Introduction to Optical Flow

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Uttendorf 2005

Zentrum für Technomathematik

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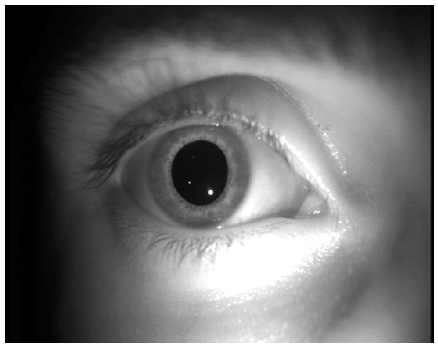
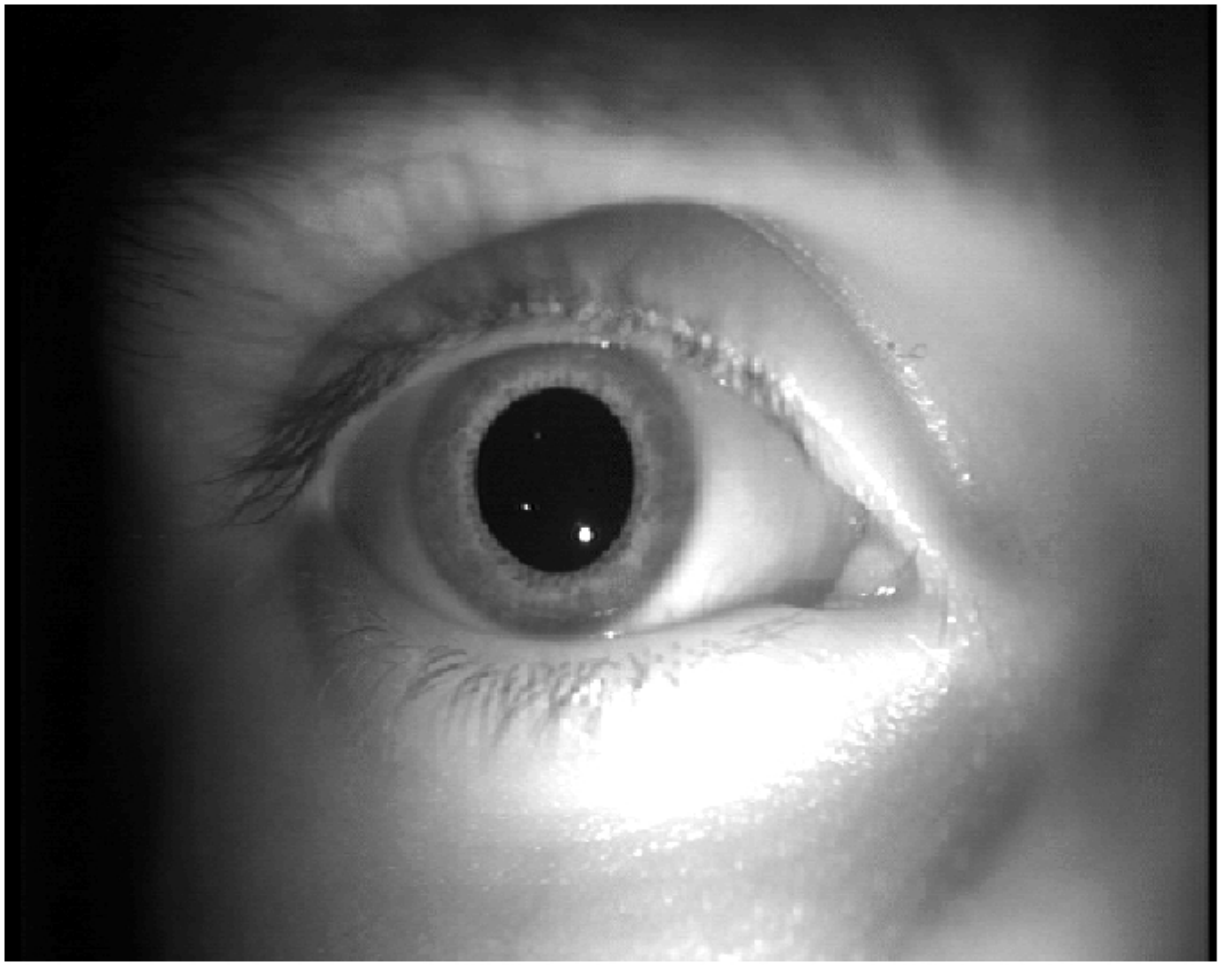
Visual cranial reflex(VCR)(?)



- Rapidly changing scene (video games, virtual reality)
- Movements are identified by brain
- Brain sends signals to the eyes
- Eyes perform opposite movements in order to equilibrate the scene

Video sequence



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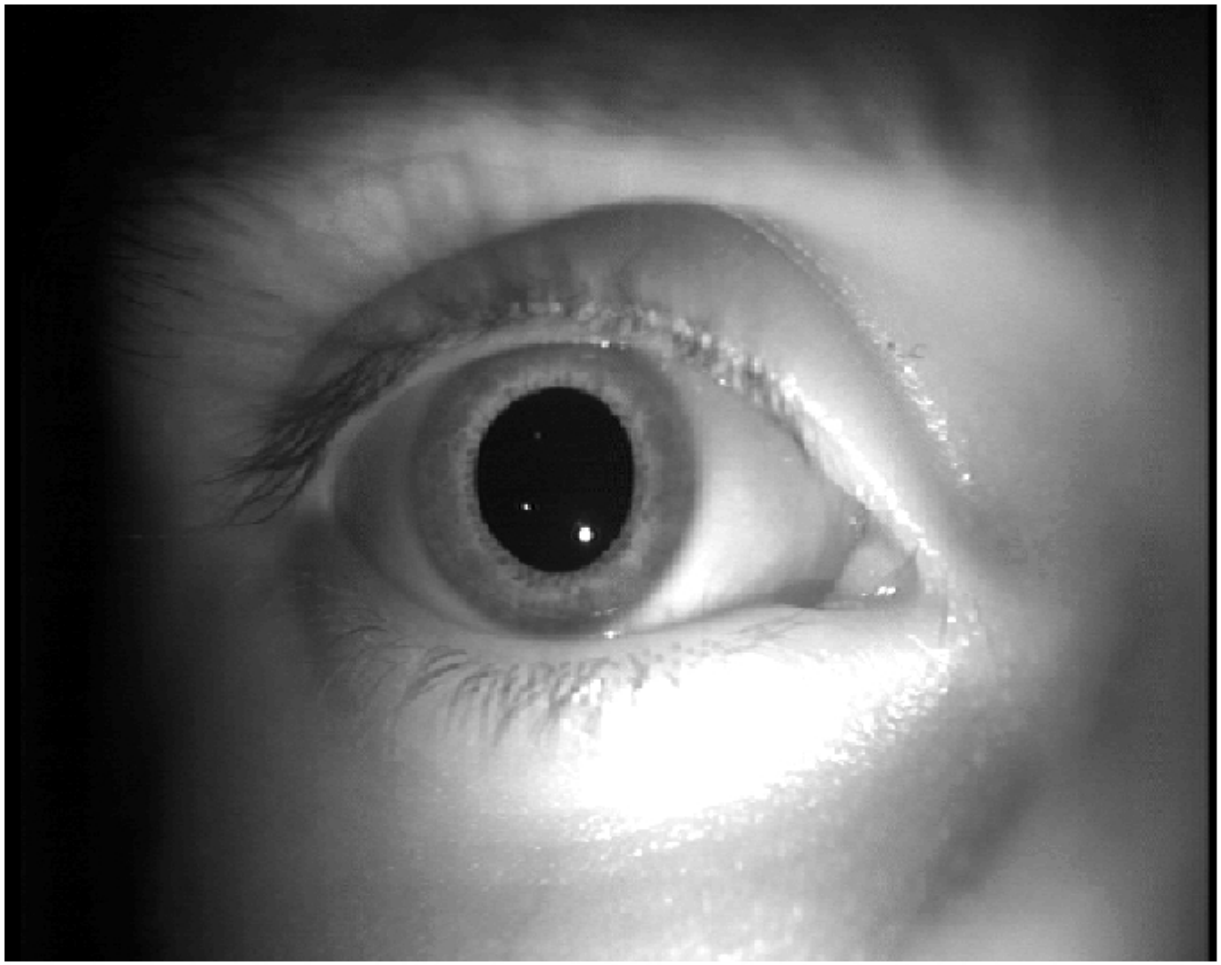

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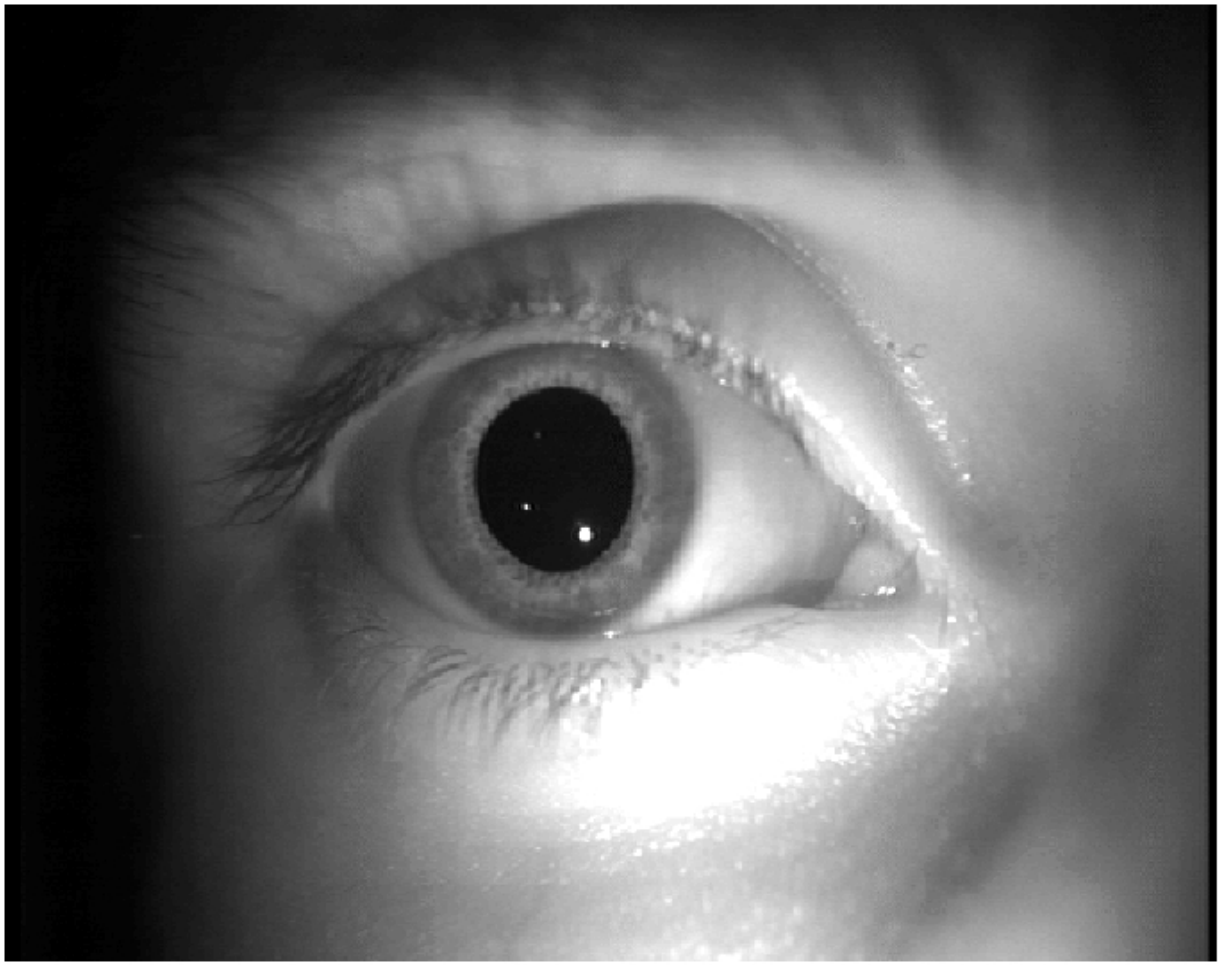

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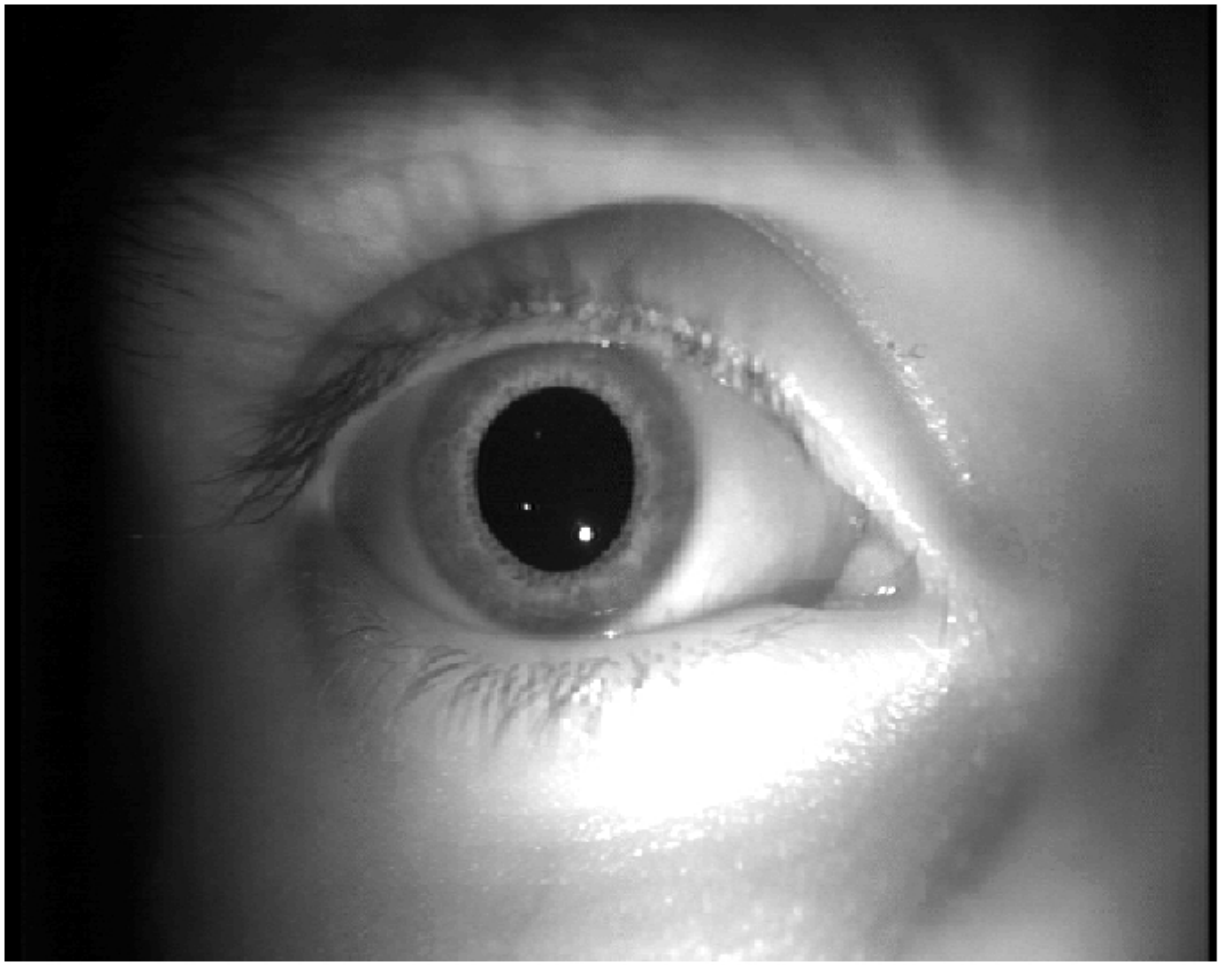

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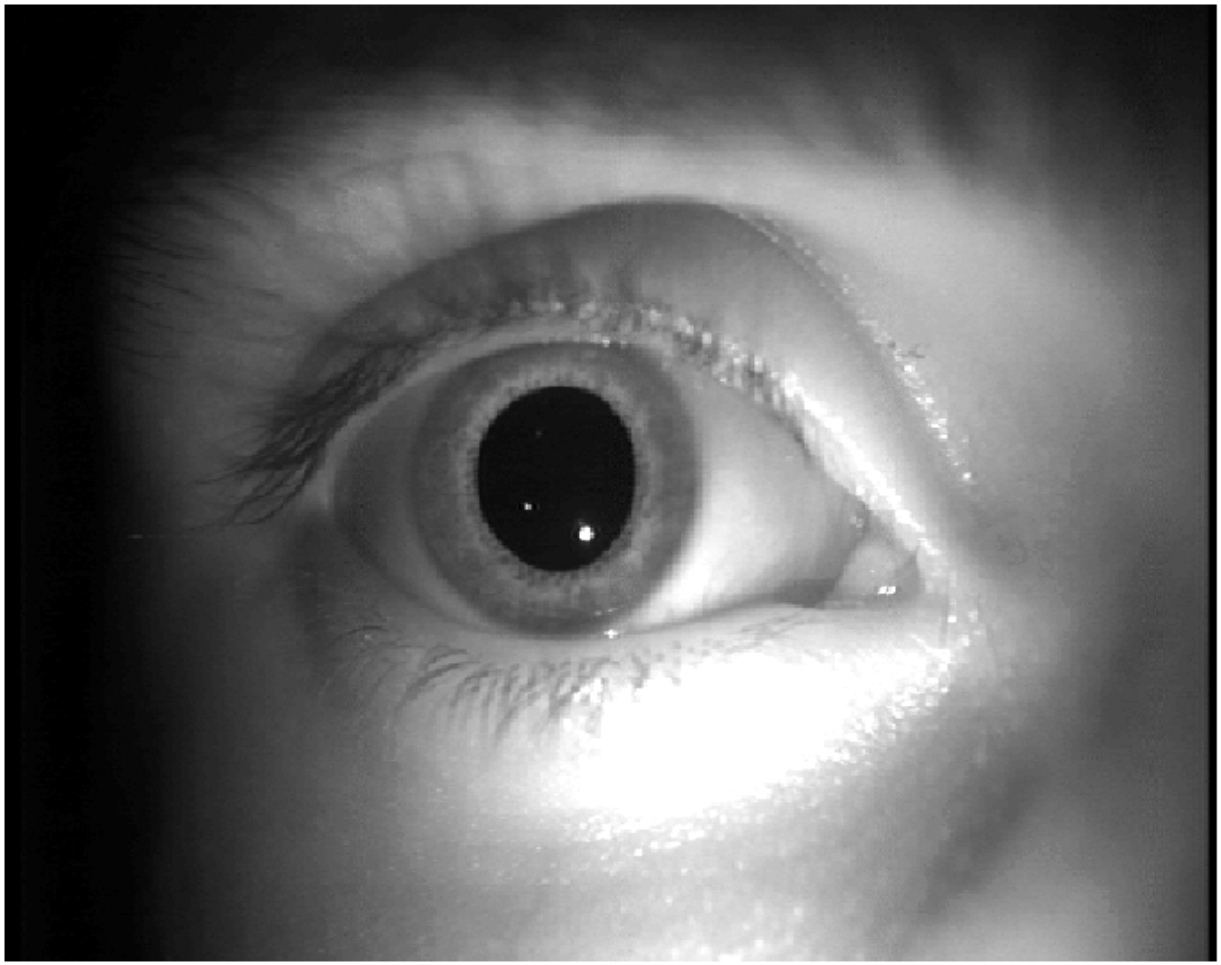

