Signal restoration through deconvolution applied to deep mantle seismic probes

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Introduction
- The Physical Problem
- The Mathematical Problem

Geophysics
- Earth
- Dataset
- D" Evidence

Mathematical Model
- Forward Model
- Inverse Problem

Experiments
- Synthetic Data
- Real Data

Conclusions and Acknowledgment
- Conclusions
- Thanks and Acknowledgment

The Physical Problem

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- In particular the interface layer of the liquid core and solid mantle
- Earth's interior is inaccessible
- Remote sampling is required
- Important steps include
  - (a) **accurate characterization** of seismic energy
  - (b) reliable estimation or measurement of seismic **wave timing**

The Mathematical Problem

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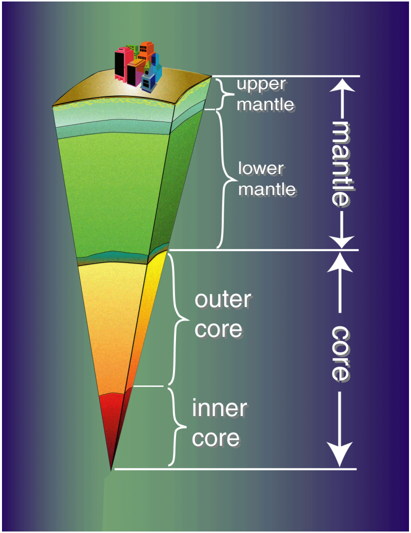
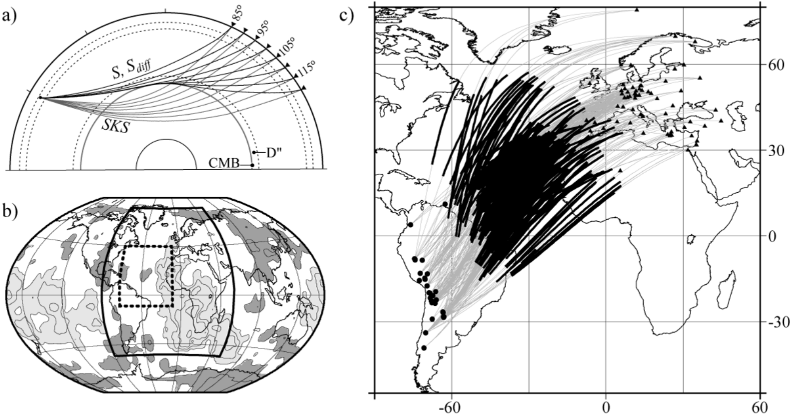
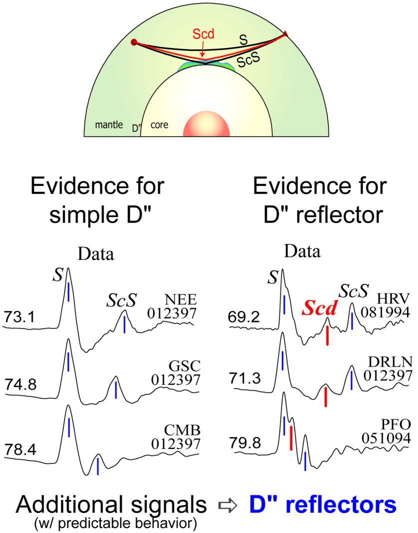
- Poor signal to noise ratio **(SNR)**
- Extremely Ill-posed


upper
mantle
lower
mantle
mantle
outer
core
inner
core
core



(Garnero, Moore, Lay, Fouch, 2004)


Scd
S
ScS
mantle
D"
core
Evidence for simple D"
Evidence for D" reflector
Data
S
ScS
73.1
NEE 012397
74.8
GSC 012397
78.4
CMB 012397
Data
S
Scd
ScS
69.2
HRV 081994
71.3
DRLN 012397
79.8
PFO 051094
Additional signals ⇨ D" reflectors
(w/ predictable behavior)

Problem Formulation

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- (b) get **timing information** to make quantitative estimates like the height of a structure.

