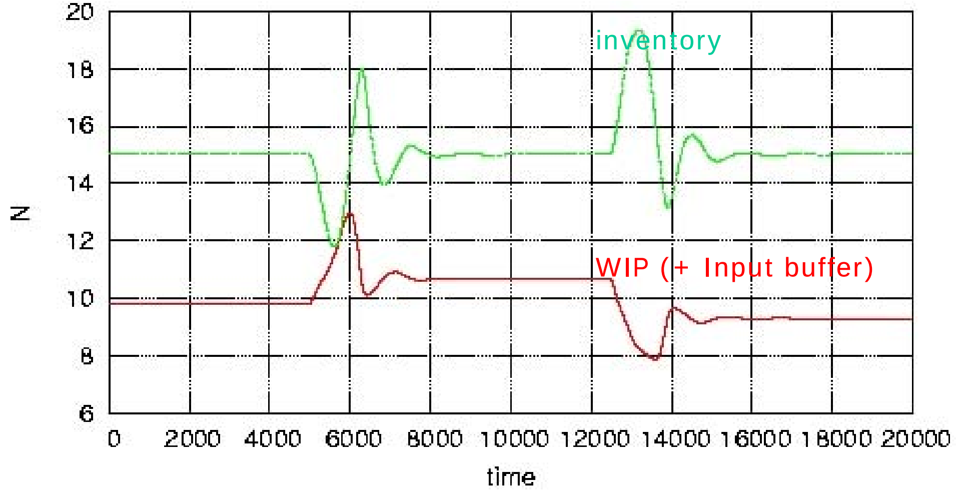
with $W_b(t) = W_b(\{N_a(t)\}, \{dN_a(t)/dt\}) = W(N_{(b)}(t))$

$N_{(b)}(t) = \sum_{c=-n}^{n} w_c \left(N_{b+c} + \Delta t \frac{dN_{b+c}}{dt} \right)$ is a **weighted mean value** of the own stock level and the

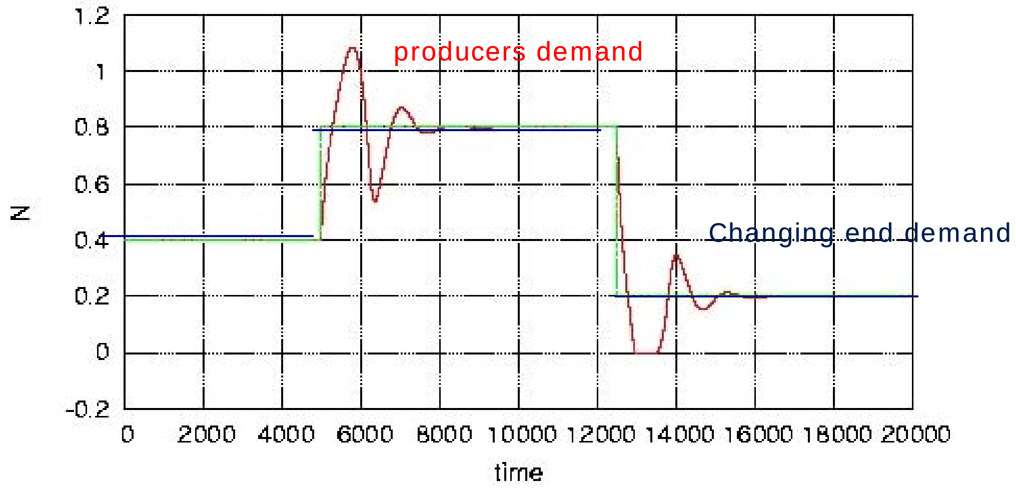
the ones of the next n upstream and n downstream suppliers. The weights w_c are normalized to one: $\sum_{c=-n}^{n} w_c = 1$.

Example: linear supply chain

Dynamics for the second producer in a chain with changes in consumer demand .