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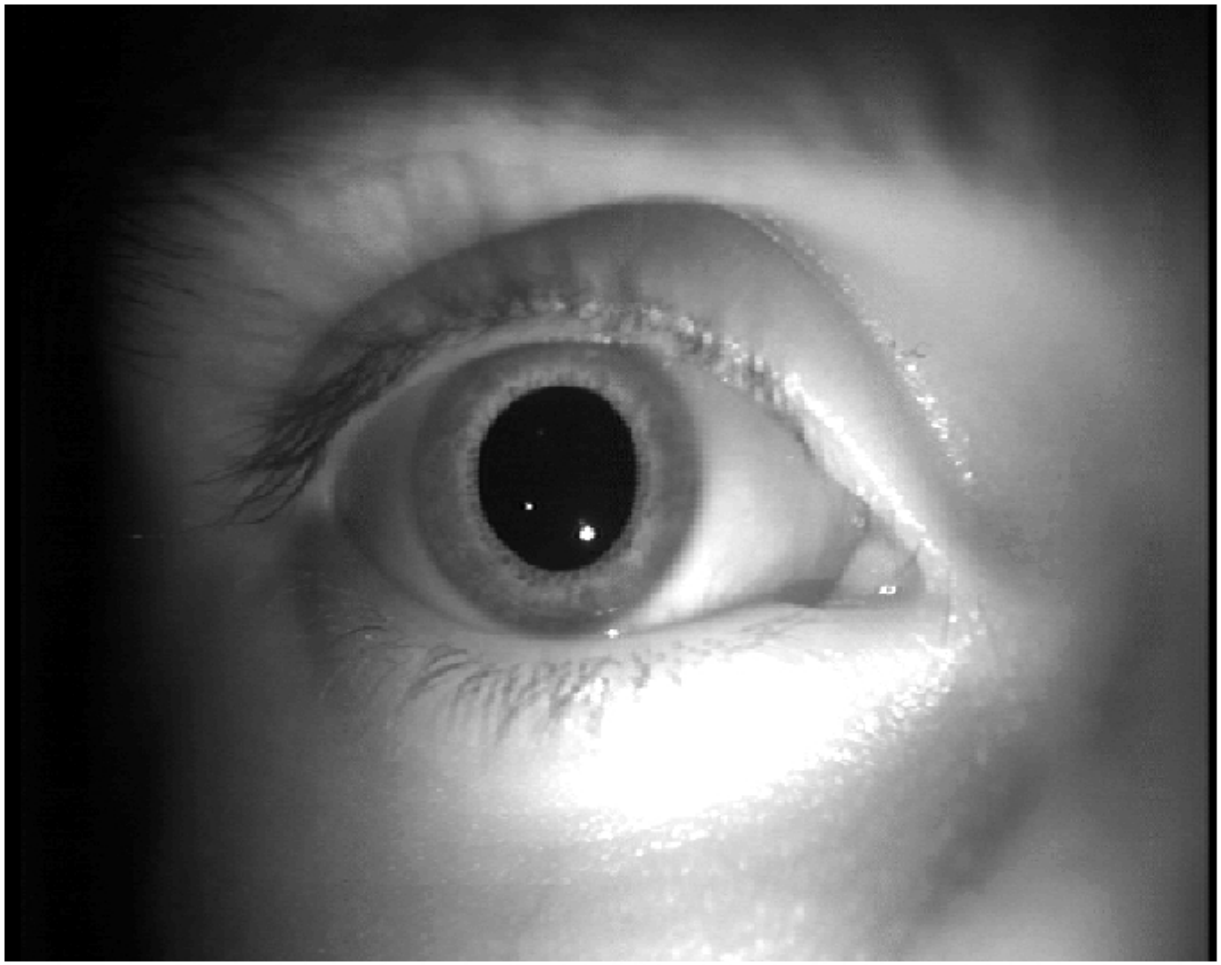

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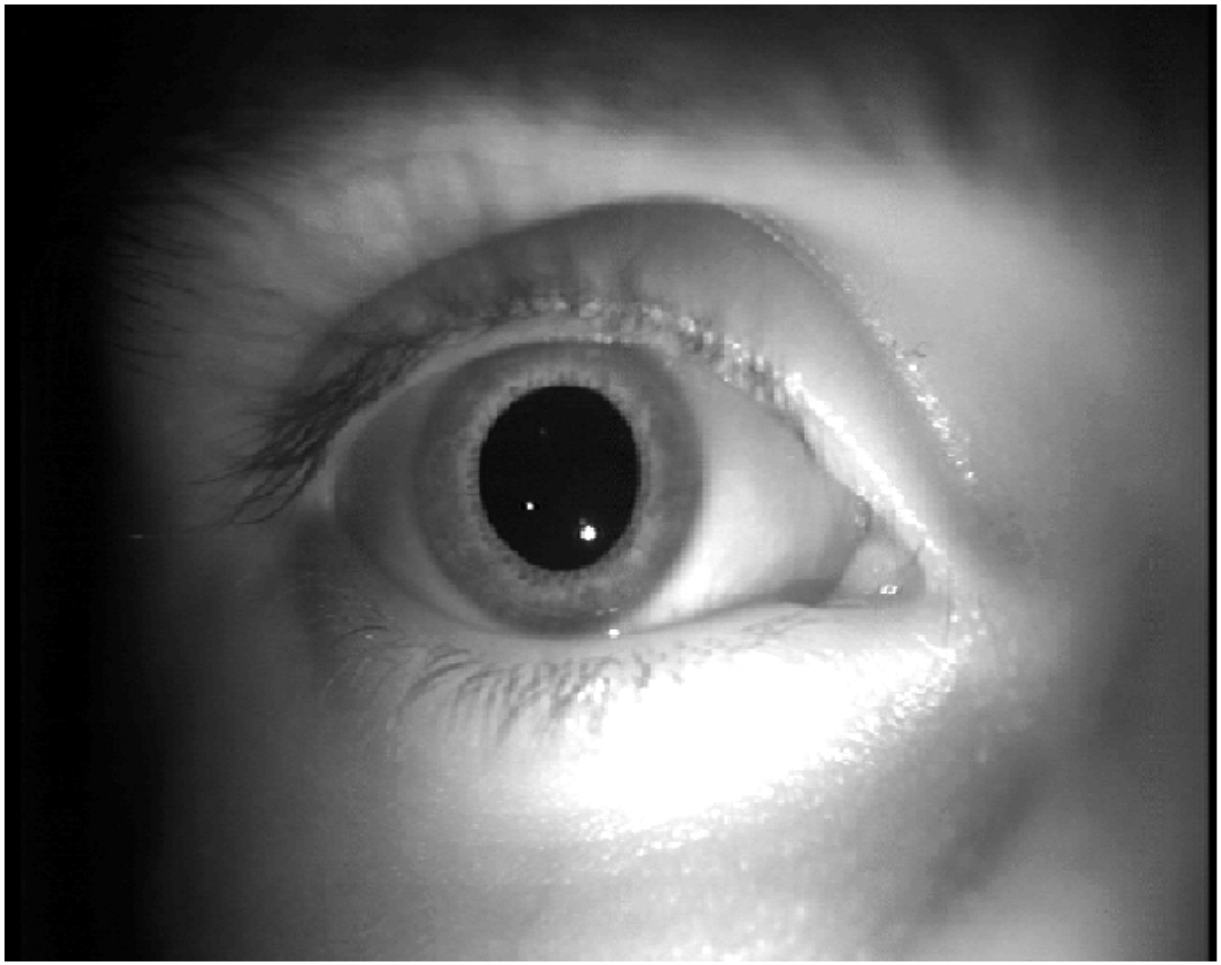

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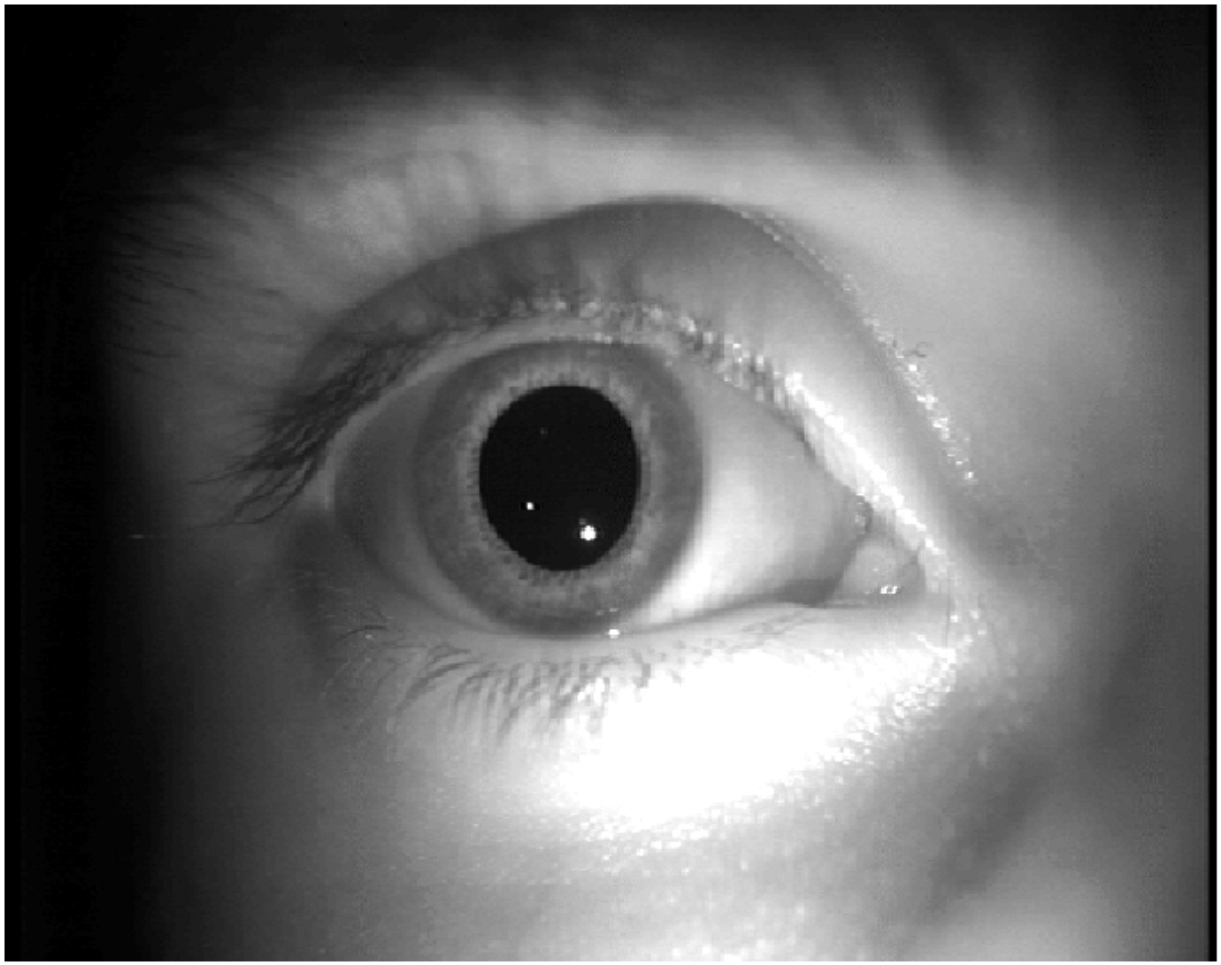

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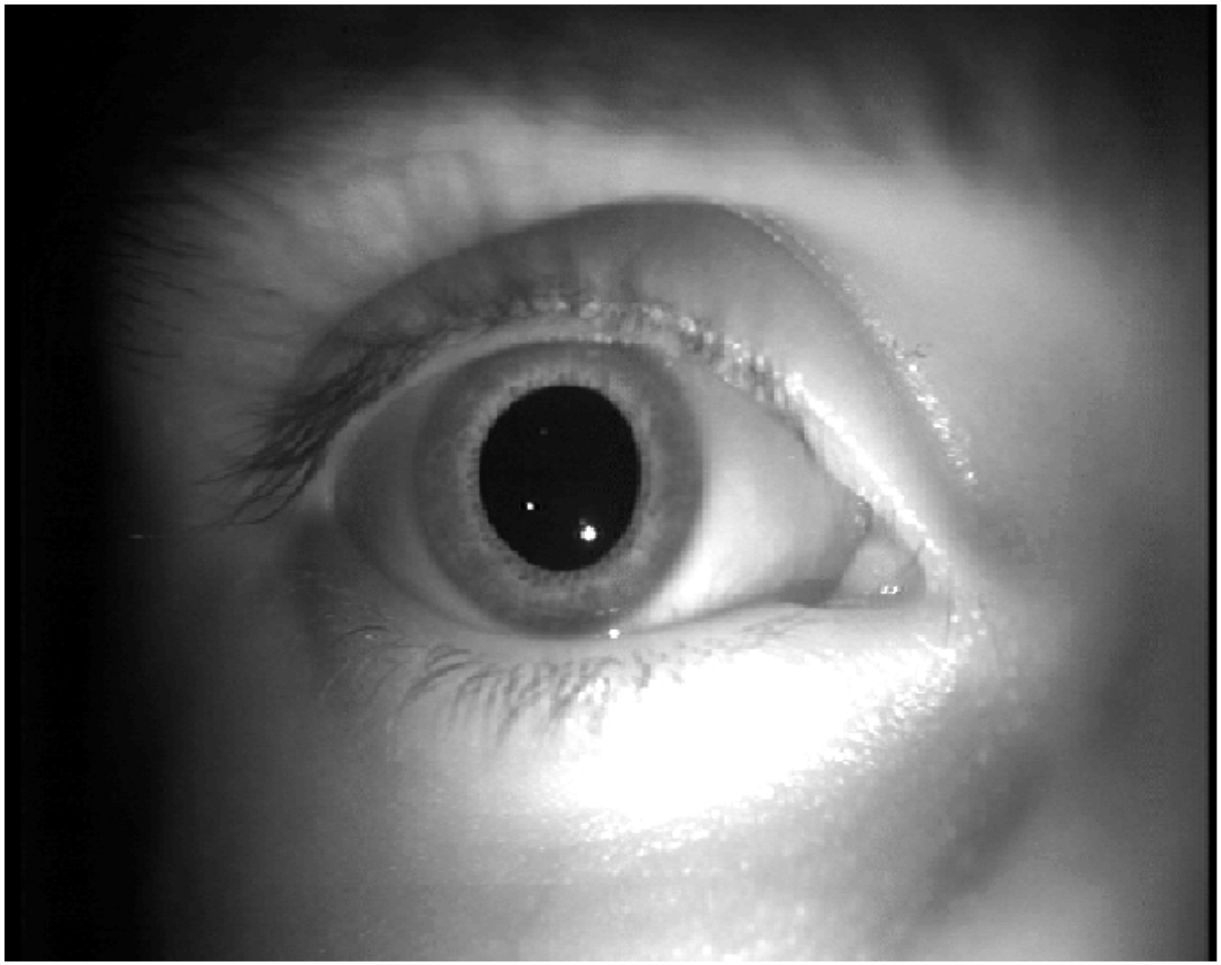

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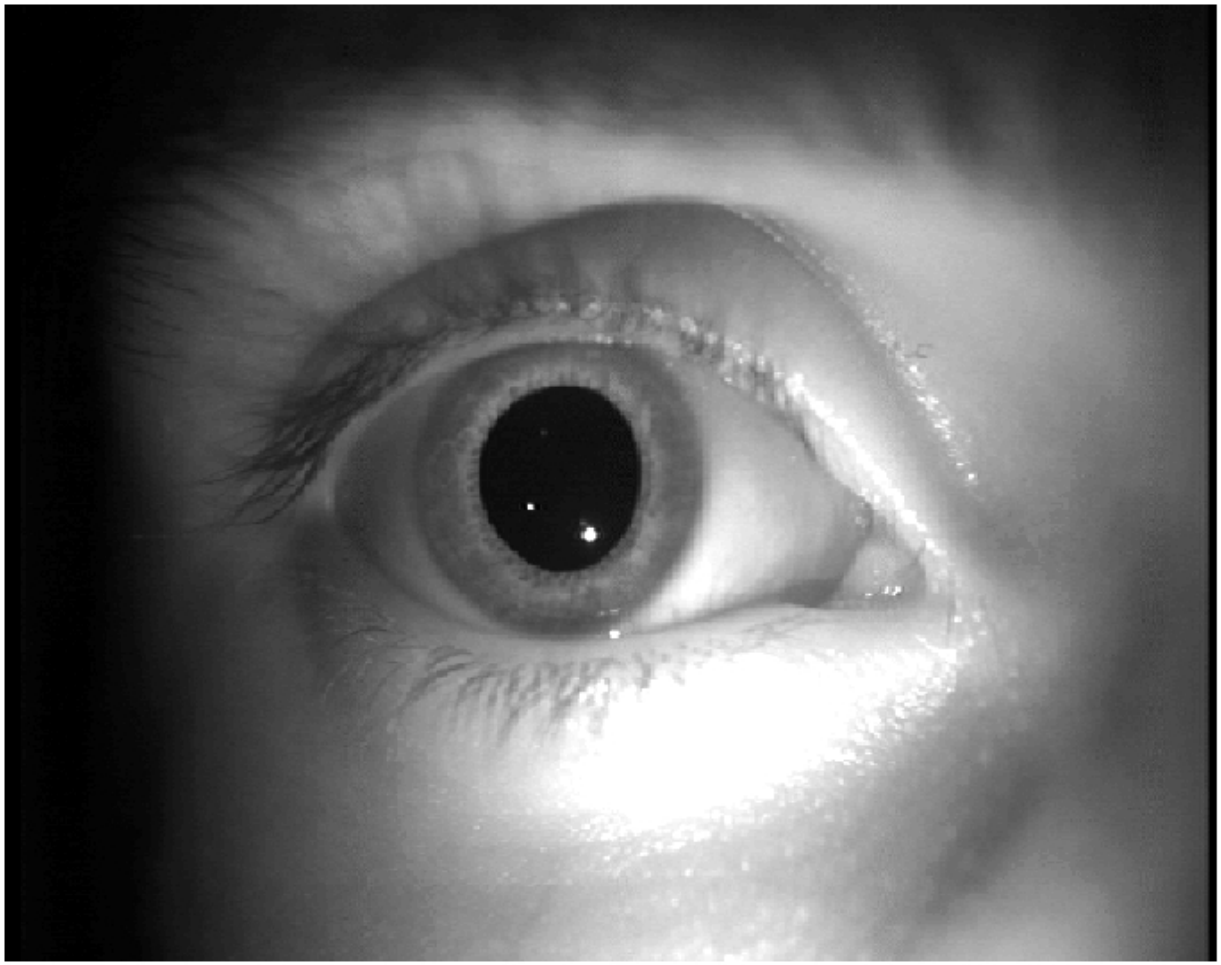

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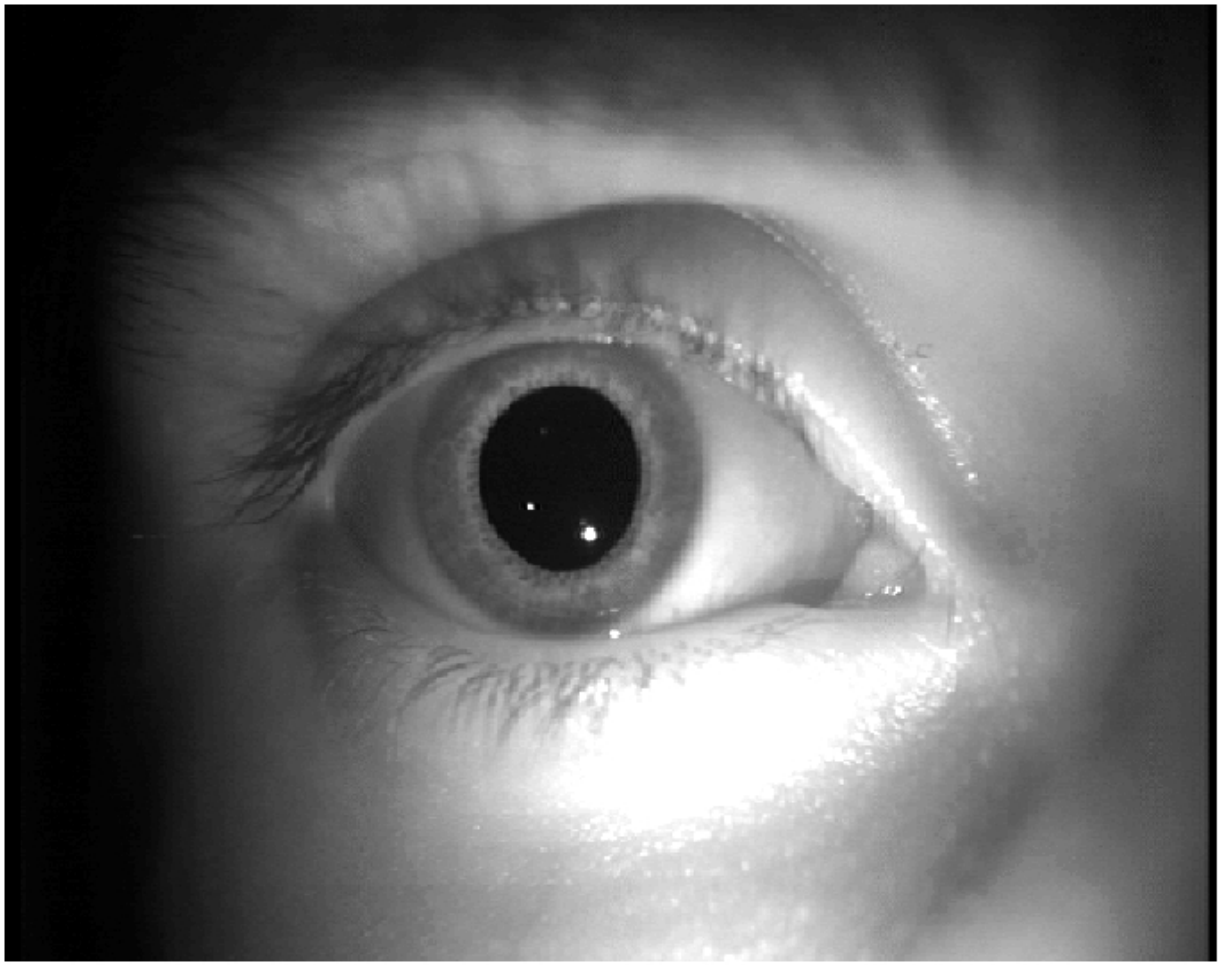