Forward Model

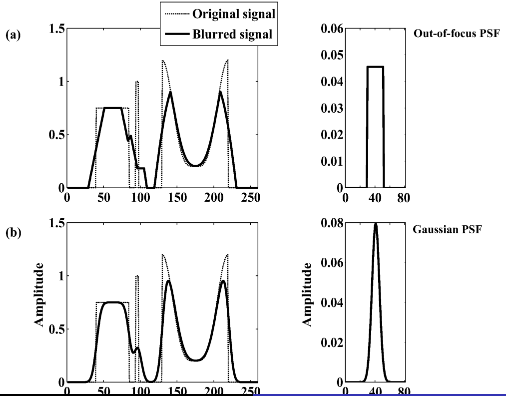
- Signal degradation is modeled as a convolution

$$g = f * h + n$$

  - where g is the blurred signal
  - f is the unknown signal
  - h is the point spread function
  - n is noise

Forward Model Example

$$g = f * h + n$$



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- **Estimations** of this PSF (in seismology wavelet) come from
  - stacking traces (problem, traces are very different)
  - estimating Earth's filter (basically a low pass filter, very difficult due to inhomogeneities)
  - use a very basic (common) shape, like a Gaussian (very rough estimate)

Gaussian Wavelet

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- σ is a width parameter, chosen such that the wavelet approximates the phase of interest.

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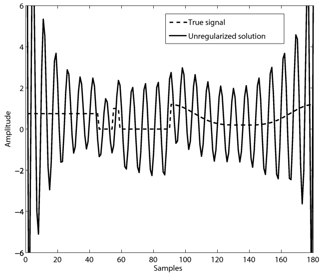
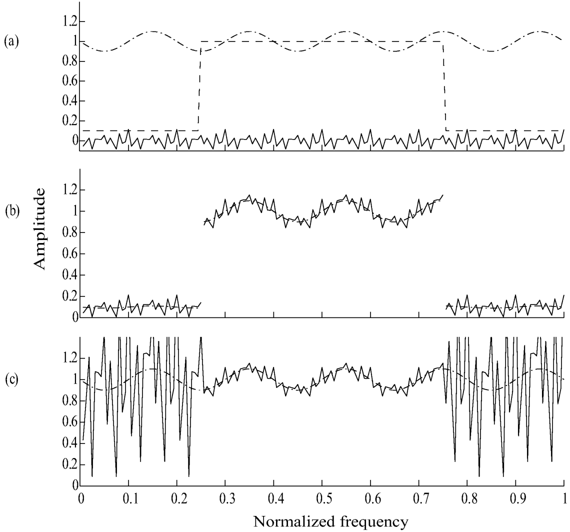
- Reconstruction with n normal distr. with $\sigma = 10^{-7}$




(a)
(b)
(c)
Amplitude
0
0.2
0.4
0.6
0.8
1
1.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Normalized frequency

Regularization

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- in latter case use a **penalty term**
- find
$$\hat{f} = \arg\min_f \{\|g - f * h\|_2^2 + \lambda R(f)\},$$
where $R(f)$ is the penalty term and λ is a penalty parameter.

Regularization Methods

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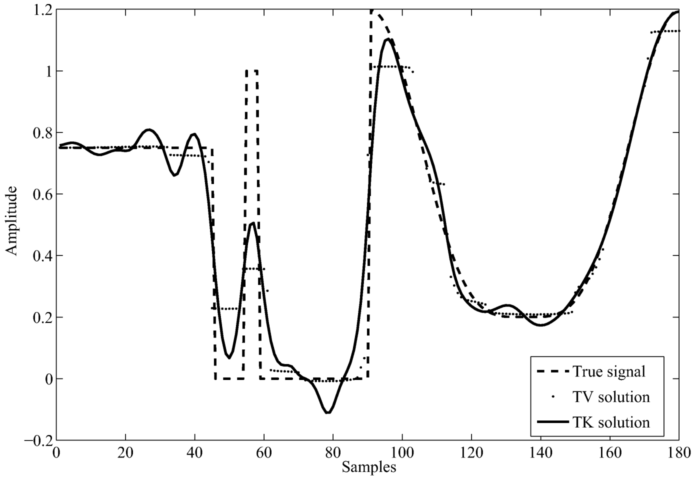
- Sparse deconvolution (L^1)