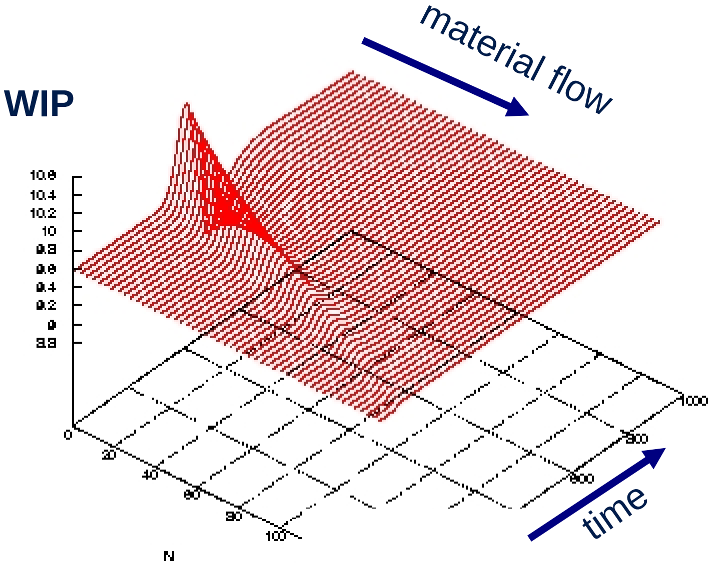
Demand jumps cause damped nonlinear oscillations in the adaption process