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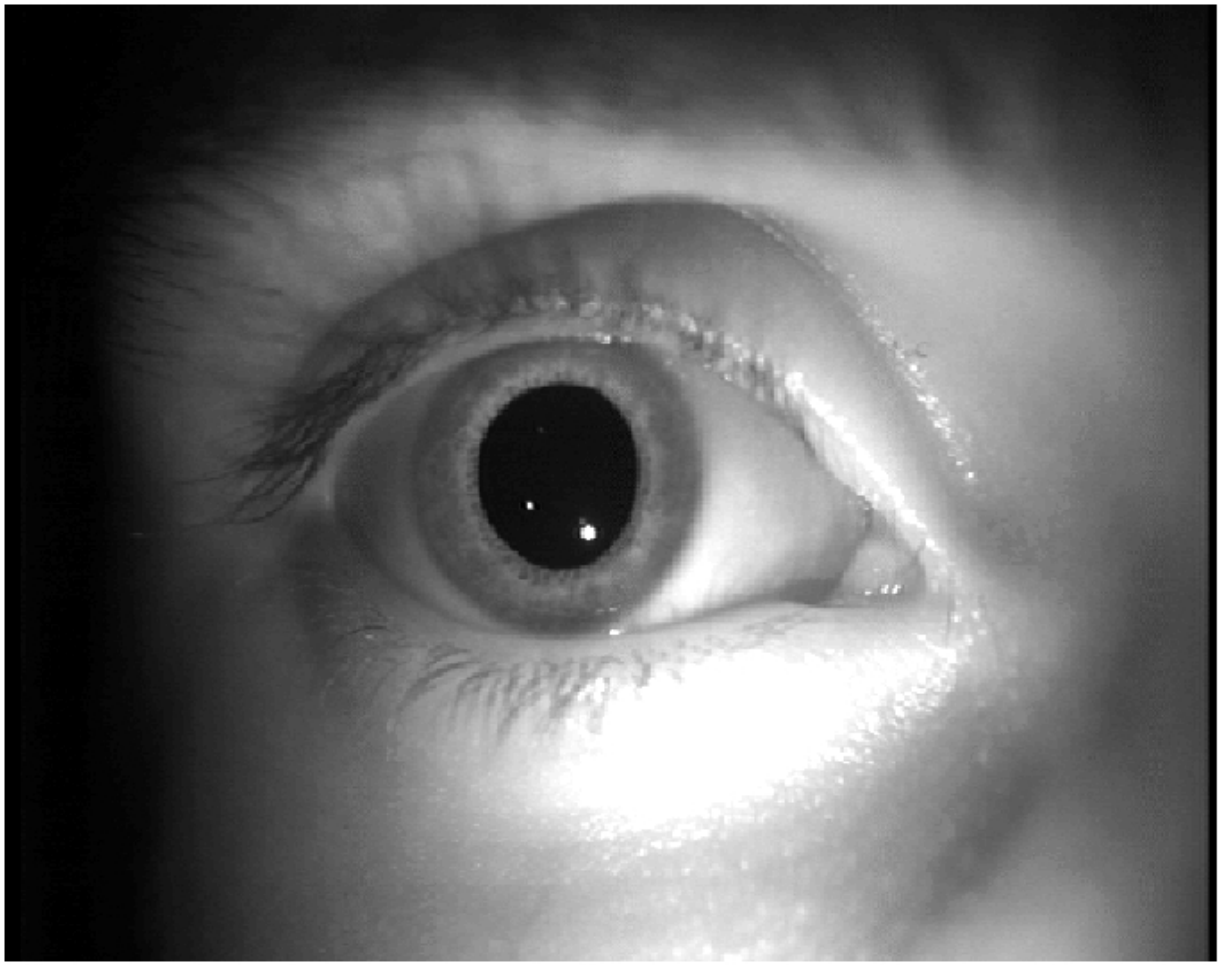

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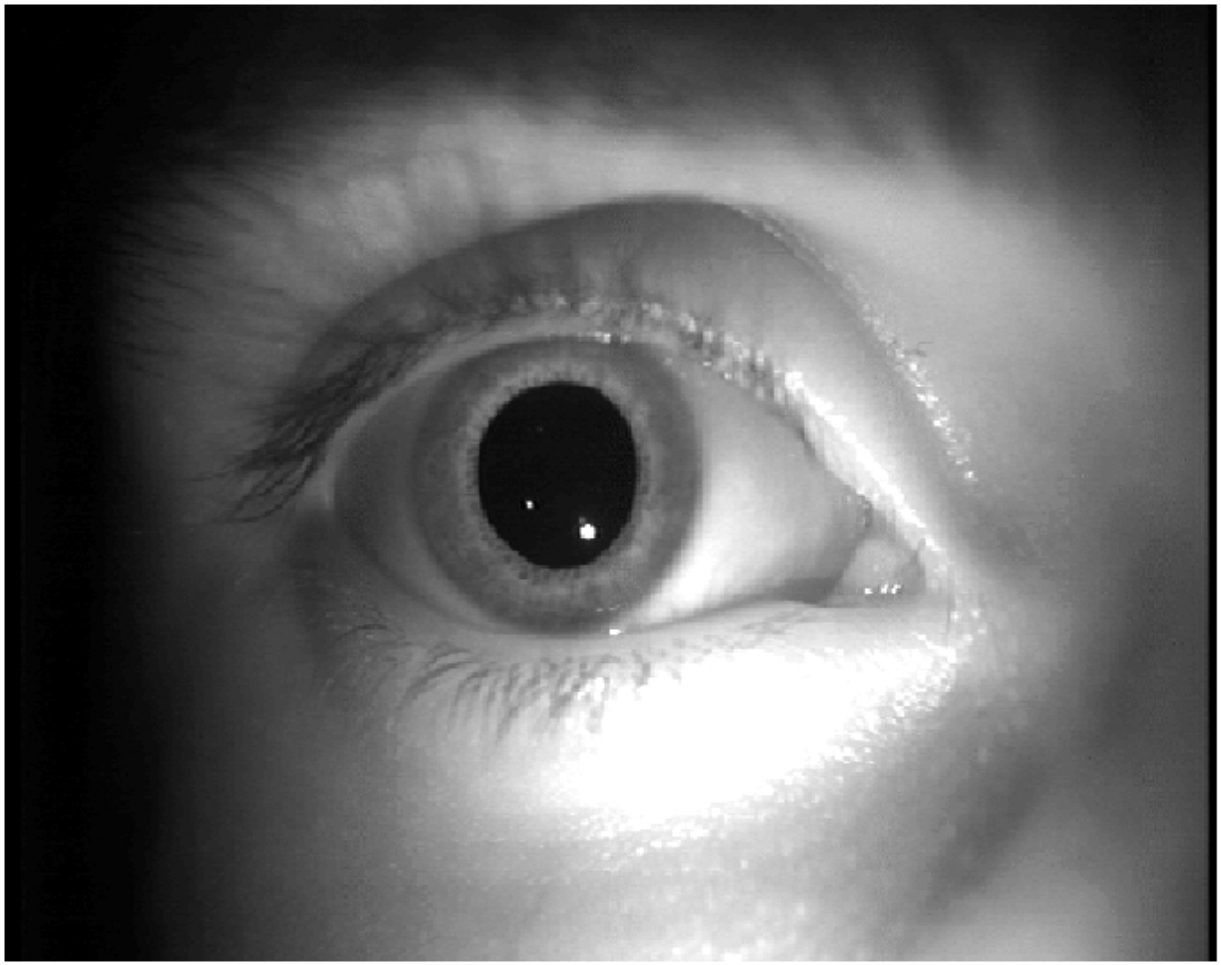

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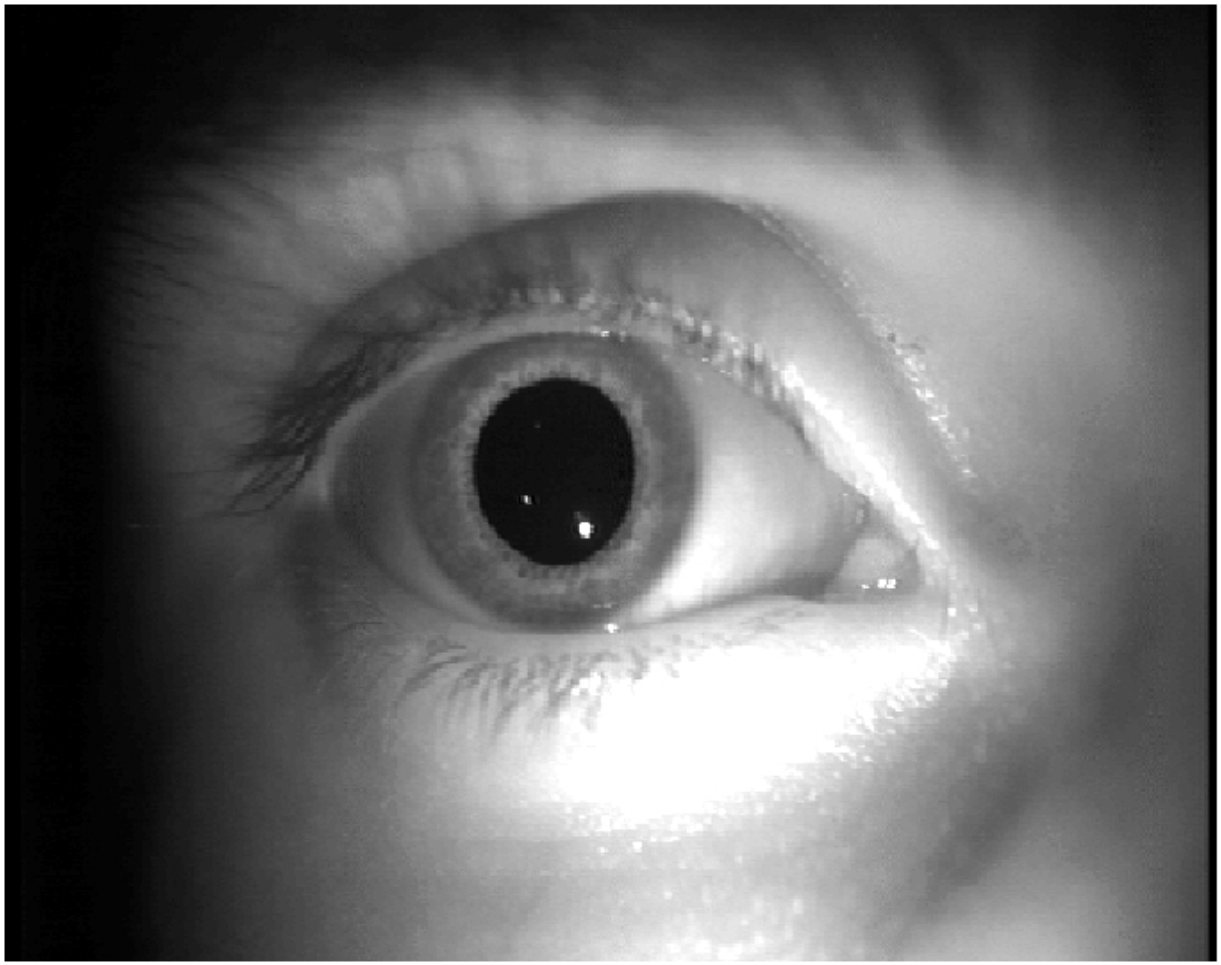

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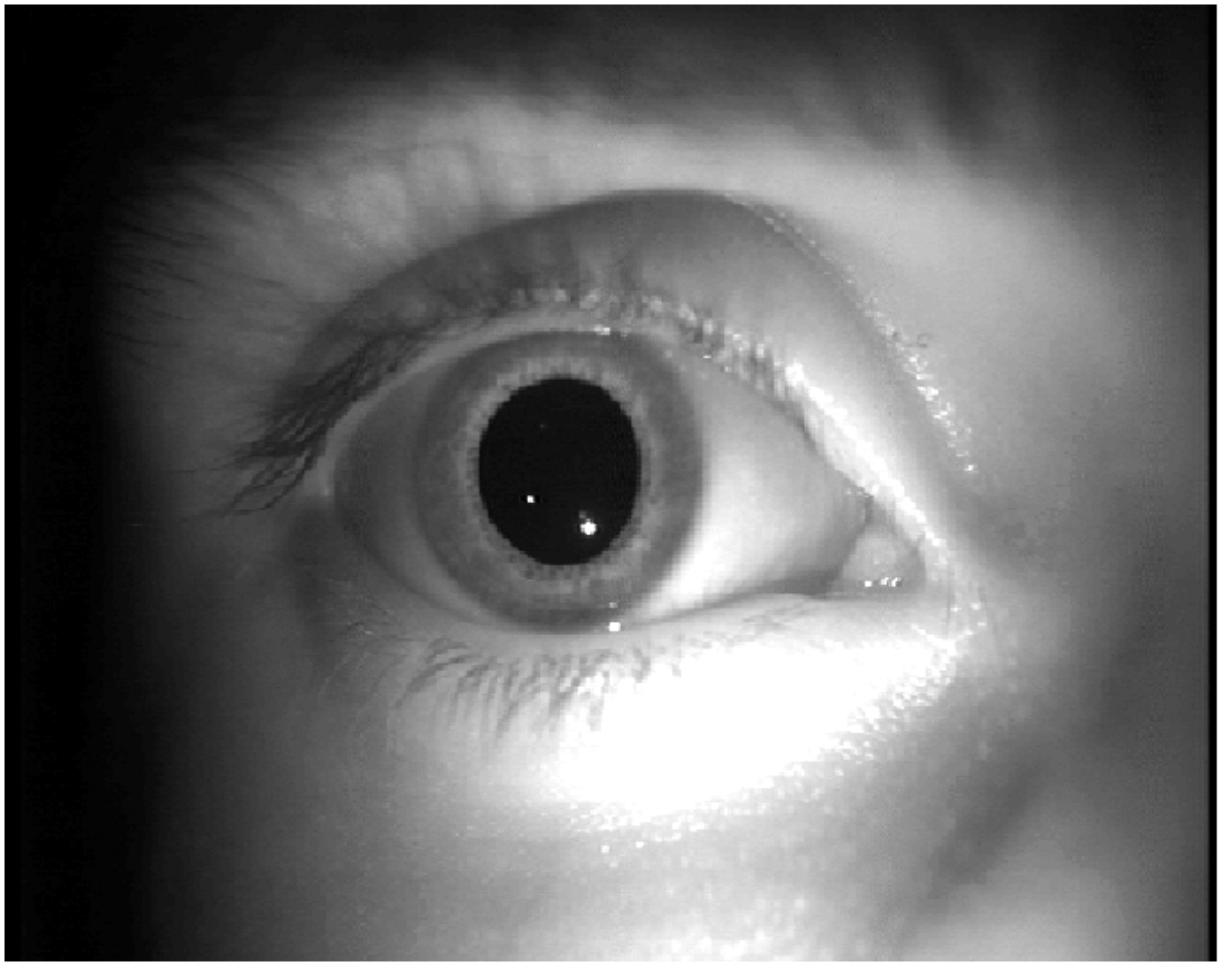

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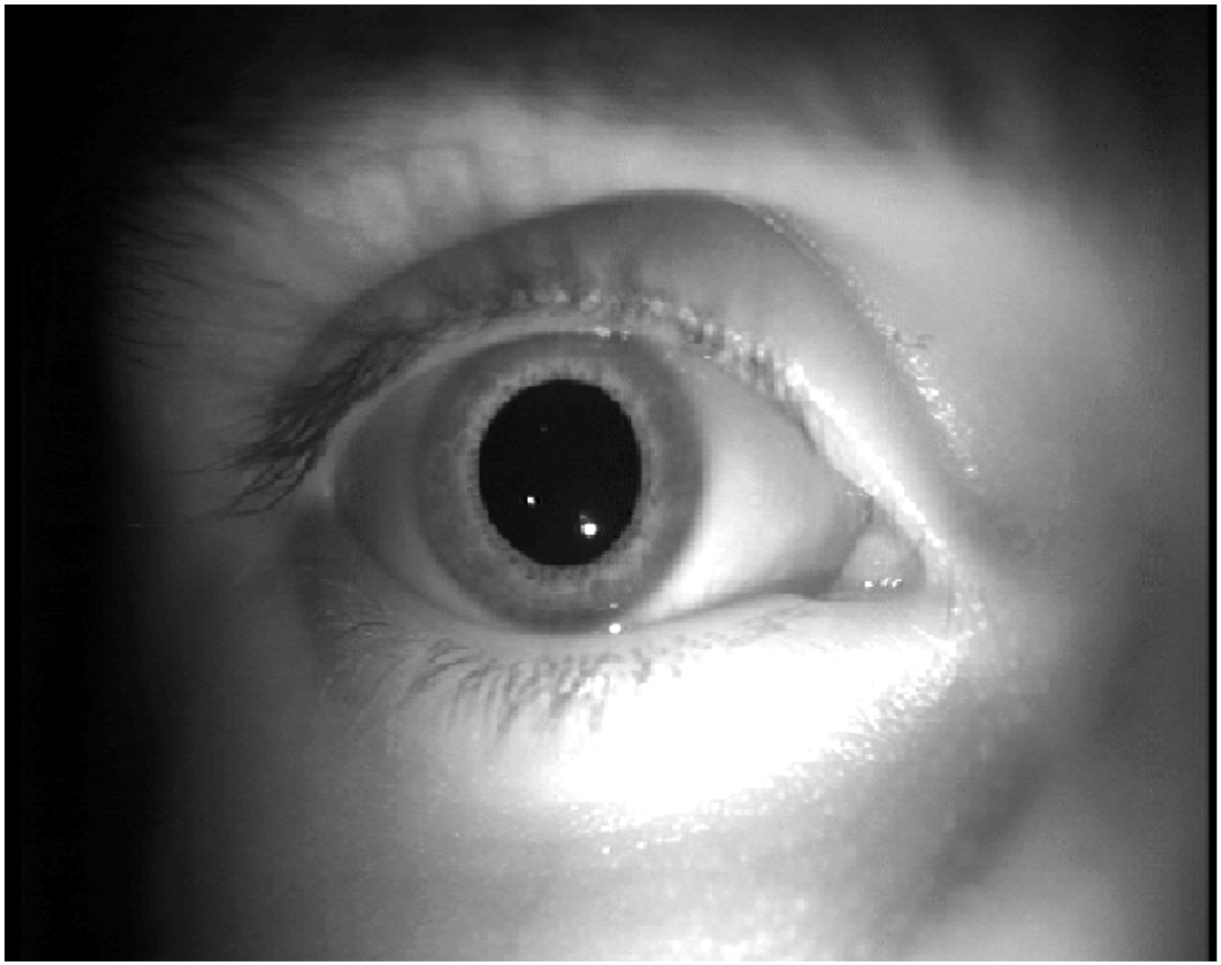

