Mathematical Models of Cutting Processes

Christina Brandt, Iwona Piotrowska

26th of February, 2009

Outline

1 Introduction

2 Face Turning
- Introduction
- Process model
- Process Machine Interaction Model

3 Milling processes

4 Grinding processes

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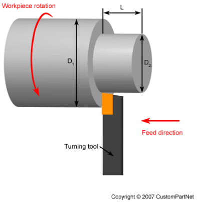
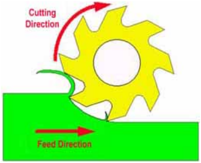
Different types of cutting processes

With defined cutting edge

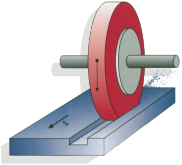
With undefined cutting edge



turning



milling



grinding

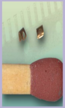
SPP1180 and SFB 747

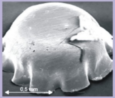
SFB 747

SPP 1180

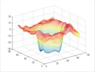
Micro cold forming

- milling
- grinding
- polishing

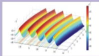




Iwona
Thi
Florian



Visualization



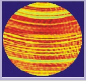
Jost

User interface

Janina

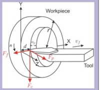
Surface models

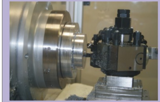
Stefan



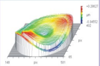
Precision balancing

- turning
- dual plane balancing





Christina
Bastian



cooperation with LFM, RICAM (Johann-Radon-Institute for Computational and Applied Mathematics), IWT

Universität Bremen

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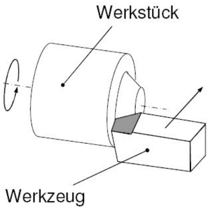
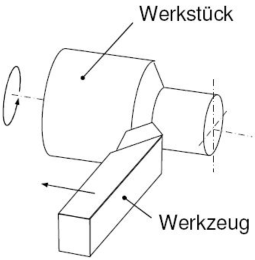
Cutting processes

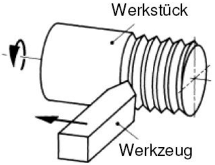
face turning



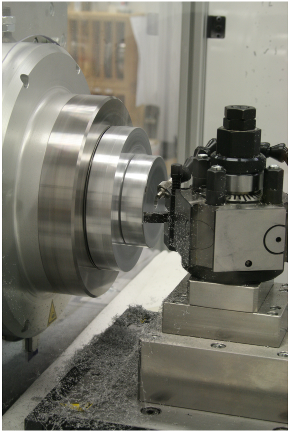
straight turning



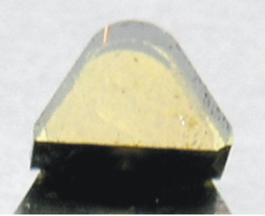
thread turning



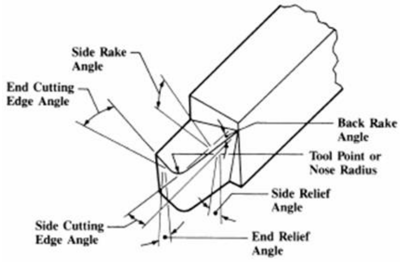
Ultraprecision turning lathe (face turning)



The tool



- Diamond tool
- very sharp cutting edge radius
  $r_\beta < 50nm$



Input paramter

machine parameter

stiffness k_{ex}, k_{ey}, k_{ez}
Radius of the workpiece r
Geometry of the tool (tool nose radius r_ϵ, cutting edge radius r_β etc)