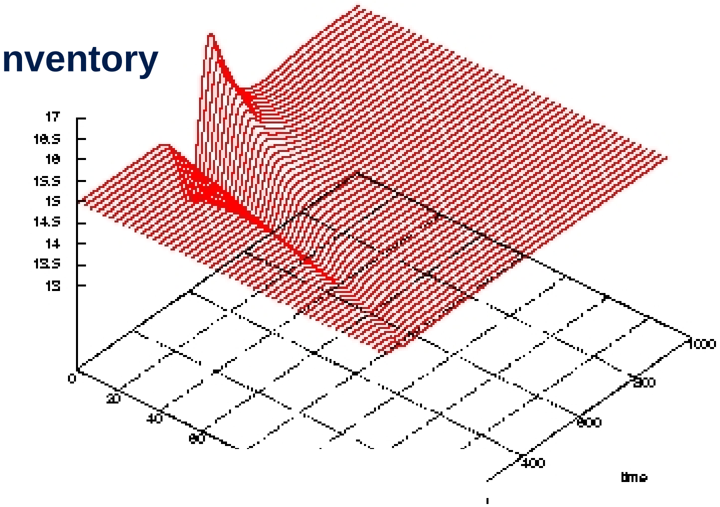
K. Peters, 2007

Bull whip Effect

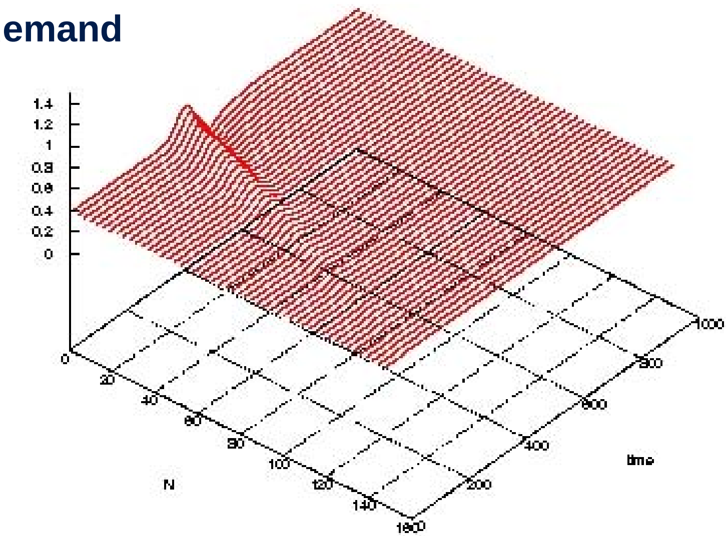
WIP



inventory



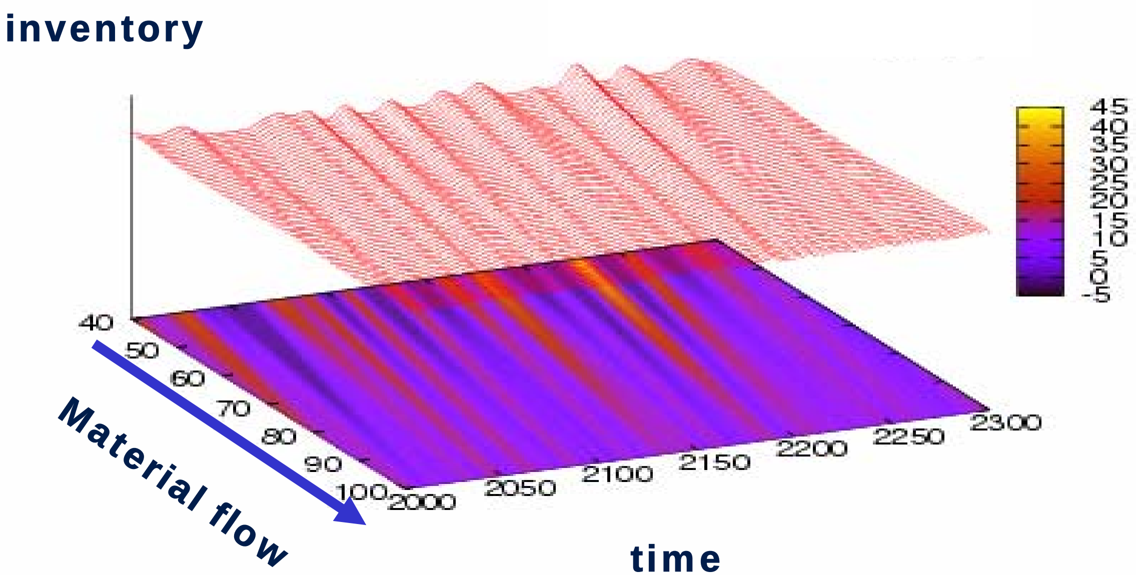
demand



K. Peters, 2007

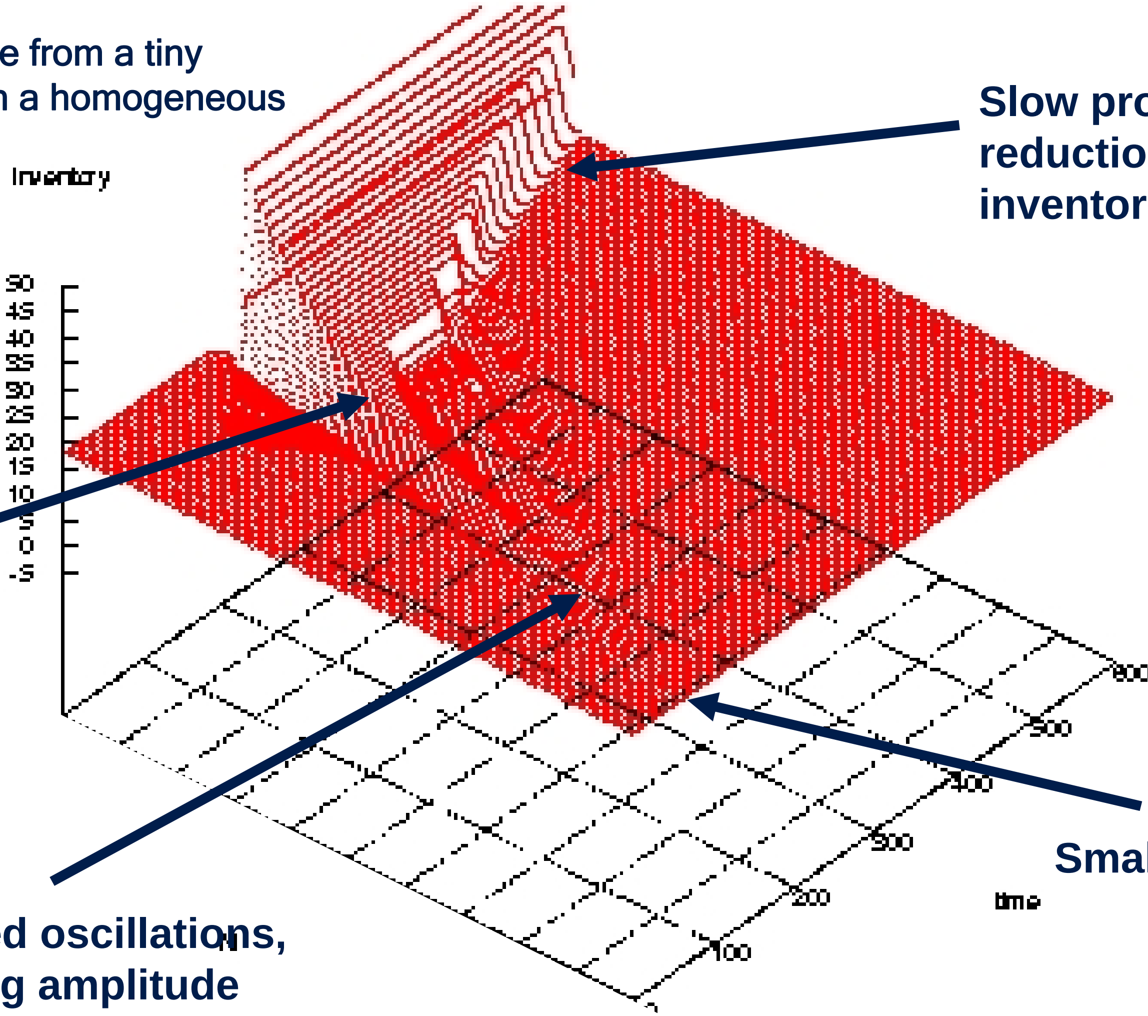
Driven by randomly varying demand ...