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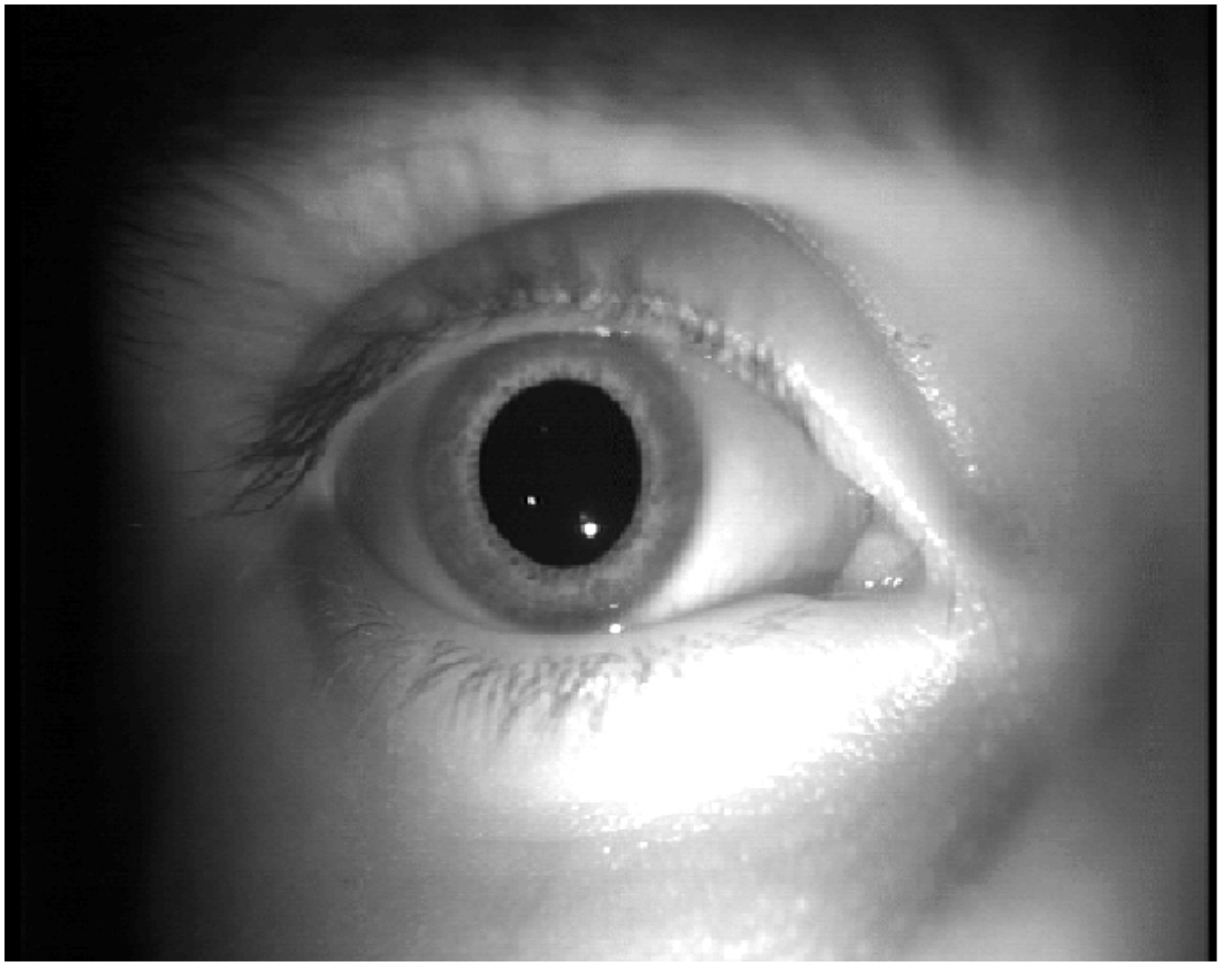

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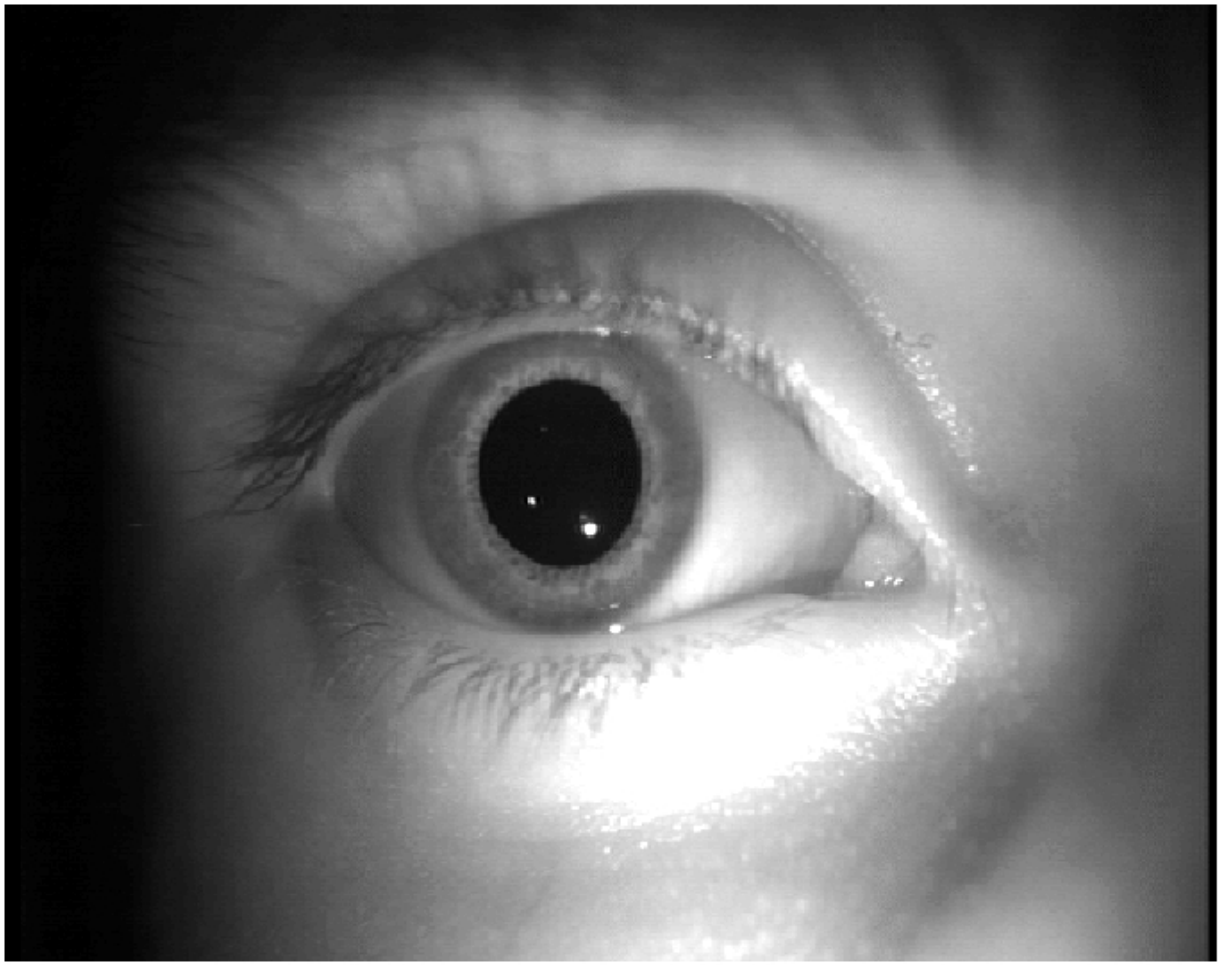

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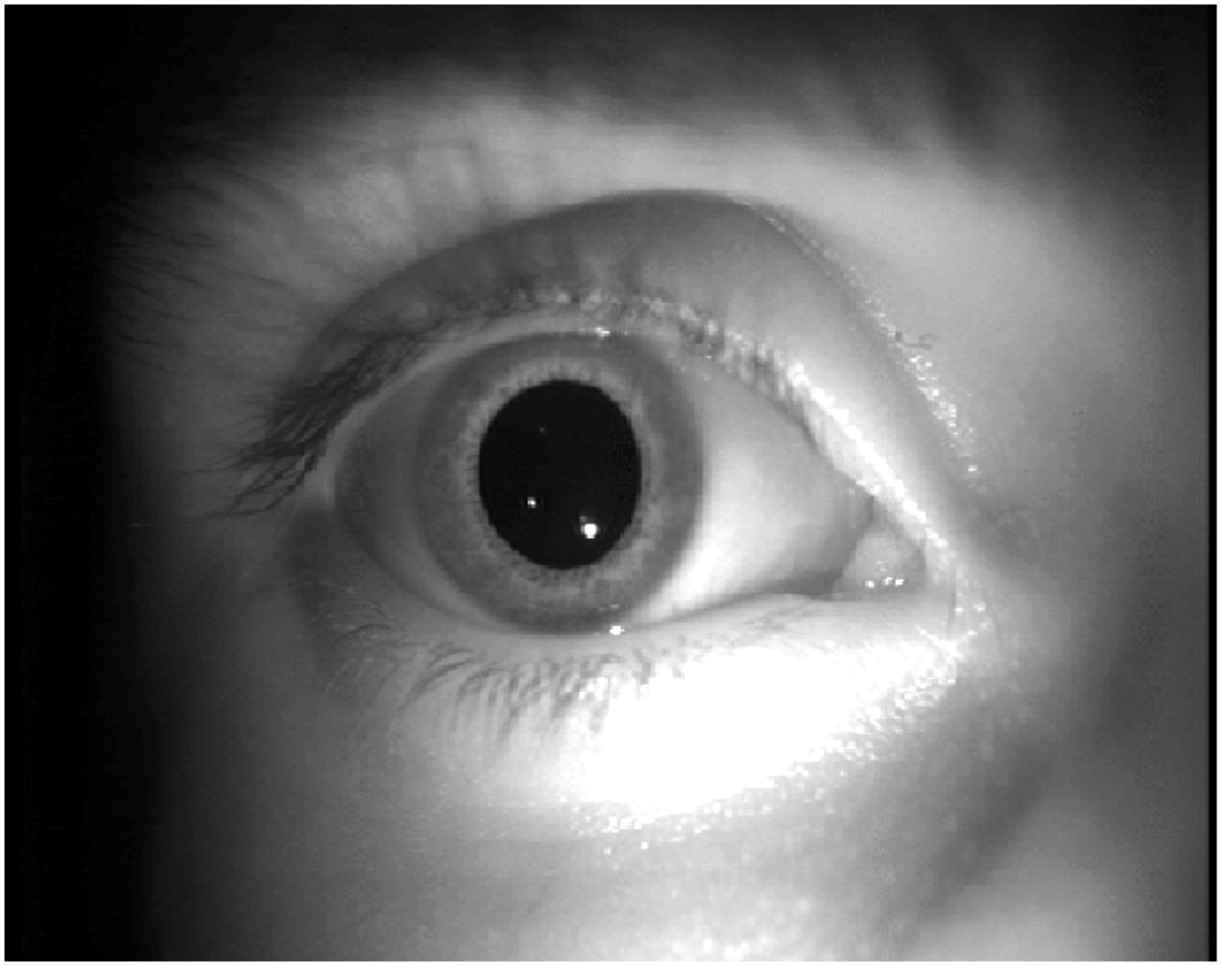

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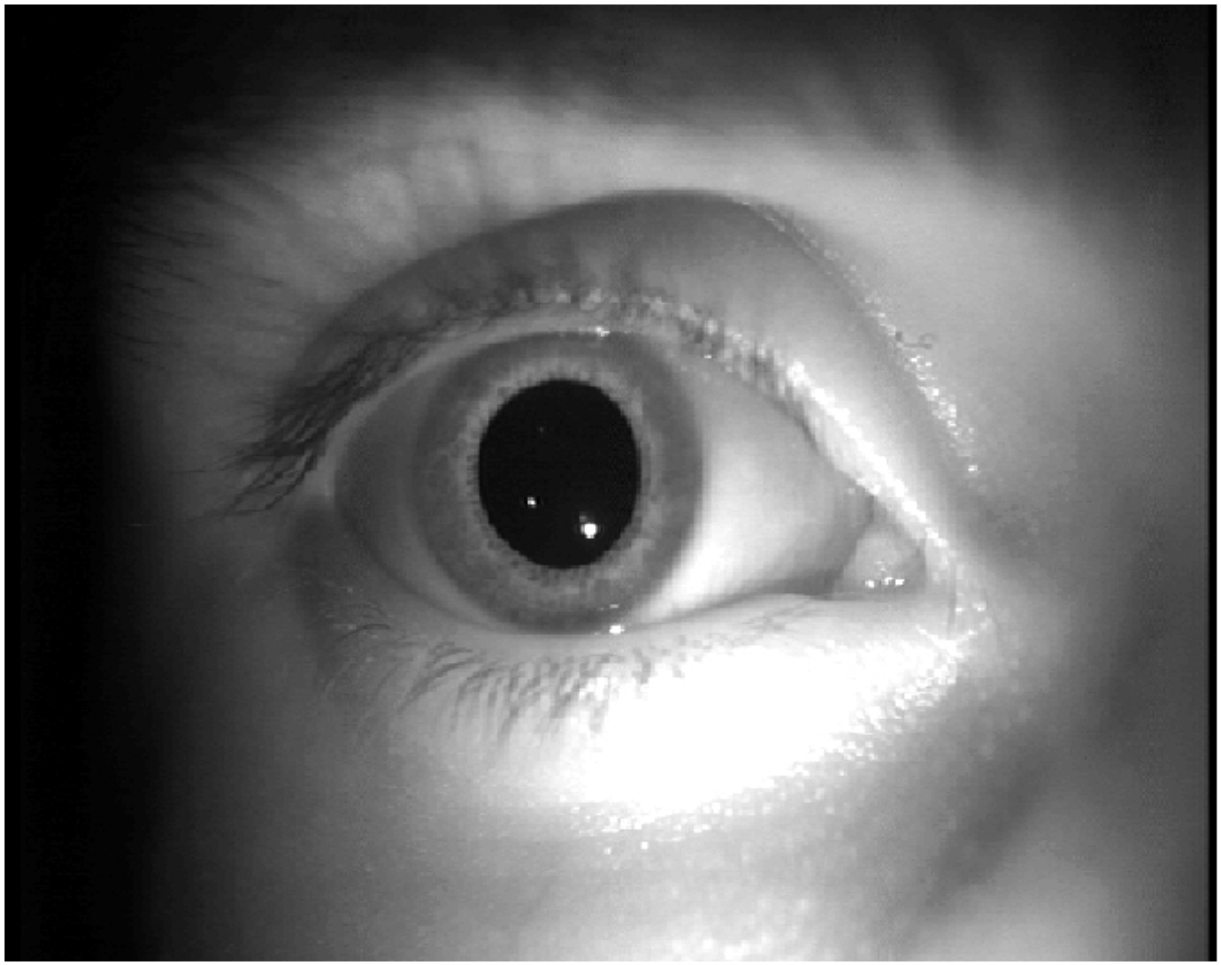

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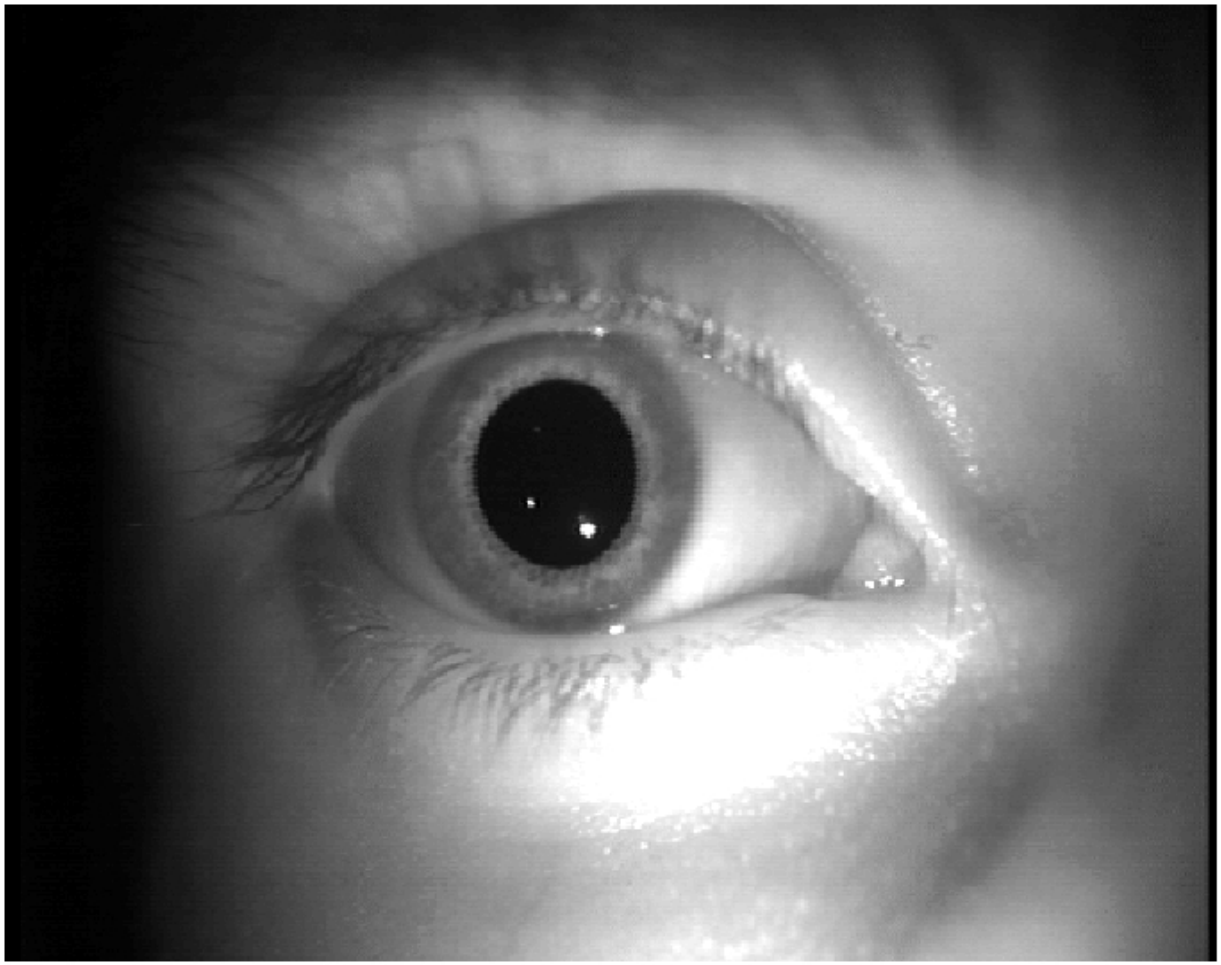