process parameter

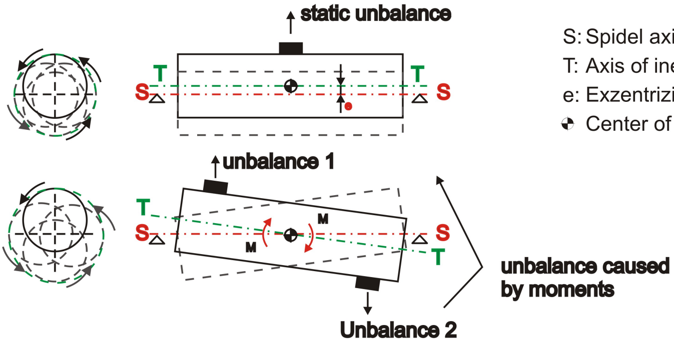
Rotational work speed n $[\frac{1}{s}]$
Feed f $[\frac{mm}{s}]$
Depth of cut a_p $[mm]$

typical parameter for ultraprecision turning

$r_\epsilon = 0.76mm$
$r_\beta < 50nm$

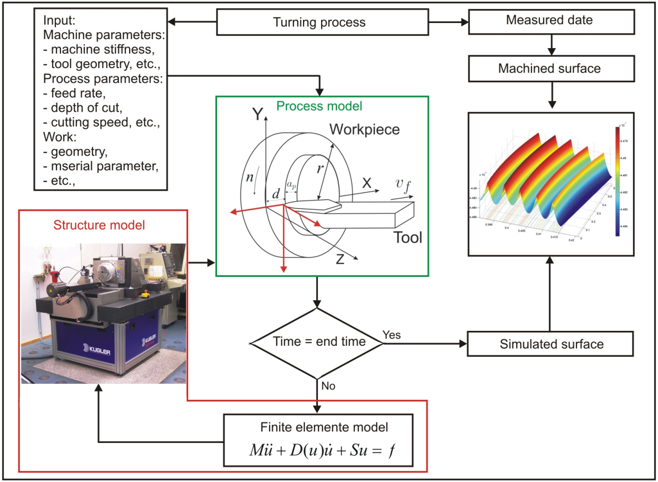
$n = 500s^{-1}$
$f = 2mm/s$
$a_p = 5\mu m$

„Mathematical methods for precision balancing for machine tools“

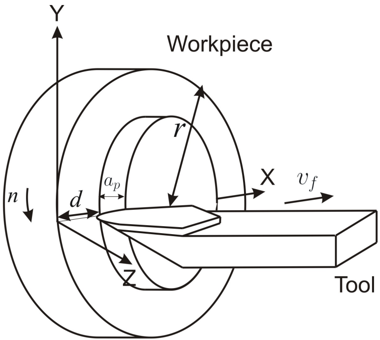


S: Spidel axis
T: Axis of inertia
e: Exzentrizität
Center of gravity

Process Machine Interaction

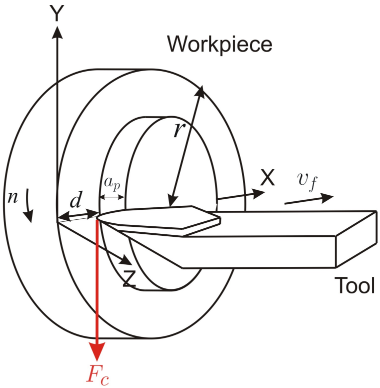


Force components



- Cutting force F_c
- Feed force F_f
- Passive force F_p

Force components



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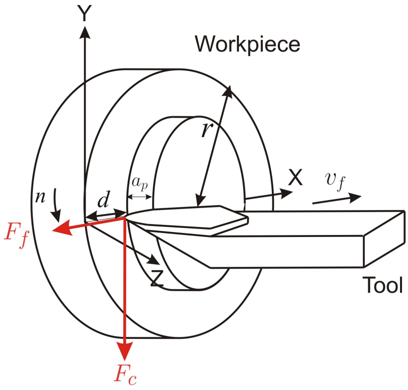
Force components