The system acts as a filter ...



Nonlinear Effects

Large inventories emerge from a tiny perturbation in demand in a homogeneous supply chain



Slow propaging reduction of inventories

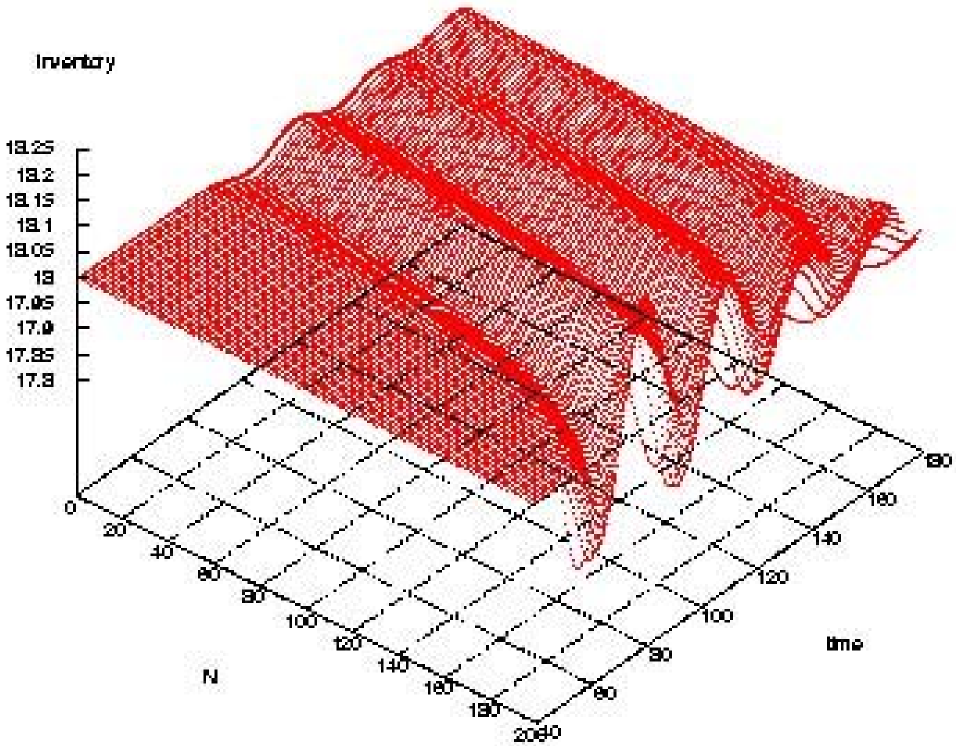
Out of supply, Large Inventories

Damped oscillations, growing amplitude

Small perturbation

Avoidance of undesired behaviour

Speed up the adaption of upstream suppliers:

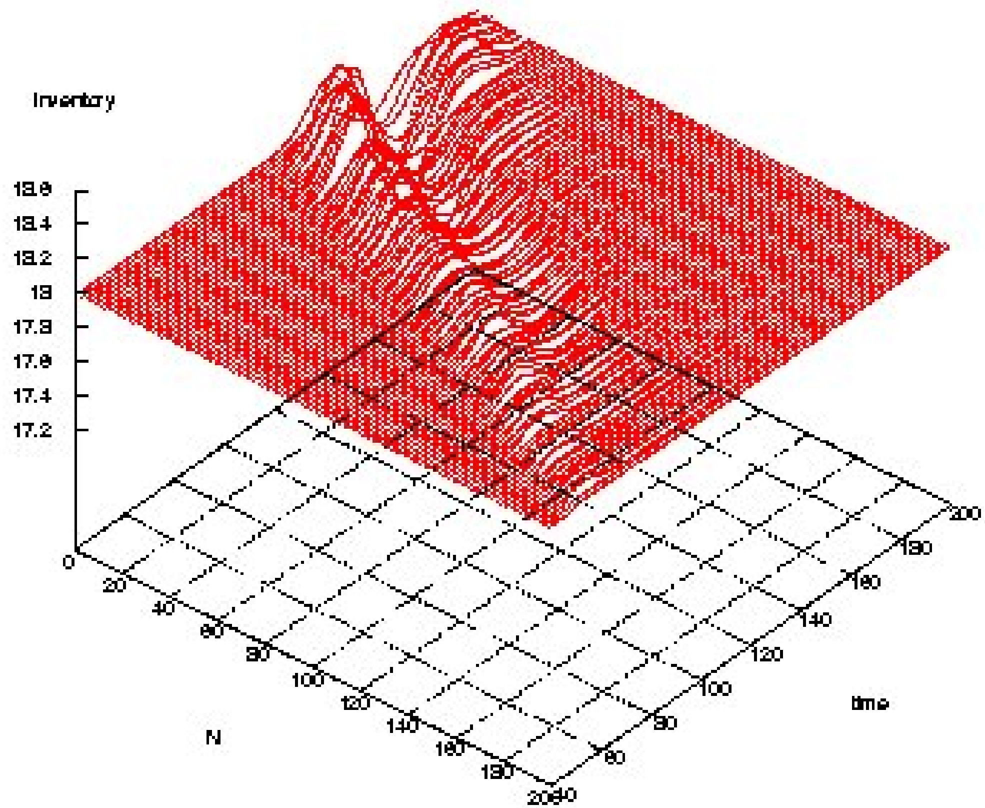


- **only possible for loop-free networks**
- **the adaption speed is limted**
- **does not prevent oscillations**

Avoidance of undesired behaviour

Same system, if different suppliers use different strategies and forecast models...

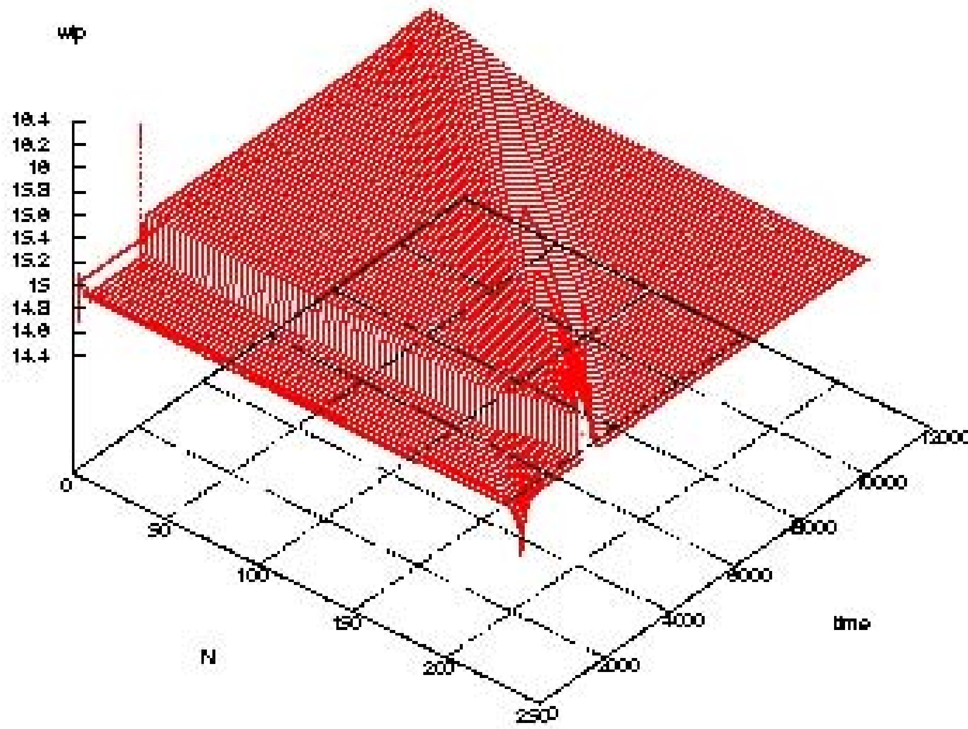
What is the best strategy mix?