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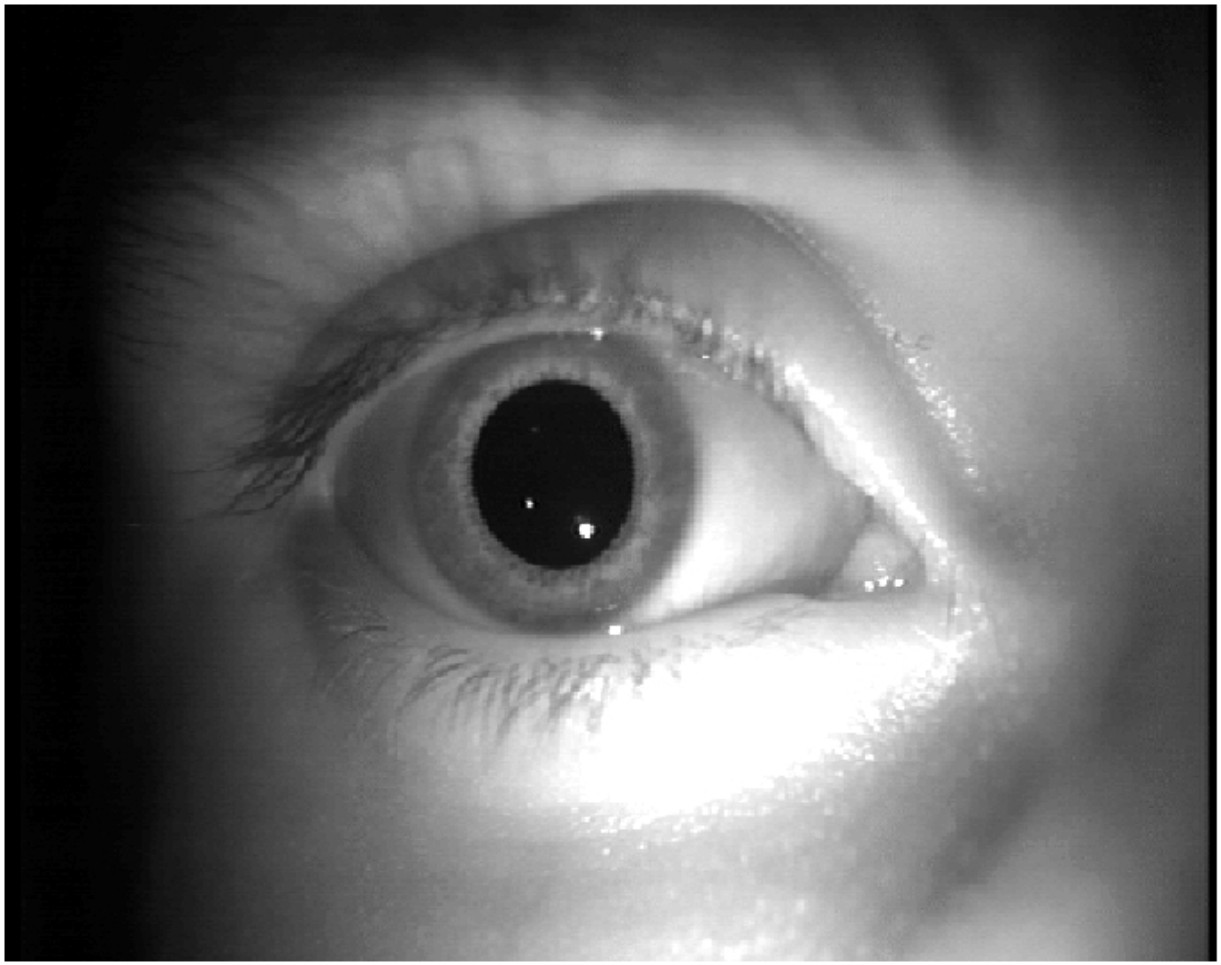

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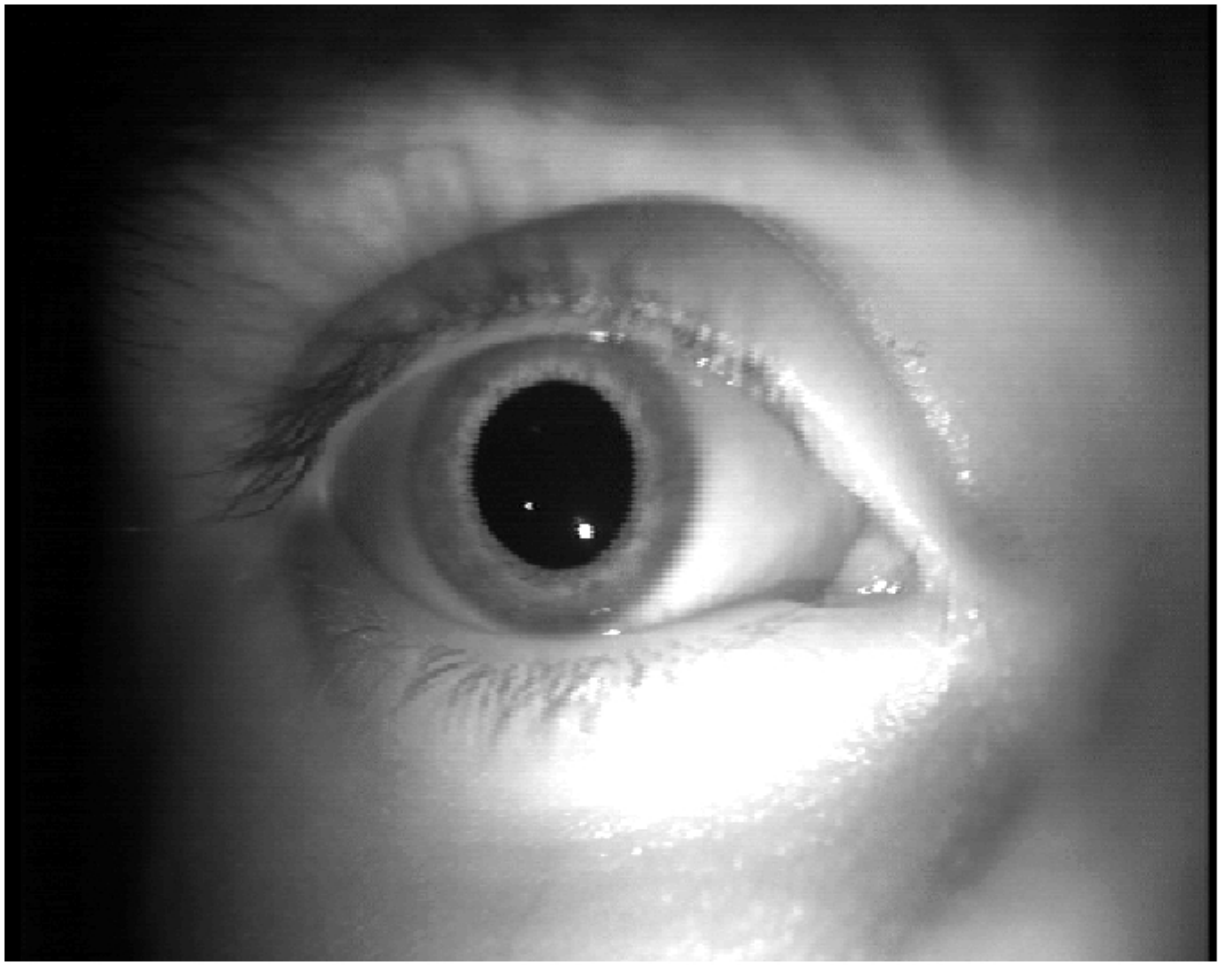

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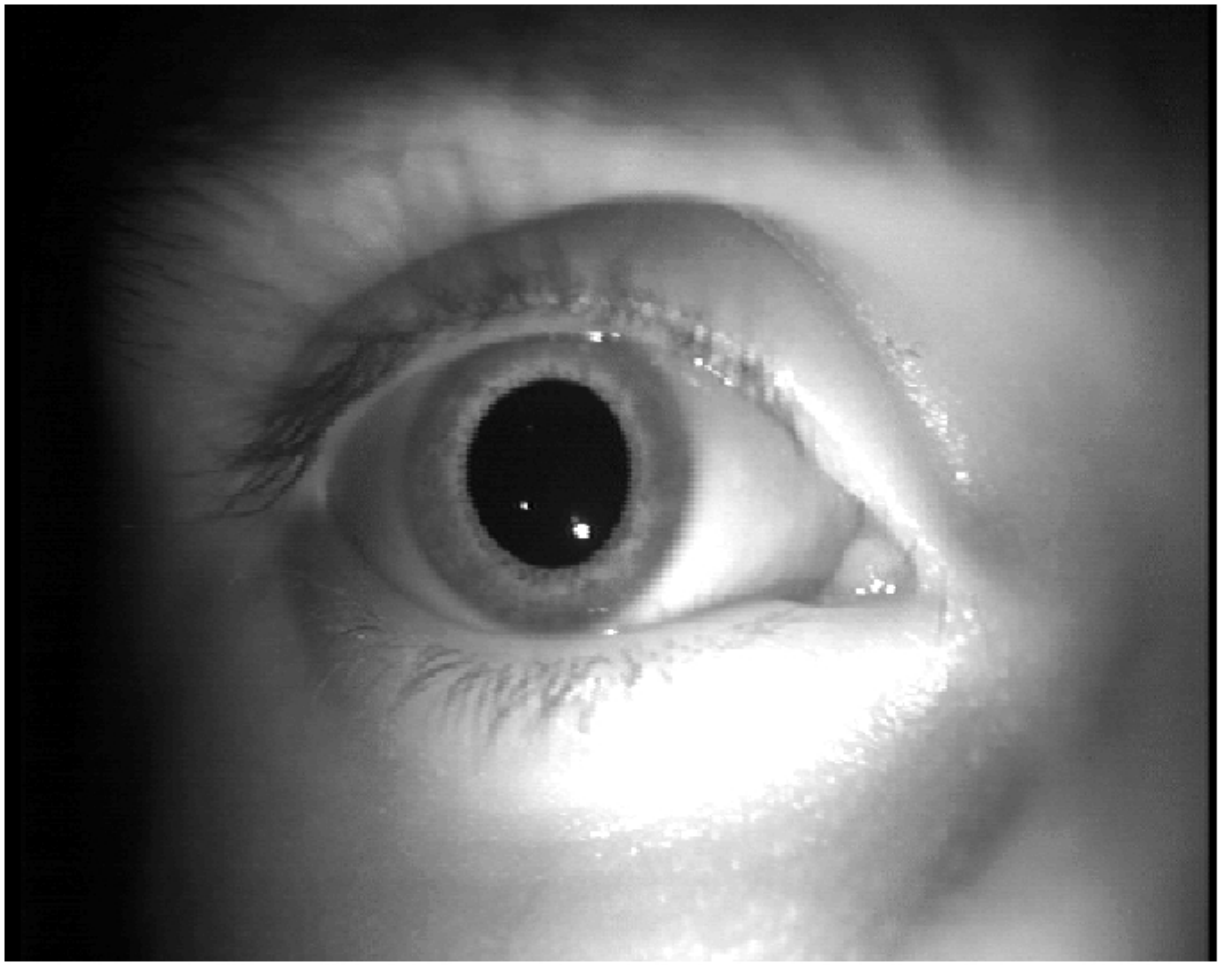

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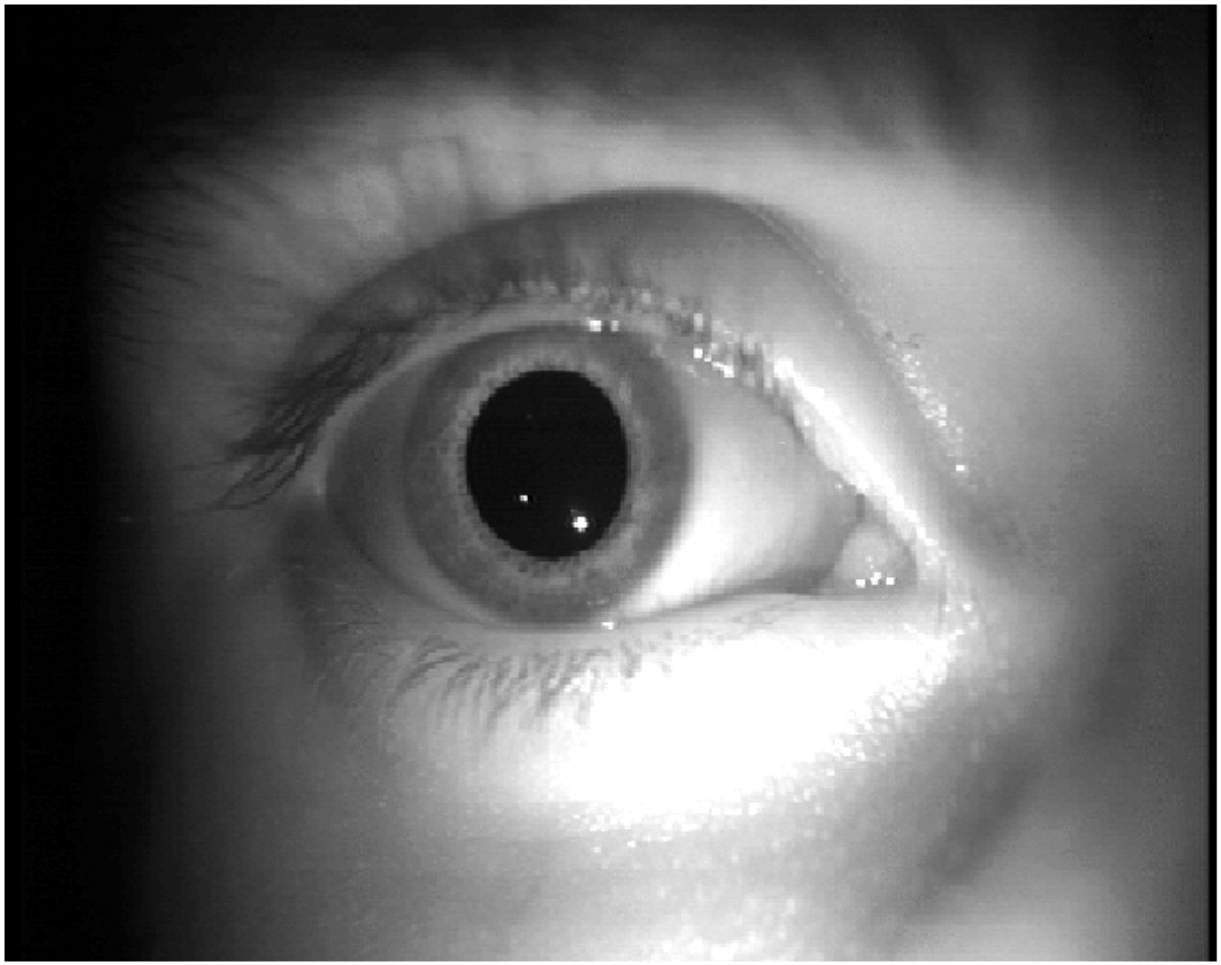

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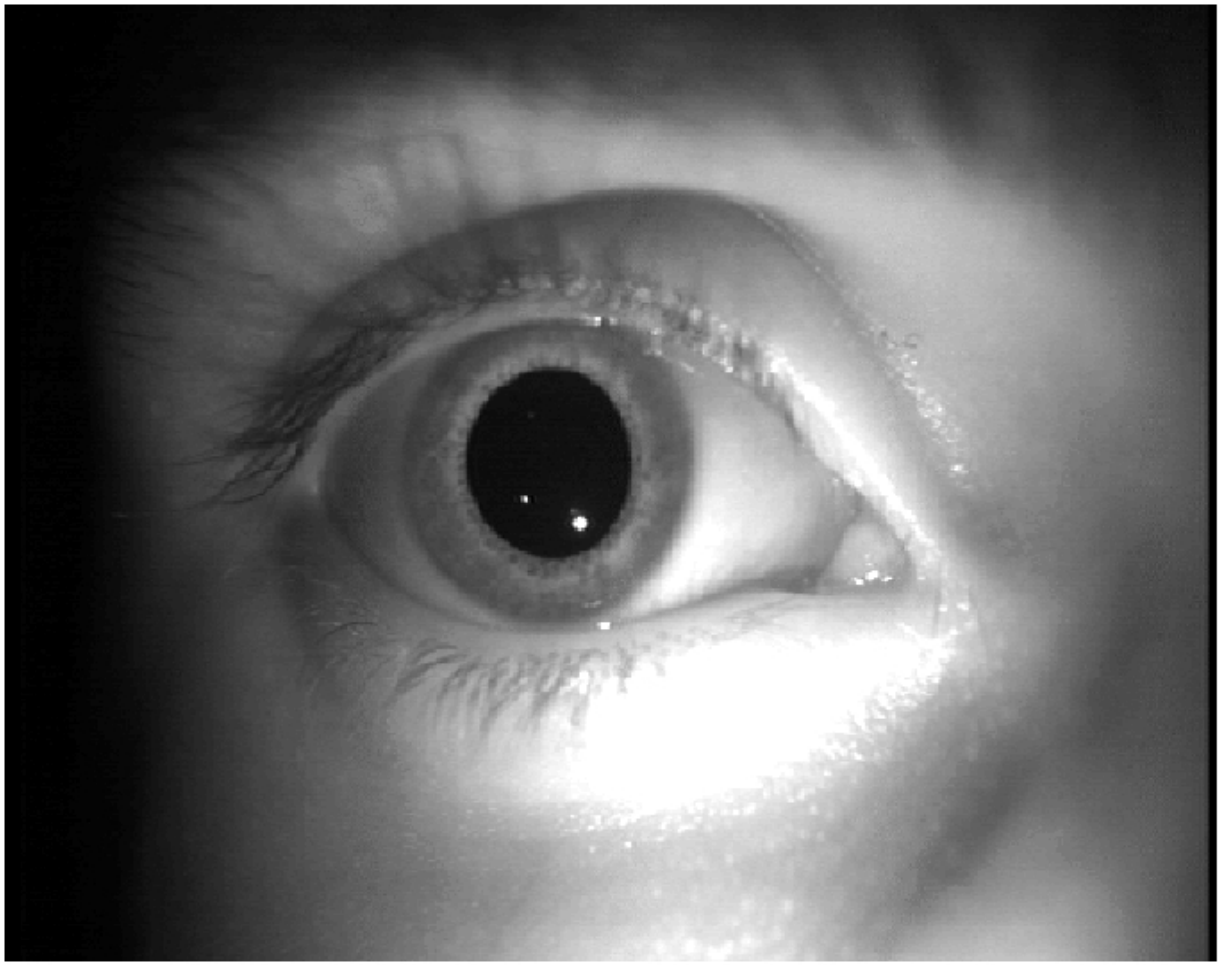
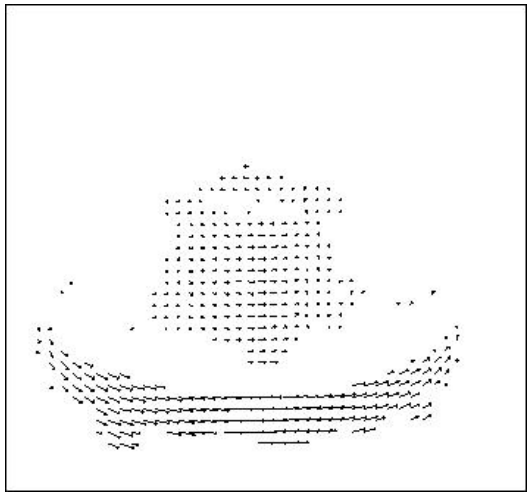


Example: Rubik Cube

Image sequence

Velocity field



Optical Flow: Assumptions

Pixel correspondence problem:

Given a pixel in the first image, look for a “nearby” pixel in the second image with the same brightness.

- Key assumptions:
  - brightness constancy
  - small motion
- Resulting flow:
  - displacement vector field
- Problems:
  - great displacements
  - changing illumination

Brightness constancy assumption

If $I(\mathbf{x}, t)$ image brightness, then

$$I(\mathbf{x}, t) \approx I(\mathbf{x} + \partial\mathbf{x}, t + \partial t),$$

where $\partial\mathbf{x}$ is the displacemente of $\mathbf{x}$ after time ∂t.

Taylor series

$$I(\mathbf{x} + \partial\mathbf{x}, t + \partial t) = I(\mathbf{x}, t) + \nabla I \cdot \mathbf{v} + \frac{\partial I}{\partial t} + H.O.T.,$$

where $\mathbf{v} = \frac{\partial \mathbf{x}}{\partial t}$

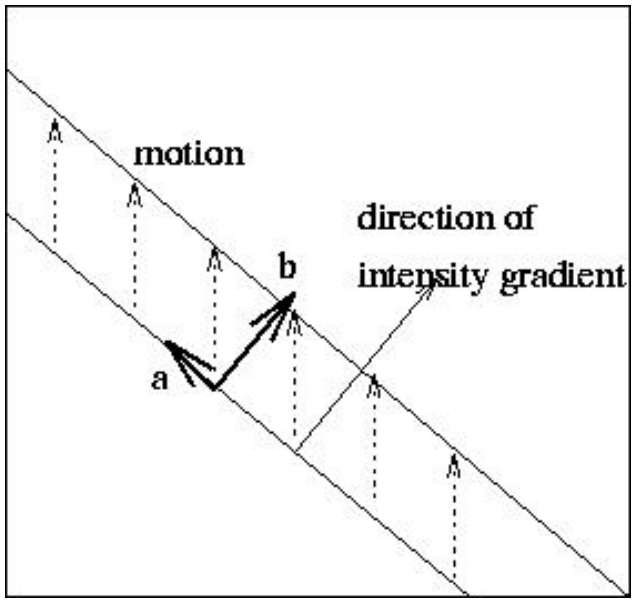
Optical Flow Constraint

Brightness constancy and small motion (vanishing H.O.T.) yuild:

$$\frac{dI(\mathbf{x},t)}{dt} = \nabla I \cdot \mathbf{v} + \frac{\partial I}{\partial t} = 0,$$

- two velocity components, one equation – underdefined problem
- so called aperture problem
- another constraint is required

Aperture Problem



Only the normal component of the velocity $\mathbf{v}$ in ∇I direction is known:

$$v_n = \mathbf{v} \cdot \frac{\nabla I}{\|\nabla I\|} = -\frac{I_t}{\|\nabla I\|}$$

the tangential component v_τ is unknown.