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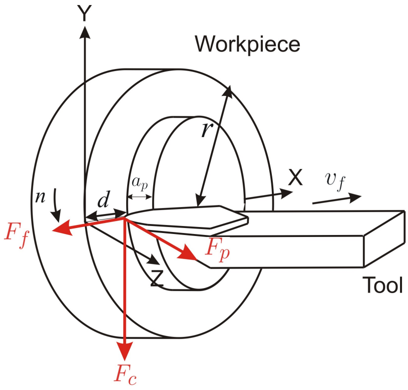
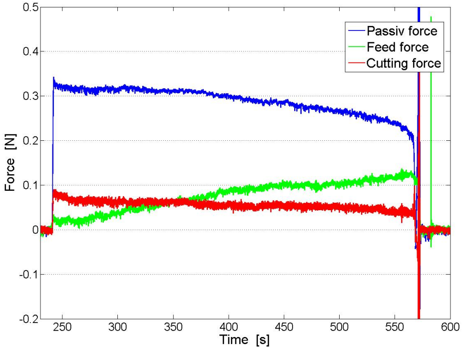


- Cutting force F_c
- Feed force F_f
- Passive force F_p

Micro-Force Measurements



- constant rotational speed n $\Rightarrow$ cutting velocity v_c decreases
- Passive force is dominant component
- Cutting force decreases with cutting velocity whereas it is increasing in conventional turning processes

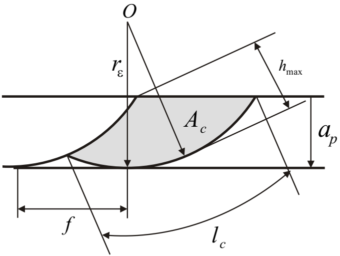
Force Model for micro-turning

General ansatz for the force:

$$F = \sum_{i=1}^{N} c_i A_i v_i^{\alpha_i}$$

with

- velocity v
- projected cross sectional area of the tool A
- constants c and α



Cutting force

$$F_c = c_1 A_c v_c + c_2 A_c$$

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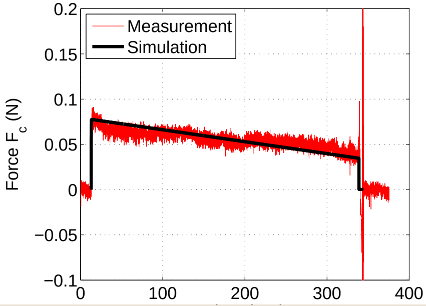


Cutting force

$$F_c = c_1 A_c v_c + c_2 A_c$$

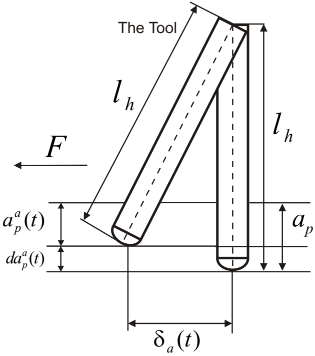
Deflection of the tool

Position of the tool tip

$$P_{s,x}(t) = v_f\, t - \delta_x(t)$$
$$P_{s,y}(t) = -\delta_y(t)$$
$$P_{s,z}(t) = -a_p^a(t)$$

Actual depth of cut

$$a_p^a(t) = a_p - da_p^a(t) - \delta_z = a_p - \frac{\delta_a(t)^2}{l_h} - \delta_z$$



System of ordinary differential equations

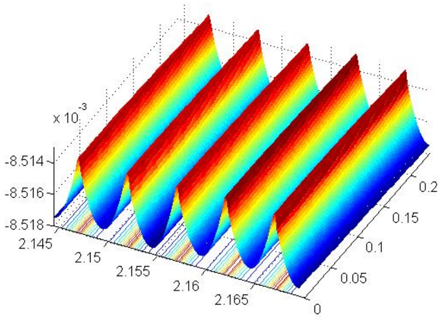
$$\begin{pmatrix} \dot{v}_f^a(t) \\ \dot{a}_p^a(t) \\ \dot{d}^a(t) \\ \dot{\delta}_x(t) \\ \dot{d}_y(t) \end{pmatrix} = P\left(v_f(t), v_f^a(t), a_p^a(t), \delta_x(t), \delta_y(t)\right)$$

with initial conditions

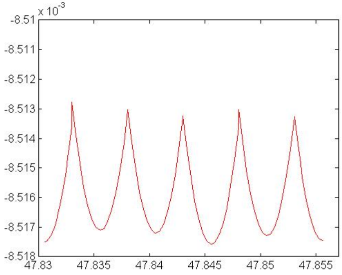
$$\begin{aligned} v_f(0) &= 0 \\ a_p^a(0) &= a_p \\ d^a &= 0 \end{aligned} \qquad \begin{aligned} \delta_x(0) &= 0 \\ \delta_y(0) &= 0 \end{aligned}$$