$$R(f) = \|f\|_1 = \int_{-\infty}^{\infty} |f(t)| dt.$$


1.2
1
0.8
0.6
0.4
0.2
0
−0.2
Amplitude
0
20
40
60
80
100
120
140
160
180
Samples
True signal
TV solution
TK solution

Regularization Notes

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- TV yields a piece wise constant reconstruction and preserves the edges of the signal
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- L^1 yields a spike train
- To find the minimum we use a limited memory BFGS method

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- **Evaluation** of the OF and its gradient **is cheap** (some FFTs and sparse matrix-vector multiplications)

- use synthetic data from 1d model

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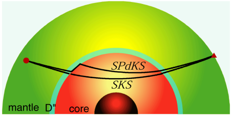
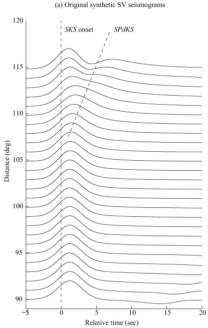
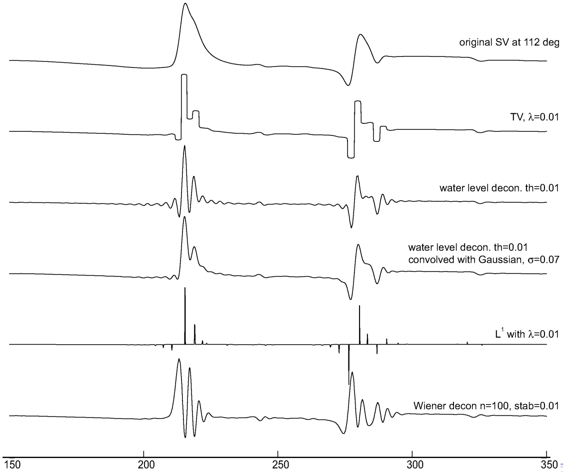
- at a critical angle of about 110 deg *SKS* starts to diffract along the core

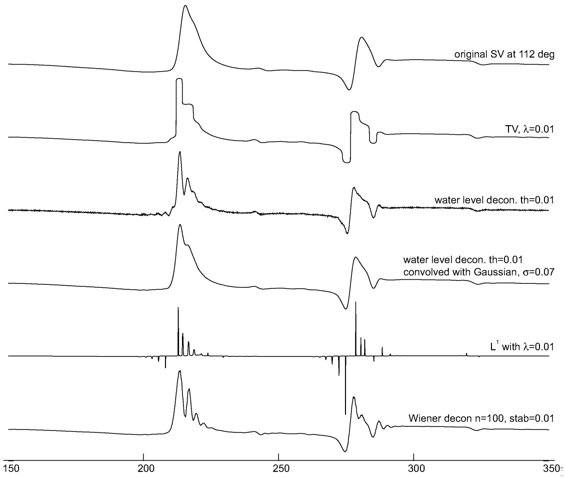


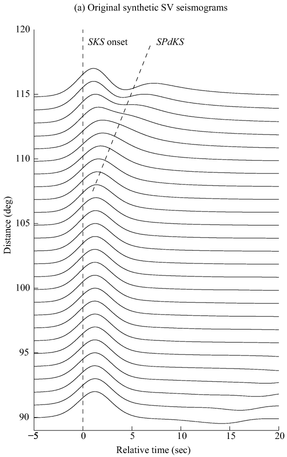


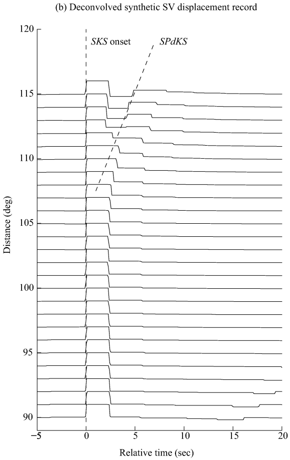
SKS at 112 deg deconvolved with SKS from 99 deg