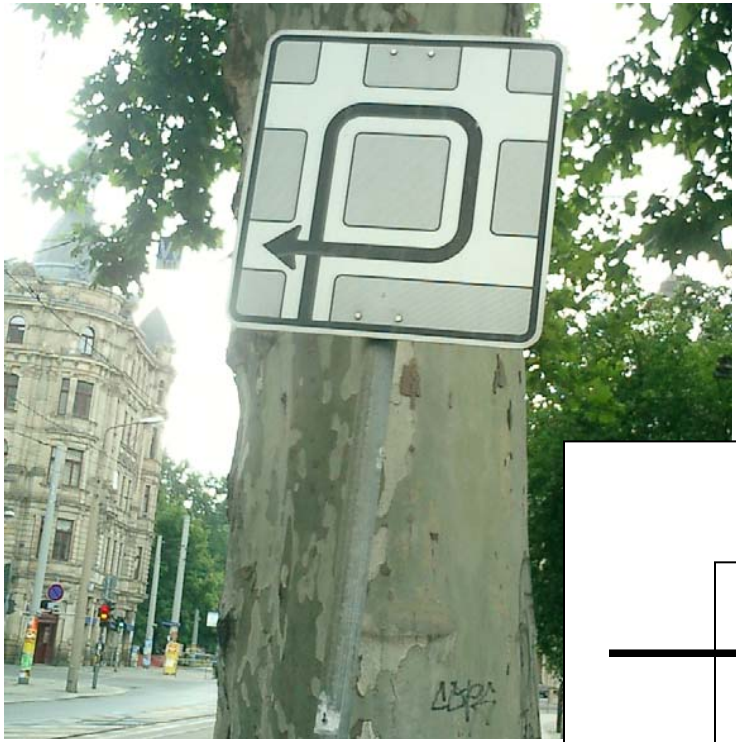
Avoidance of undesired behaviour

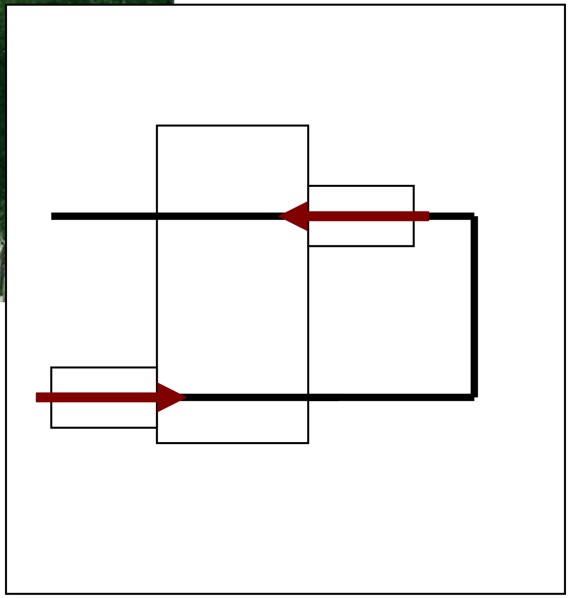
Introduce a global coupling through information distribution.

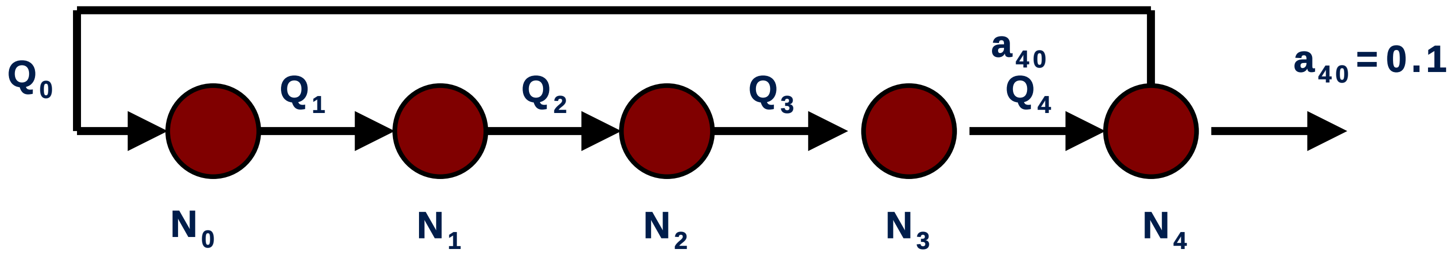
- Providing end to end demand information turns the pull policy into a partial push policy for the material flow
- The material flow network is often unchangable
- The information network can be desingned
- What is the optimal information network for a given supply newtork?