Fazit: locally we can not see the tangential motion.

Optical Flow Techniques

Classification:

- Differential techniques
  - Global methods
    - Horn and Schunck (1st order)
    - Nagel (2nd order)
  - Local methods (Lucas and Kanade)
- Region-based matching
- Frequency-based methods

Note: All OF techniques can use the hierarchical (coarse-to-fine) refinement.

Horn-Schunck Method

Regularisation: the optical flow constraint

$$\nabla I \cdot \mathbf{v} + \frac{\partial I}{\partial t} = 0$$

combined with a smoothness assumption based on:

$$\|\nabla v_x\|^2 + \|\nabla v_y\|^2$$

(Another measure of smoothness: $\Delta v_x + \Delta v_y$)

Look for $\mathbf{v} = (v_x, v_y)$ minimizing functional:

$$E(\mathbf{v}) = \int_\Omega \left(\nabla I \cdot \mathbf{v} + \frac{\partial I}{\partial t}\right)^2 + \lambda(\|\nabla v_x\|^2 + \|\nabla v_y\|^2) d\mathbf{x}$$

Horn-Schunck Method

Minimization of $E(\mathbf{v})$:

Variational calculus:

$$\begin{aligned} I_x(I_x\bar{v}_x + I_y\bar{v}_y + I_t) + \lambda\Delta v_x &= 0 \\ I_y(I_x\bar{v}_x + I_y\bar{v}_y + I_t) + \lambda\Delta v_y &= 0 \end{aligned}$$

Discretising of derivatives via finite differences:

$$\Delta v = v - \bar{v},$$

where

$$\bar{v} = v * M,$$

are local averages with mask

$$M = \begin{pmatrix} 1/12 & 1/6 & 1/12 \\ 1/6 & 0 & 1/6 \\ 1/12 & 1/6 & 1/12 \end{pmatrix}$$

Horn-Schunck Method

Discretized equations in separated form:

$$\begin{aligned}(\lambda + I_x^2 + I_y^2)(v_x - \bar{v}_x) &= -I_x(I_x\bar{v}_x + I_y\bar{v}_y + I_t)\\ (\lambda + I_x^2 + I_y^2)(v_y - \bar{v}_y) &= -I_y(I_x\bar{v}_x + I_y\bar{v}_y + I_t)\end{aligned}$$

Gauss-Seidel Iteration:

$$\begin{aligned}v_x^{n+1} &= \bar{v}_x^n - I_x\frac{I_x\bar{v}_x^n + I_y\bar{v}_y^n + I_t}{\lambda + I_x^2 + I_y^2}\\ v_y^{n+1} &= \bar{v}_y^n - I_y\frac{I_x\bar{v}_x^n + I_y\bar{v}_y^n + I_t}{\lambda + I_x^2 + I_y^2}\end{aligned}$$

with $\bar{v}_x, \bar{v}_y$ local averages of v_x, v_y.

Horn-Schunck Method

- Advantages
  - smooth flow
  - global information
  - accurate time derivatives, using more then two frames, possible
- Disadvantages
  - iterative method: slow
  - unsharp boundaries

Lucas-Kanade Method

Assume the velocity $\mathbf{v} = (v_x, v_y)$ to be constant over a small neigbourhood Ω_x of every $\mathbf{x} \in \Omega$.

Minimize for all $\mathbf{x} \in \Omega$:

$$\sum_{\mathbf{y} \in \Omega_x} W(\mathbf{y})[\nabla I(\mathbf{y}, t) \cdot \mathbf{v} + I_t(\mathbf{y}, t)],$$

where $W(\mathbf{x})$ is a weight function.

Solution via normal equation:

$$A^T W A \mathbf{v} = A^T W \mathbf{b},$$

where

$$\begin{aligned} A &= [\nabla I(\mathbf{x}_1), \ldots, \nabla I(\mathbf{x}_n)]^T \\ W &= \mathsf{diag}[W(\mathbf{x}_1), \ldots, W(\mathbf{x}_n)] \\ \mathbf{b} &= -[I_t(\mathbf{x}_1), \ldots, I_t(\mathbf{x}_n)] \end{aligned}$$

Lucas-Kanade Method

- Advantages
  - easy and fast calculation
  - accurate time derivatives
- Disadvantages
  - errors on boundaries

Best combination between accuracy and speed.

Region-based Matching

Define velocity $\mathbf{v}$ as shift $\mathbf{d} = (d_x, d_y)$.

Consider sum-of-squared difference between two frames I_1 and I_2:

$$\begin{aligned} SSD(\mathbf{x}, \mathbf{d}) &= \sum_{\mathbf{y} \in \Omega_x} W(\mathbf{y} - \mathbf{x})[I_1(\mathbf{y}) - I_2(\mathbf{y} + \mathbf{d})]^2 \\ &= W * [I_1(\mathbf{x}) - I_2(\mathbf{x} + \mathbf{d})]^2, \end{aligned}$$

where W 2-dim window function, $\mathbf{d}$ integer.

Ω_x is a 3×3, 5×5 etc. square with $\mathbf{x}$ in the middle.

Note: Minimizing SSD is equal to maximizing the cross-correlation, which is the sum over products $I_1(\mathbf{x})I_2(\mathbf{x} + \mathbf{d})$

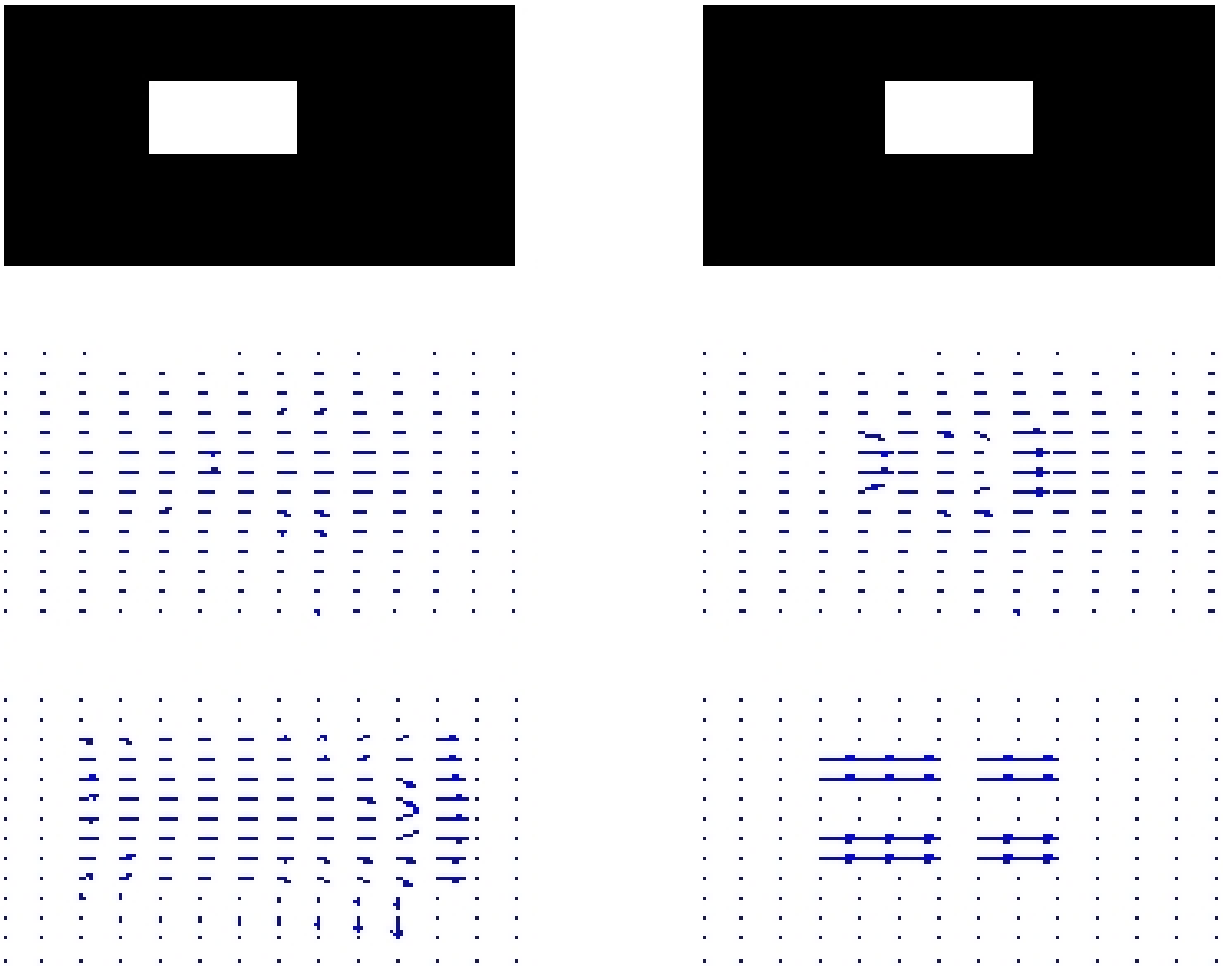
Region-based Matching

- Advantages
  - easy to calculate
- Disadvantages
  - only integer displacements: innacurate
  - only local information used
  - two-frame time derivatives: inaccurate

All methods described here work better with presmoothed data.

Simple Example: Square

Unsmoothed sequence:



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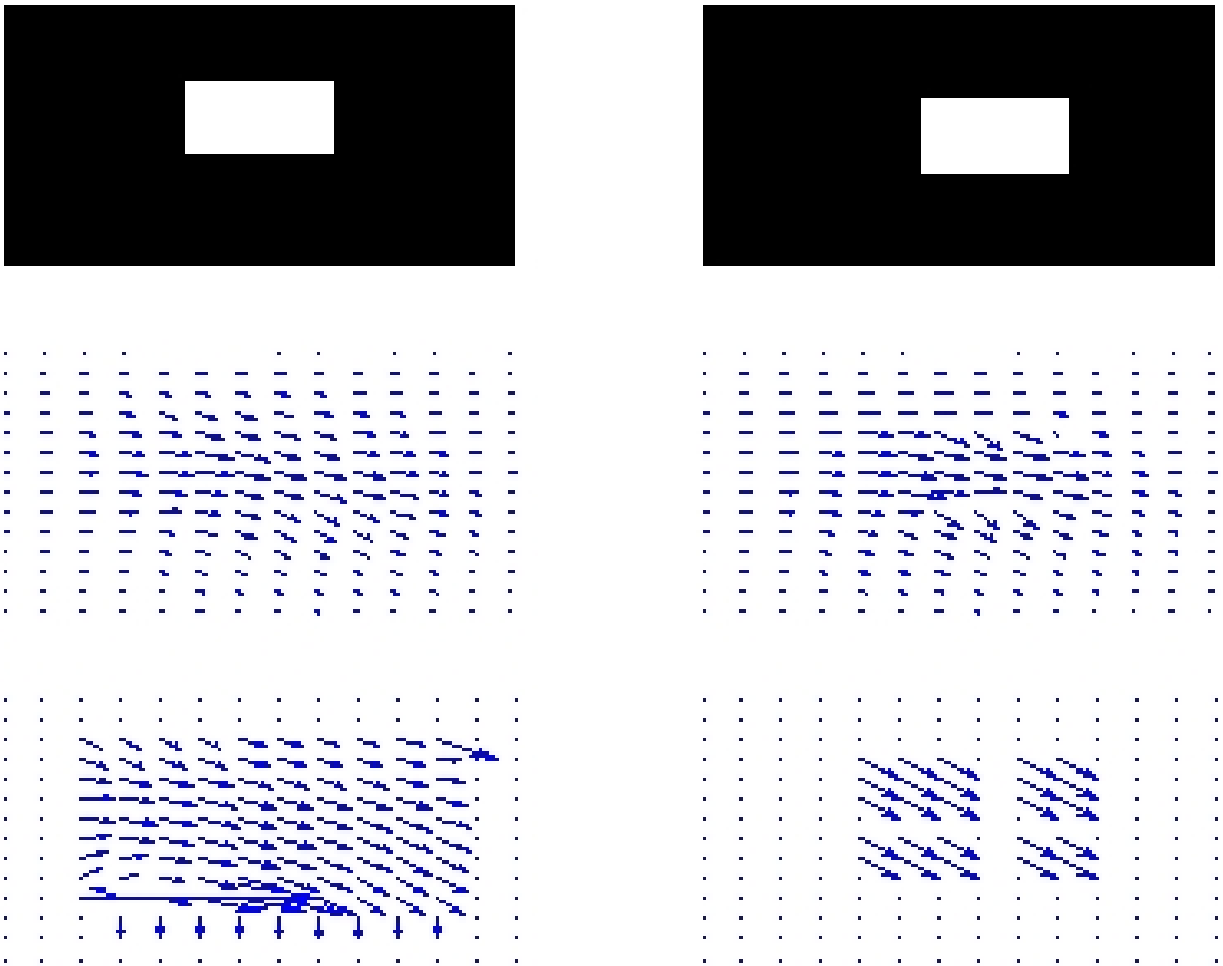

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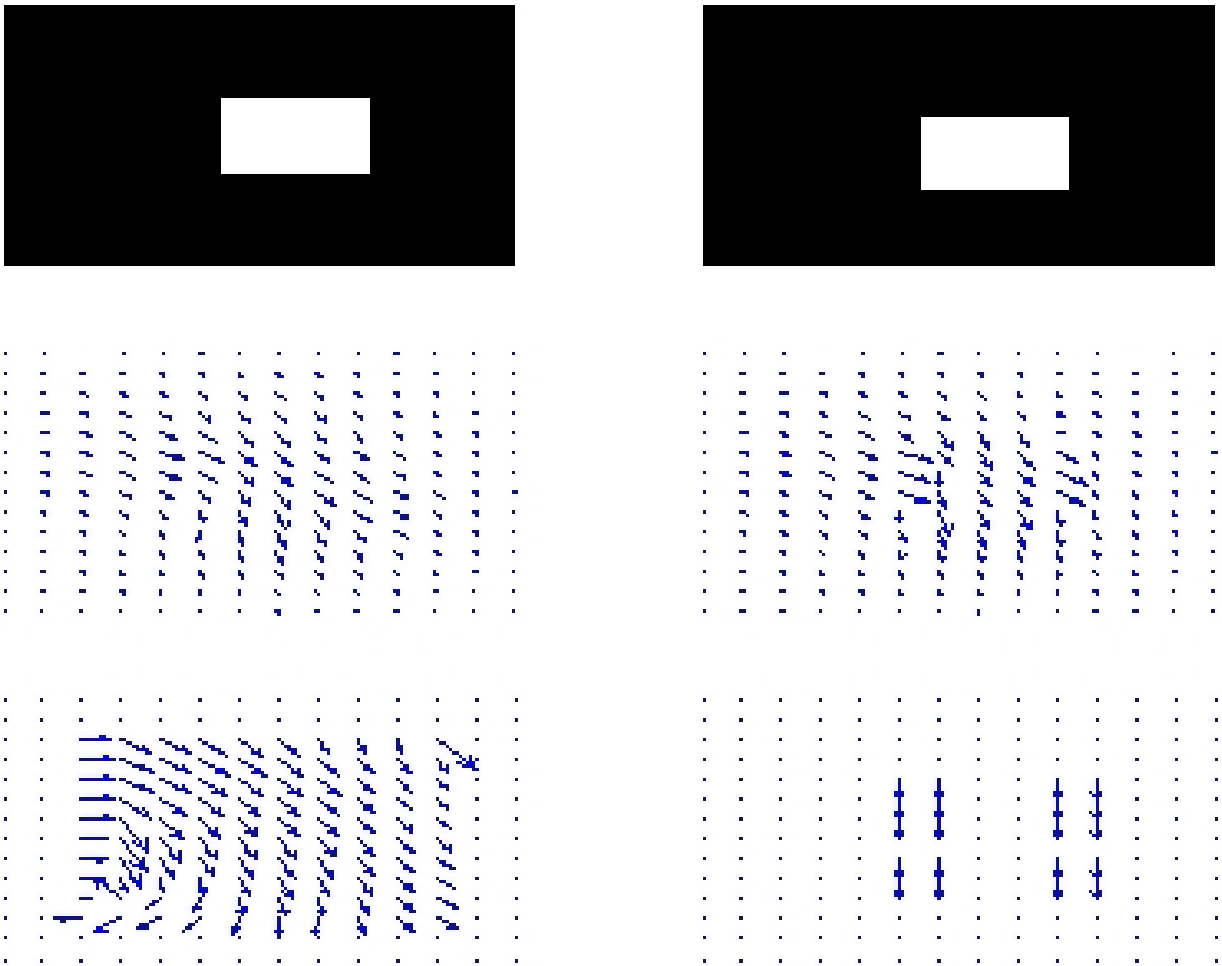

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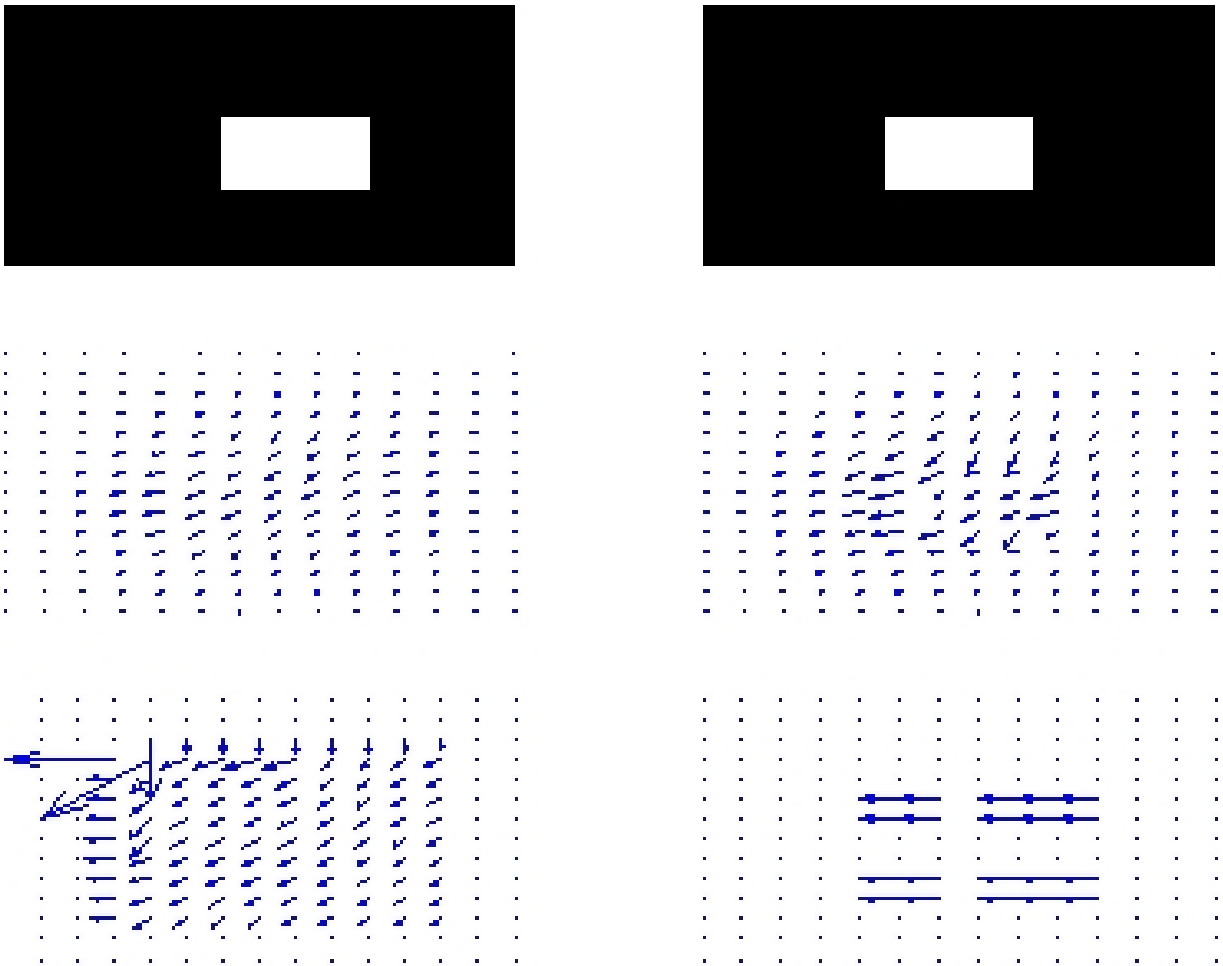