Simulation of the surface

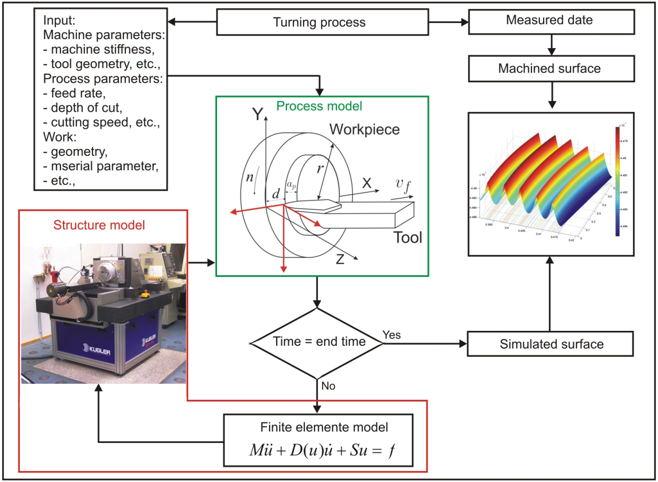
Simulated surface



Radial cross section



Process Machine Interaction



Coupling of process and machine model

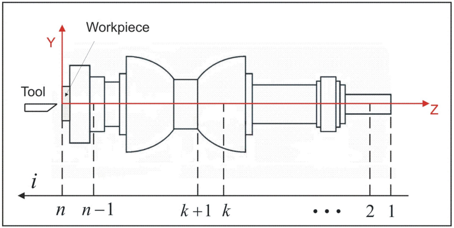
Structure model ⇒Process model

- Unbalances cause vibrations
  ⇒changing position of the workpiece
- changing projected cross sectional areas $A_c(t)$, $A_f(t)$,$A_p(t)$ and new position of th tool tip on the workpiece
  ⇒ changing forces

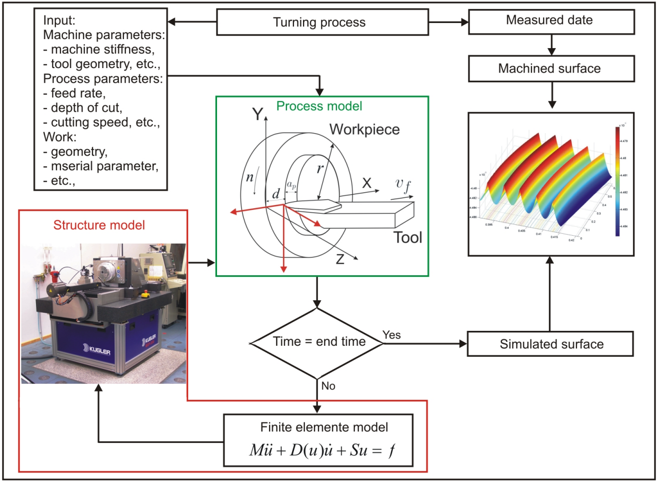
Coupling of process and machine model

Process model ⇒ Structure model

- Process force acts at the tool tip position on the workpiece ⇒additional force and moment on the workpiece



Process Machine Interaction

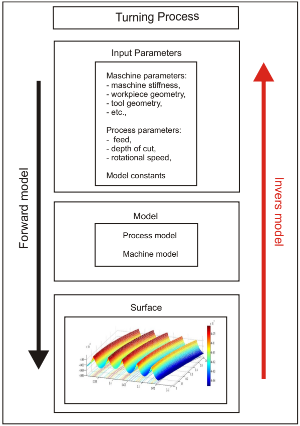


How to use the interaction model?