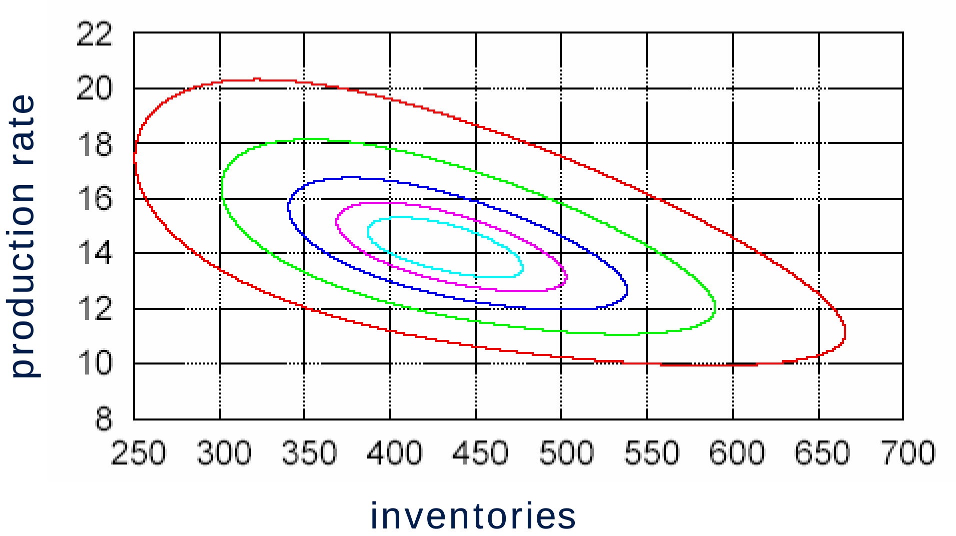


Re-entrant Systems





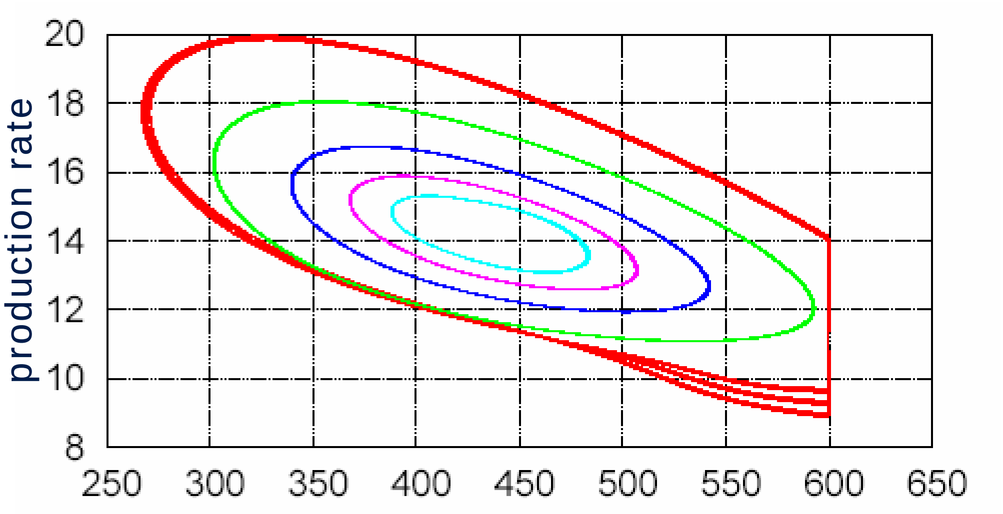




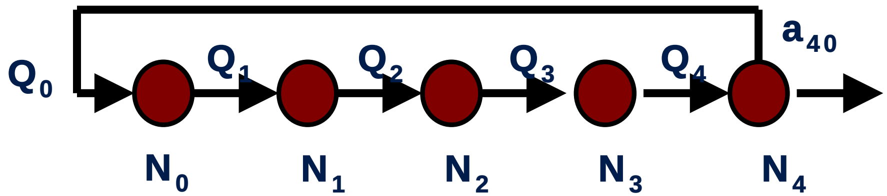
feedback loops result in Limit cycle oscillations