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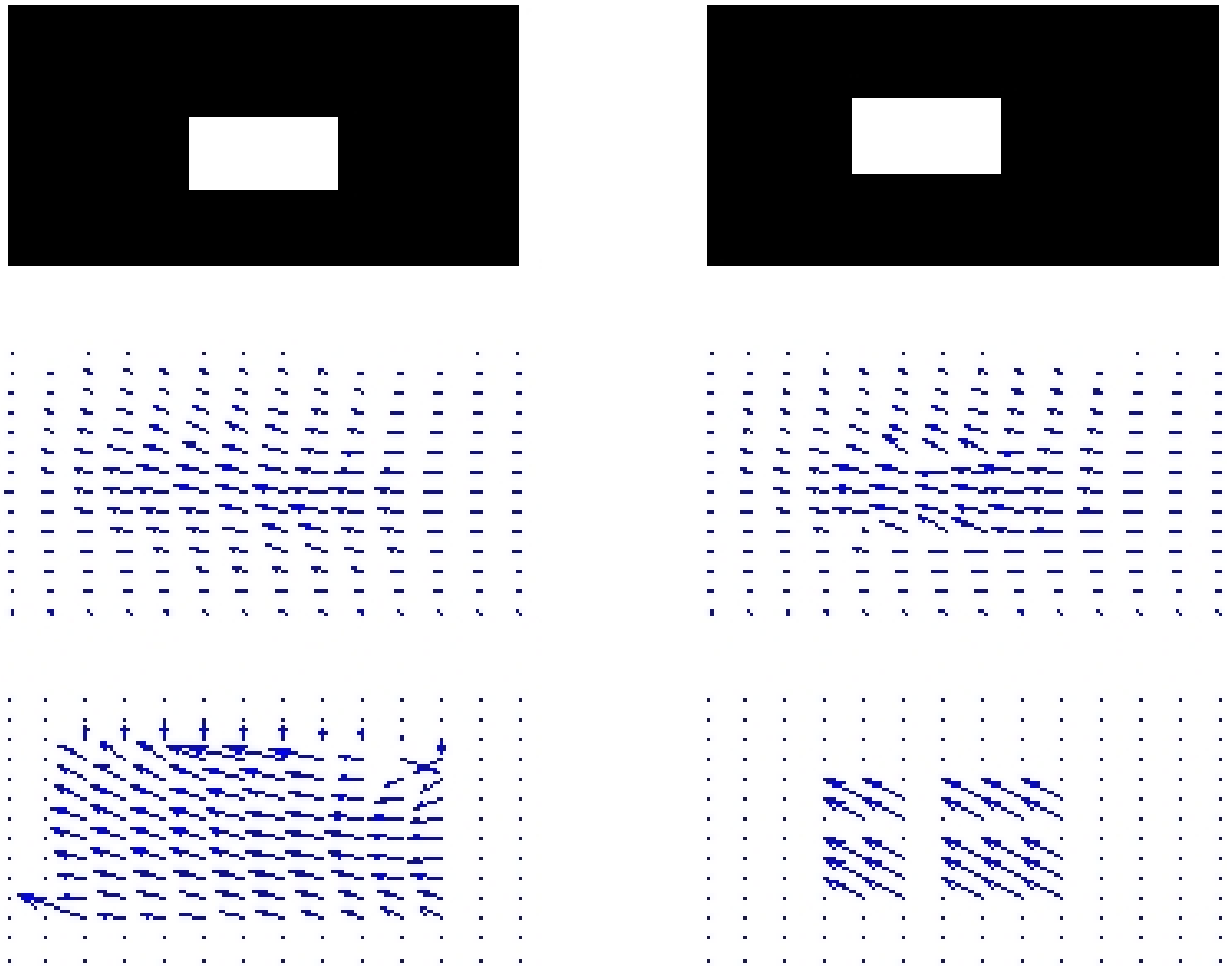
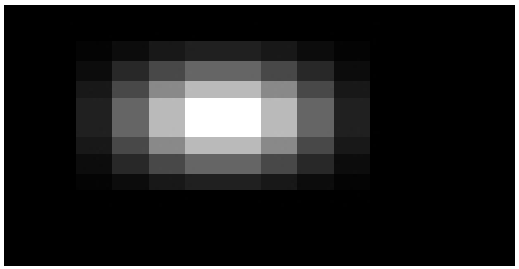
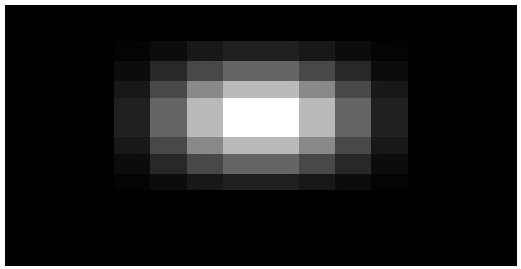
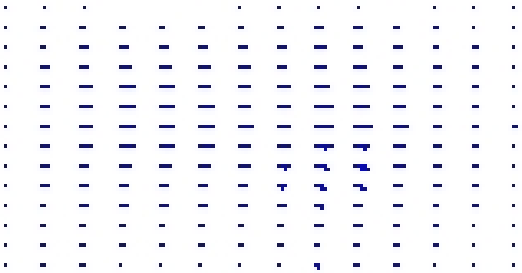
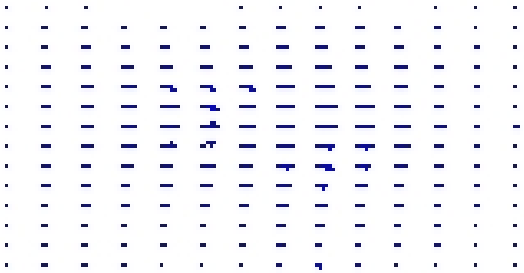
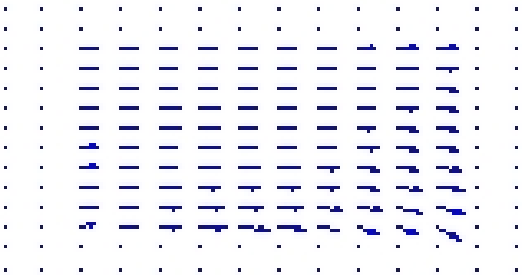
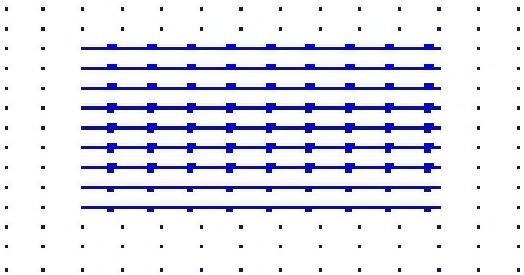


Simple Example: Square

Presmoothed sequence:

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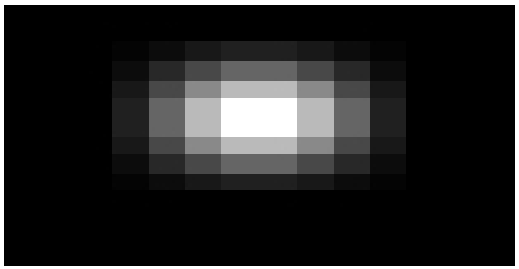
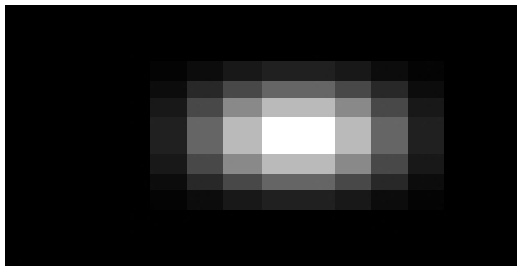
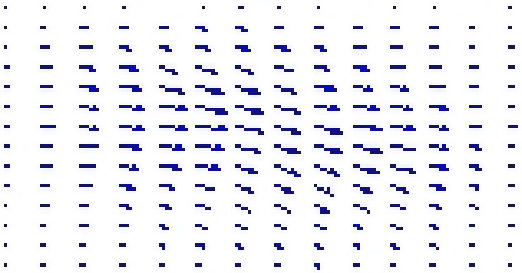
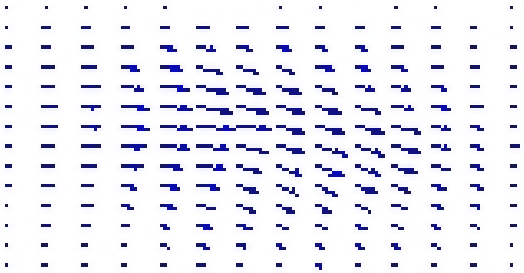
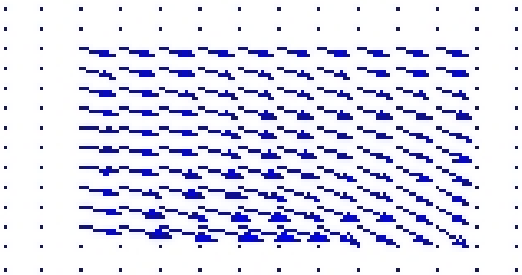
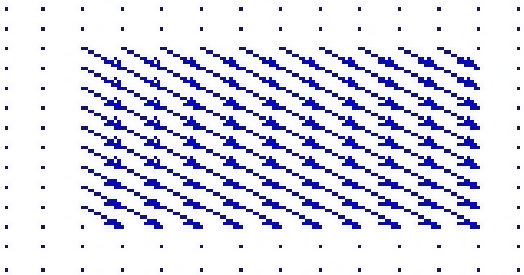
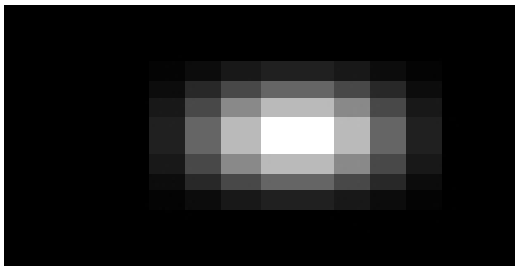
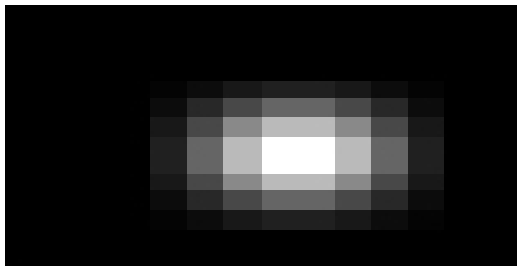
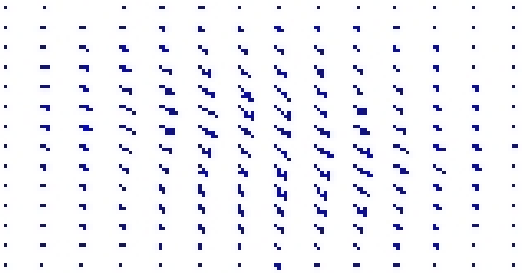







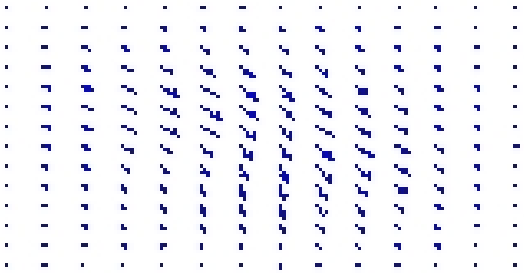
Simple Example: Square

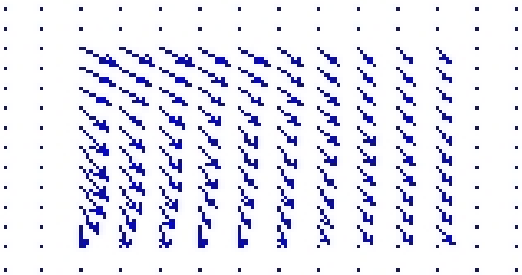
Presmoothed sequence:

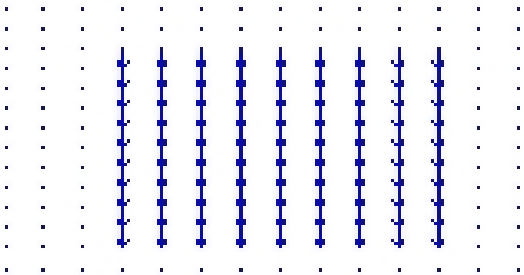






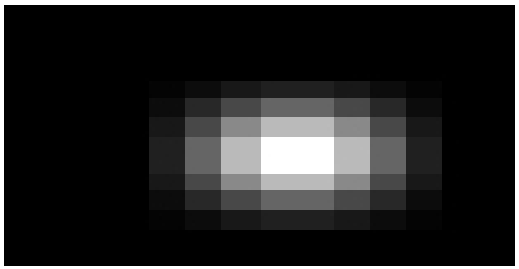
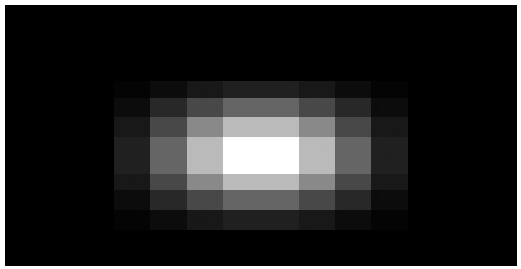
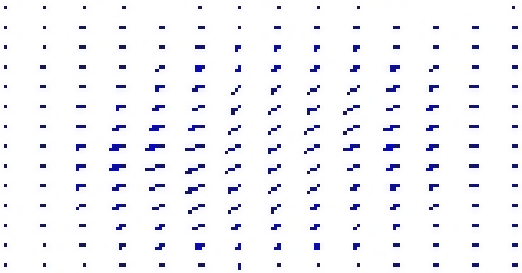
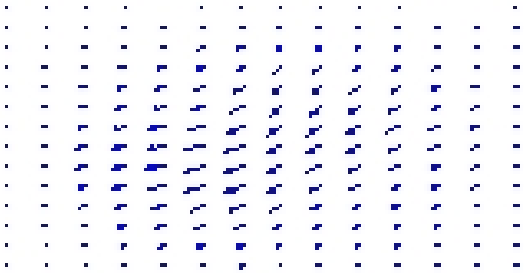
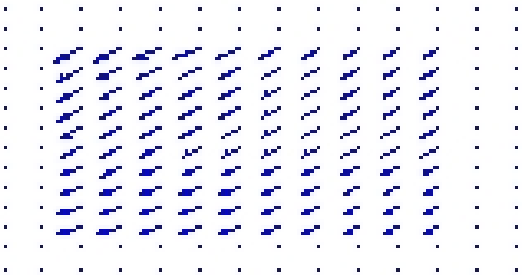
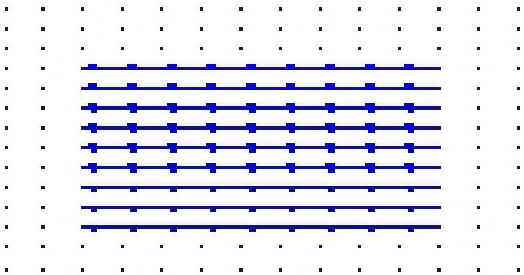






Simple Example: Square

Presmoothed sequence:

Simple Example: Square

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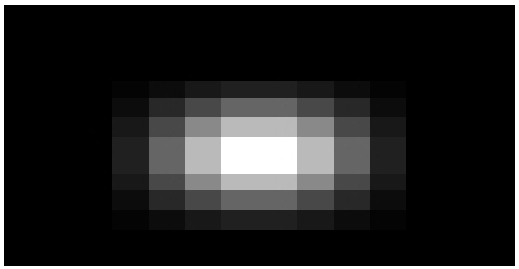
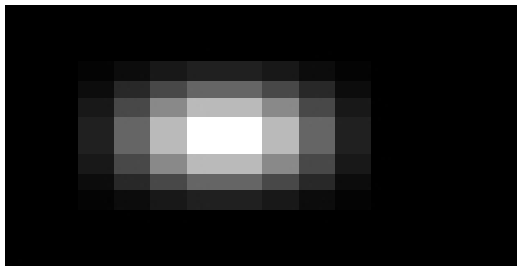
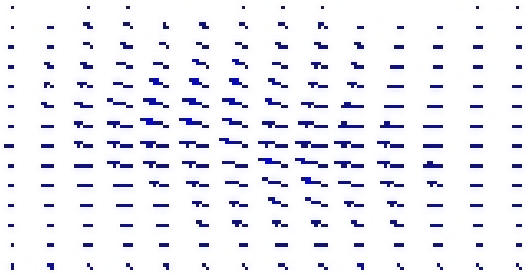
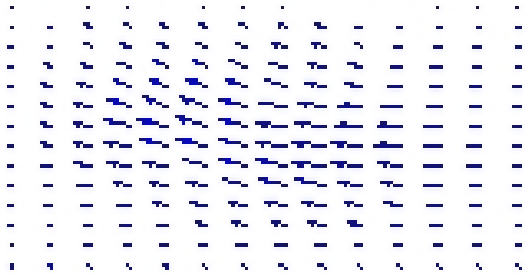
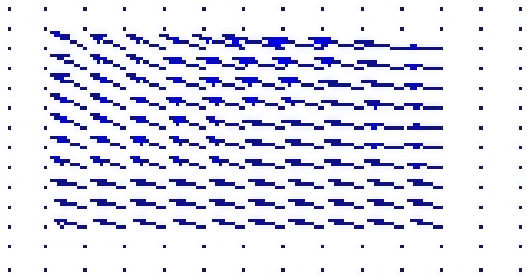
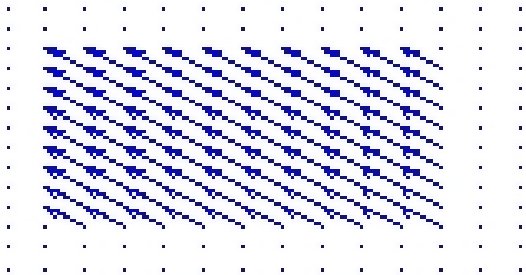
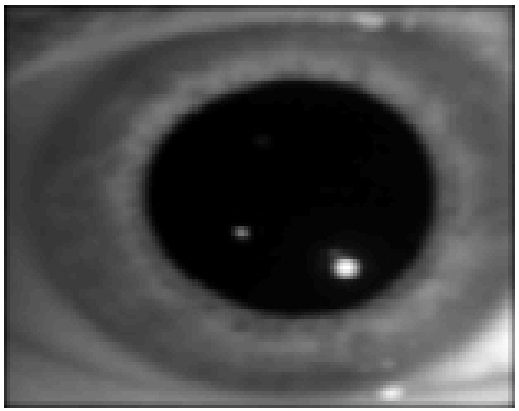
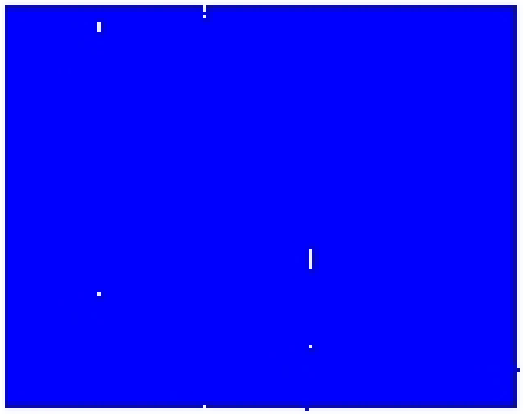
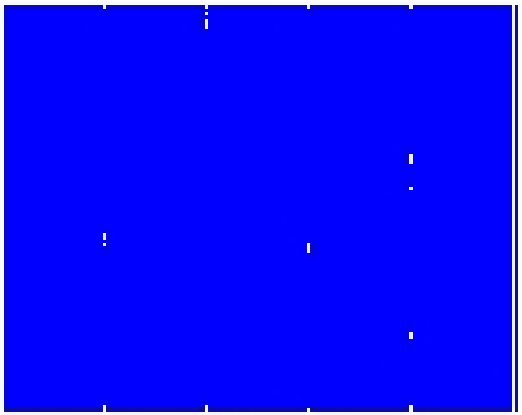
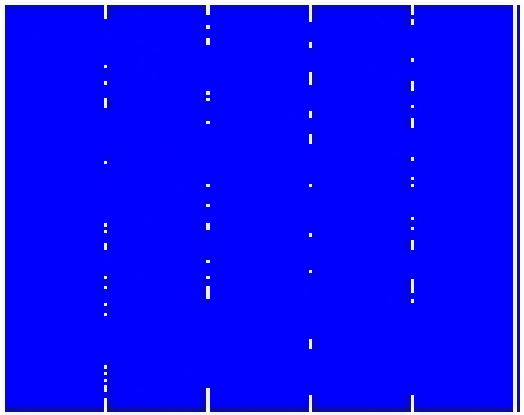