Inverse problem

-
$$\min_{p} \|O_{sim}(p) - O_{machined}\|$$

-
$$\min_{a_p, f, n} \|O_{sim}(a_p, f, n) - O_{ideal}\|$$



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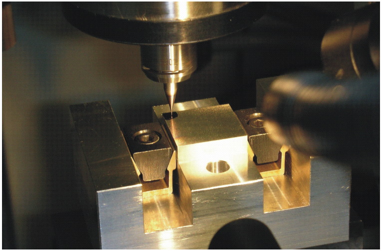
Milling processes

- defined cutting edge
- micro ball-end-milling



Model for the actual position of the tool tip

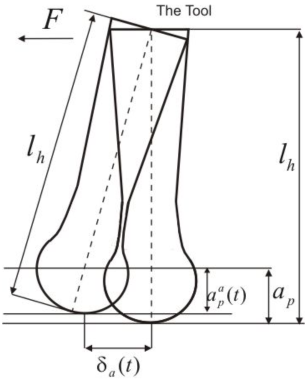
Position of the tool tip

$$P_{s,x}(t) = v_f\, t - \delta_x(t)$$
$$P_{s,y}(t) = -\delta_y(t)$$
$$P_{s,z}(t) = -a_p^a(t)$$

Deflection

$$\delta = \frac{F}{k_e}$$

k_e – stiffness



Dynamic forces model

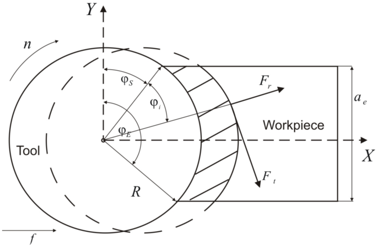
Force model

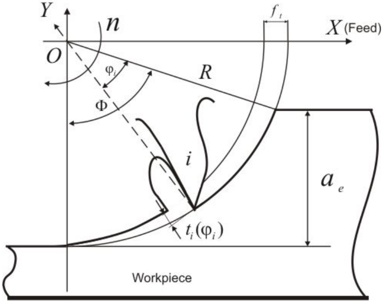
$$F_r(t) = K_{rc}a(t)h(t) + K_{re}a(t) \qquad \text{Radial force}$$
$$F_t(t) = K_{tc}a(t)h(t) + K_{te}a(t) \qquad \text{Tangential force}$$
$$F_a(t) = K_{ac}a(t)h(t) + K_{ae}a(t) \qquad \text{Axial force}$$

$a(t)$ – cutter edge length

$h(t)$ – chip thickness

Dynamic forces model

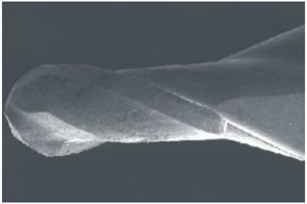




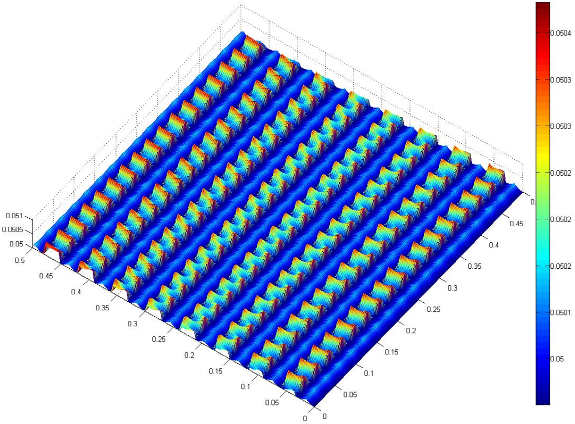
Simulation results

Parameters:

- Ball End Mill
- $a_p = 0.05$[mm]
- $v_f = 1800$[mm/min]
- $n = 30000$ [rev/min]
- Workpiece: 0.5[mm] x0.5[mm] x0.1[mm]
- $R = 1$[mm]