Here: homogeneous supply chain

In inhomogeneous supply chains or networks these oscillations are more complicated



inventories



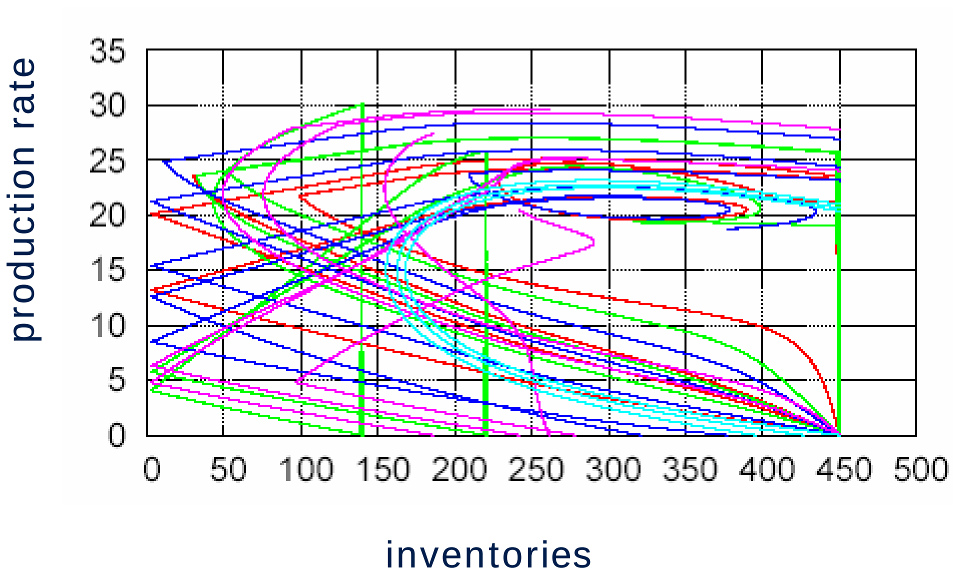
Inventories are restricted

For too large invetories the production must stop

alternatively other policies are applied to control the inventories.

Policies may imply non-smooth dynamical systems

Reaching of thresholsd implies Border Collision Bifurcations

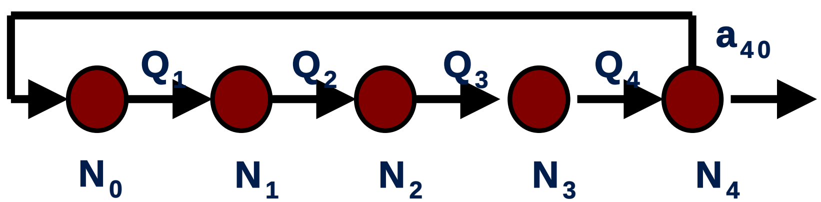


Out of supply effects:

If the inventory of a predecessor is empty the processing rate of the follower drops down

Limits in adaption of production rates:

limits for maximum production rates (technology, investment etc.)



Even in simple supply chains complicated dynamics may occur

Effects for complex supply networks?

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Summary

- Modeling of material and information flows in networks may require hybrid models
- Hybrid systems have a binary state space: continous + discrete (symbolic) variables
- In non-smooth and hybrid dynamical systems Border Collision Bifurcations are the generic bifurcations
- Thus a rich variety of dynamics must be expected if non-smoothness or discrete controller are involved in the systems

Complex Systems and Logistics

- **What is Complex Systems Science about?**

Modelling, **understanding** (and optimization) **of Complex Systems** in all Domains ranging from Physics over Biology, Sociology and Business to technical systems using methods from Natural sciences, in particular **physics and mathematics**.

- **Topic:** blurry, basically **defined by what the community is doing.**

- **Objective:** obtain **new knowledge!** Explain Phenomena!

Any unexpected observation may be a lucky chance.

Complex Systems and Logistics

- **What is Logistics about?**

Organization and control of efficient material- and information flows in manufacturing and the economic systems using methods from more basic sciences.

- **Topic:** blurry, basically **defined by what the community is doing.**

Objectives: 6 x R

- the right good
- in the right quantity
- with the right quality
- at the right place
- and right time
- for the right price

Any unexpected event may be failure.

Conclusion

Data from Logistics:

- We have to understand **what data** we have and **how the processes work** that are used in the real world system.
- In order to test a hypothese we must **exclude the effects** that are not essential but **caused by the information handling** and algorithms usually used in technical systems or include these processes in our models.
- **Be careful with data from logistics, these data are completely different compared to measurements in physics!**

Conclusions

There is **no unique model** or method to deal with the rich variety of complex behaviour in logistic systems

But **methods** from nonlinear dynamics, network theory or in general complex systems can help to **understand the dynamical phenomena**

Such methods can help to **improve the efficiency** of material flow systems

Complex systems theory may also help to **design more appropriate systems**.

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Thank you very much for your attention!

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