Results: Moving Eye

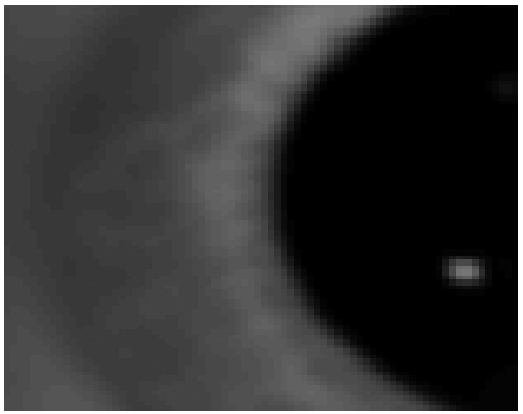


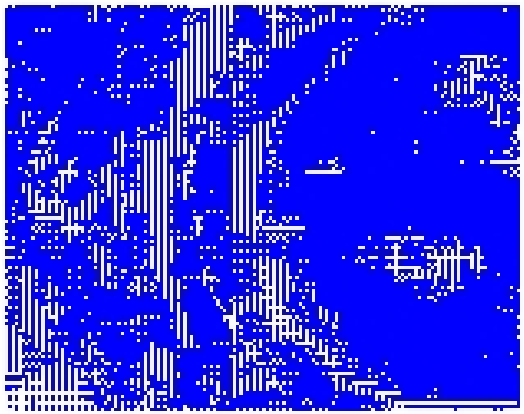


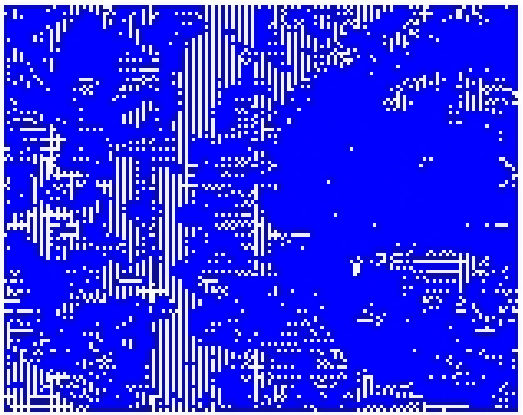


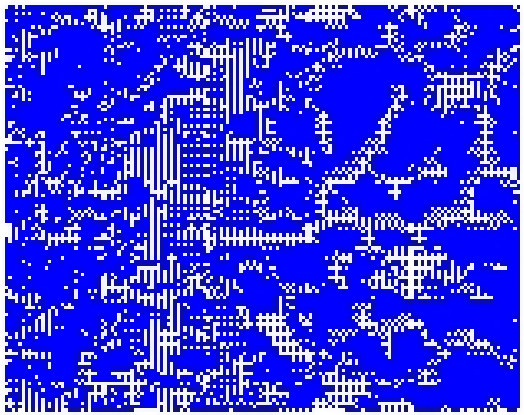


Results: Moving Eye

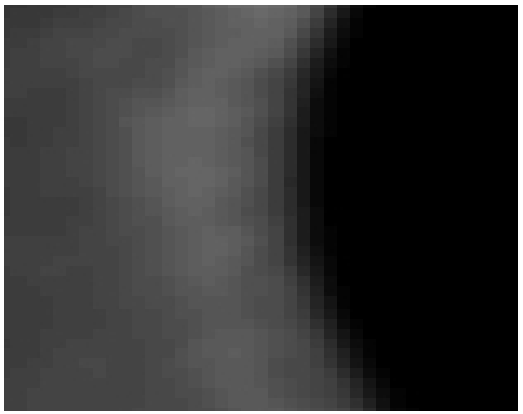


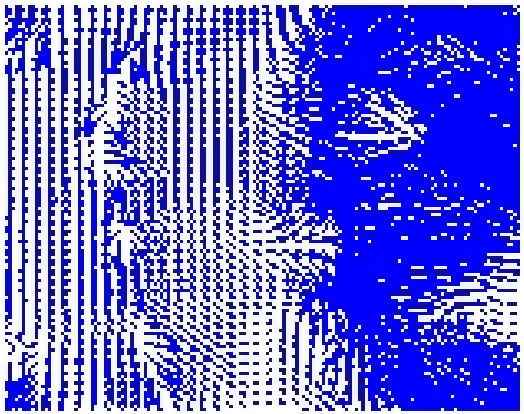


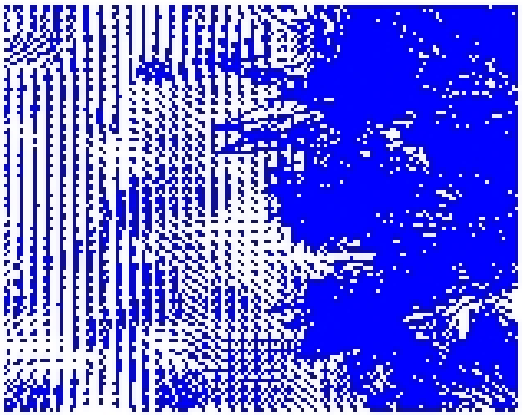


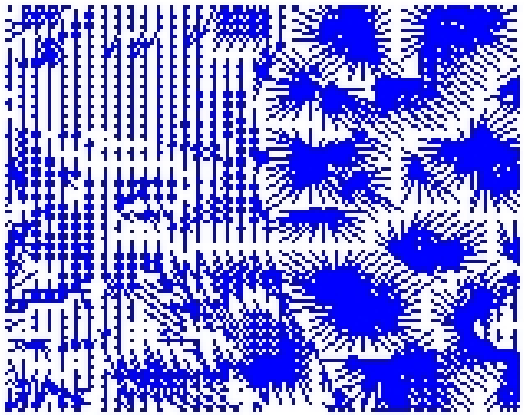


Results: Moving Eye

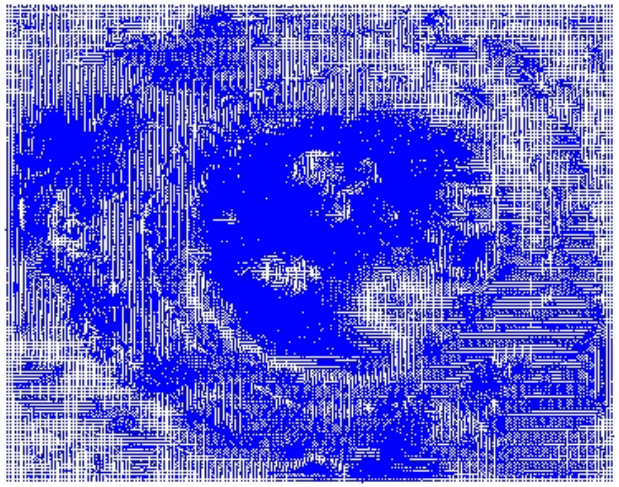


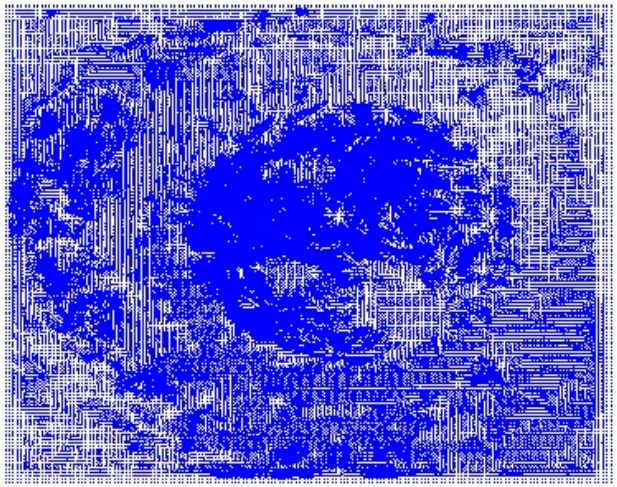




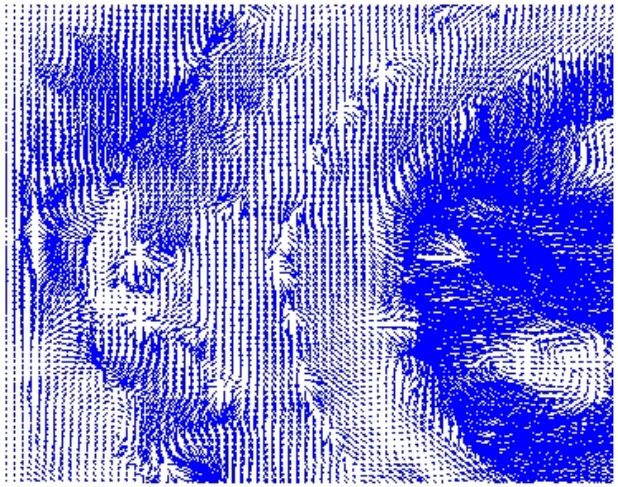


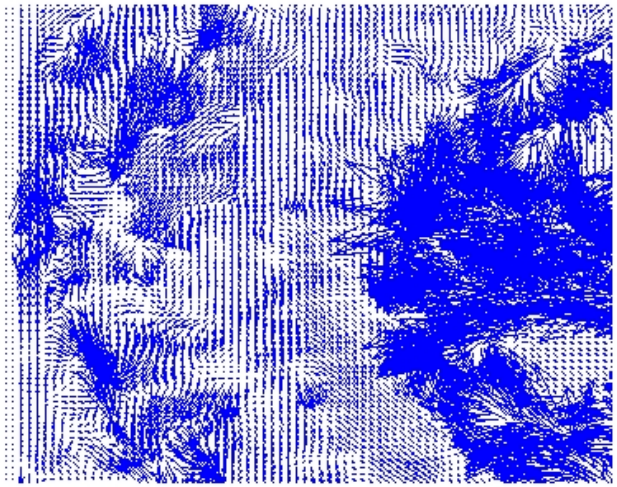
Horn-Schunck vs Lucas-Kanade



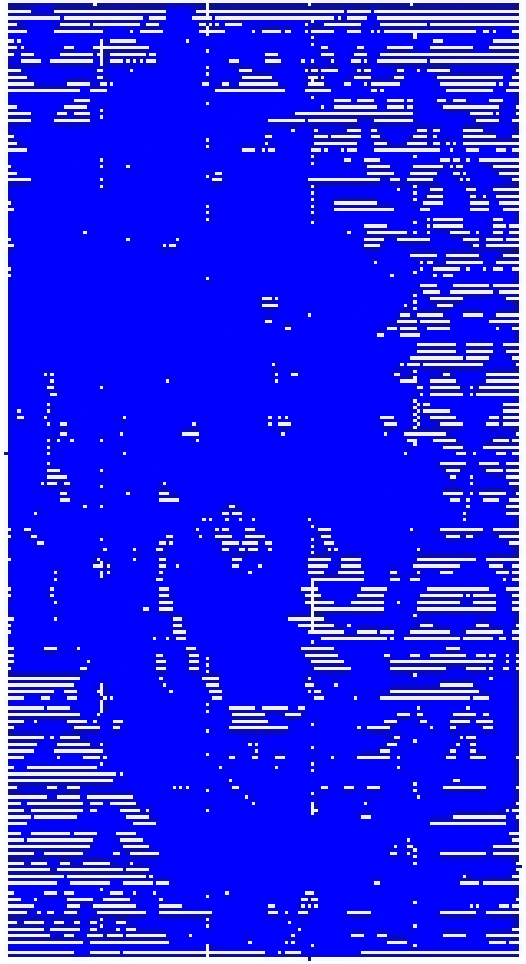


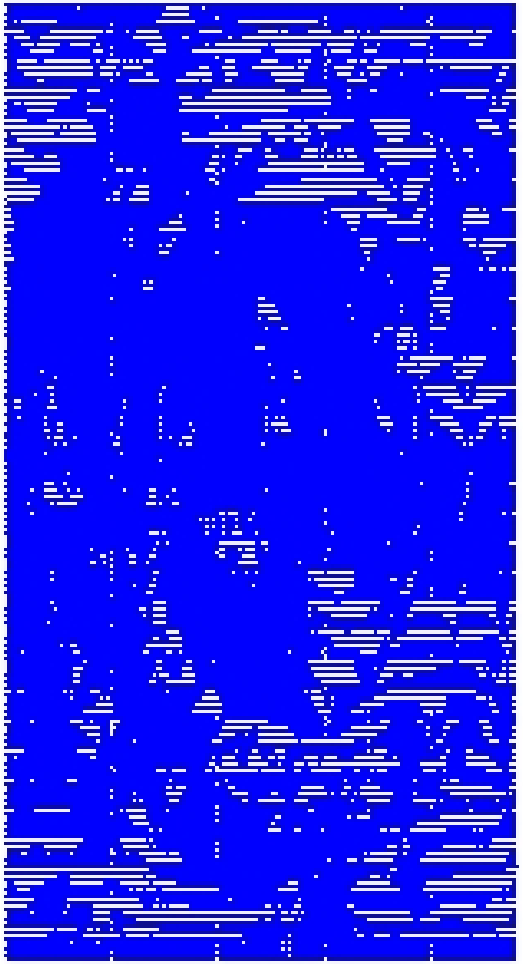
Horn-Schunck vs Lucas-Kanade



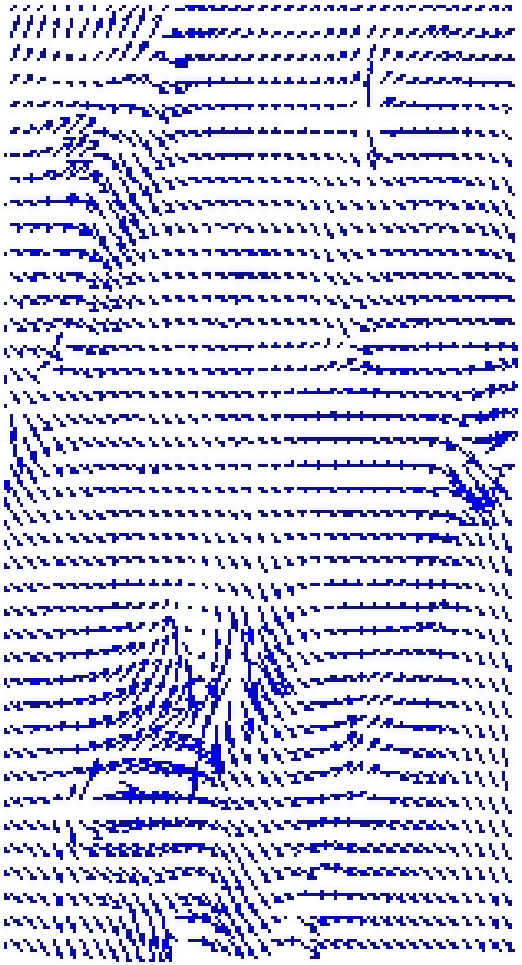


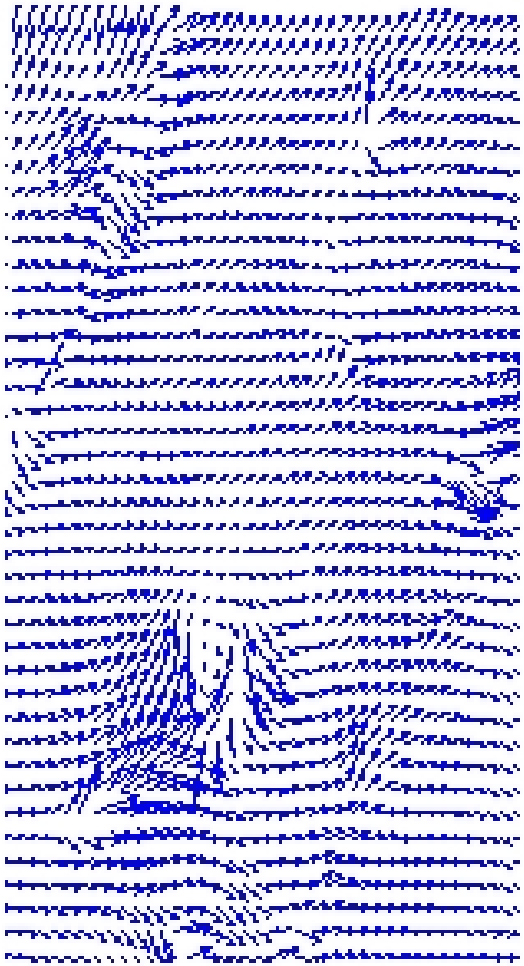
Horn-Schunk: Rotation



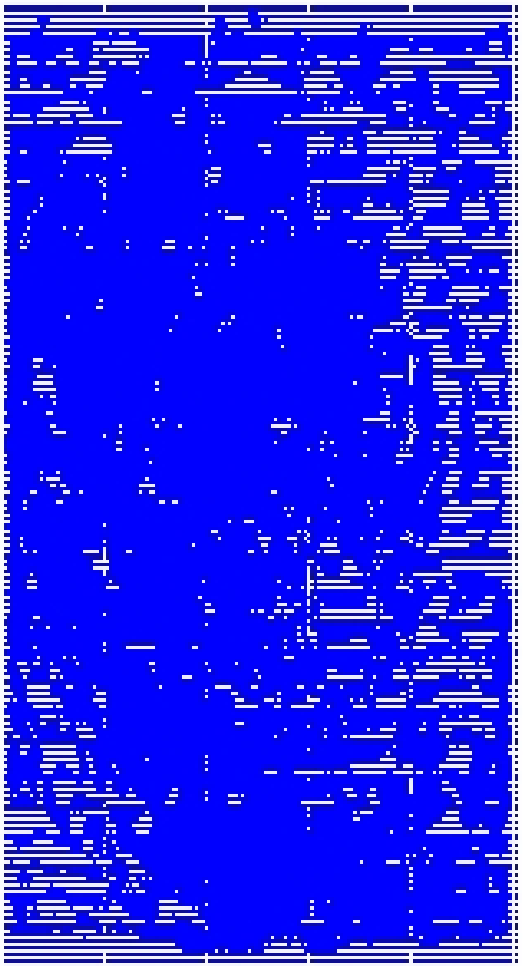


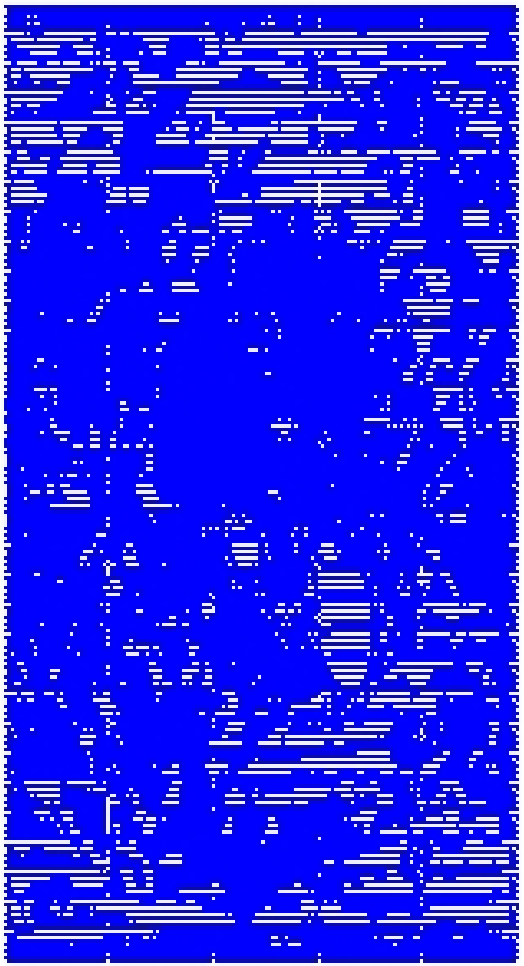
Horn-Schunk: Rotation





Lucas-Kanade: Rotation





Lucas-Kanade: Rotation