Simulation results



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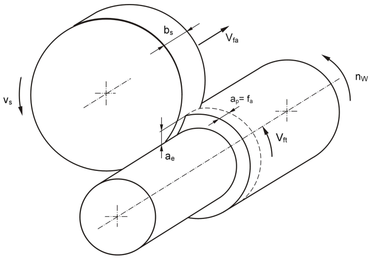
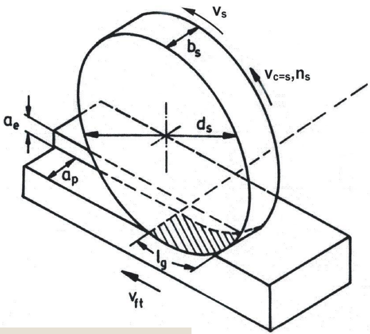
Grinding processes

- undefined cutting edge
- cylindrical plunge grinding
- plane grinding





Model for the actual position of the tool tip

Actual position model

$$v(t) = u(t) - \frac{\mathrm{d}\delta}{\mathrm{d}t} - w(t)$$

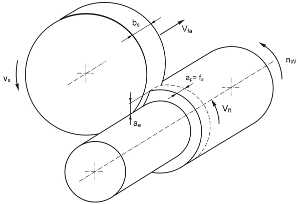
S. Malkin: "Model based simulation of Grinding Processes" (2006)

$u(t)$	the radial infeed rate
$\delta(t)$	the deflection
$w(t)$	the radial wear rate of the wheel

Dynamic forces model

Normal force

$$F_n(t) = F_{ch}(t) + F_{pl}(t) + F_{sl}(t)$$



Force model

$$F_{ch}(t) = c_1 \frac{u_{ch} v_w b a(t)}{v_s}$$ Chip formation force

$$F_{pl}(t) = const$$ Plowing force

$$F_{sl}(t) = c_2 p_c A_{eff} b (d_e a(t))^{0.5}$$ Sliding force

Differential equation

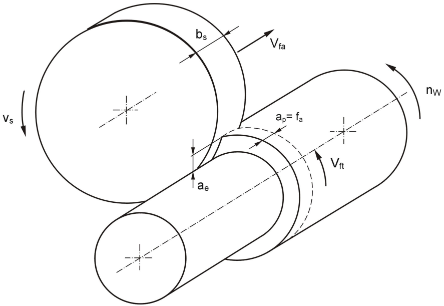
$$\dot{a}(t) = \frac{(u - (c_4 + n_w)a(t))}{c_1 + c_2 \frac{1}{\sqrt{d_e a(t)}} + c_3 \frac{1}{\sqrt{d_e a(t)}} \int_0^t \sqrt{d_e a(\tau)}\, \mathrm{d}\tau}$$

d_e	the equivalent diameter
$a(t)$	the actual wheel depth of cut
n_w	the rotational speed of the wheel

Simulation results

Parameters:

- Cylindrical plunge grinding
- $d_s = 70[\text{mm}]$
- $d_w = 96.22[\text{mm}]$
- $v_s = 30\ [\text{m/sec}]$
- $n_w = 70\ [\text{rev/min}]$
- $b = 12[\text{mm}]$
- $k_e = 6500\ [\text{N/mm}]$

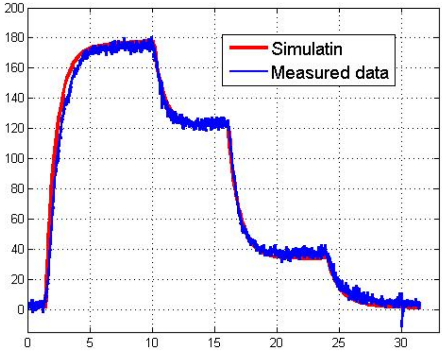


Simulation results

Parameters:

Four-stage infeed grinding

- Rough
- Finish
- Fine-finish
- Spark-out



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