SKS at 112 deg deconvolved with a Gaussian

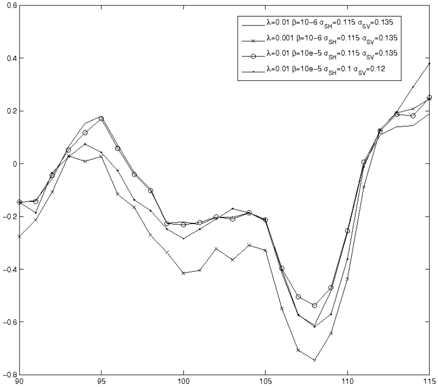



(a) Original synthetic SV seismograms
SKS onset
SPdKS
Distance (deg)
120
115
110
105
100
95
90
−5
0
5
10
15
20
Relative time (sec)

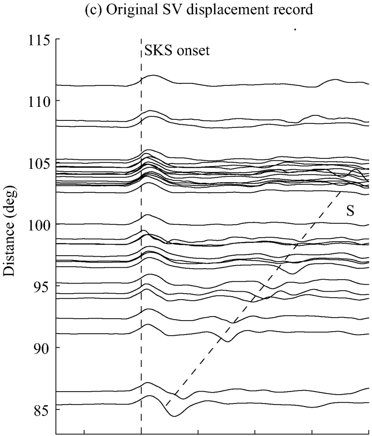

(b) Deconvolved synthetic SV displacement record
SKS onset
SPdKS
Distance (deg)
120
115
110
105
100
95
90
−5
0
5
10
15
20
Relative time (sec)

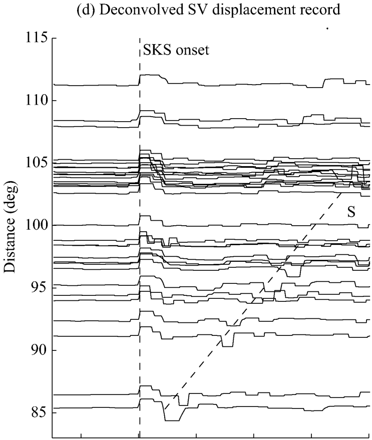
Error estimates

- Arrival time from an edge detection vs. ray theory prediction

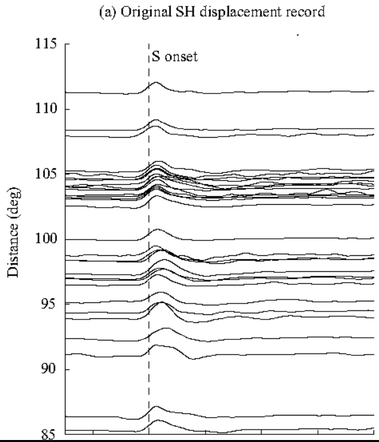


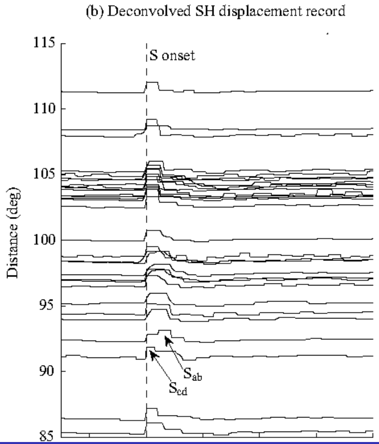
Real Data (SV) from an earthquake in South America



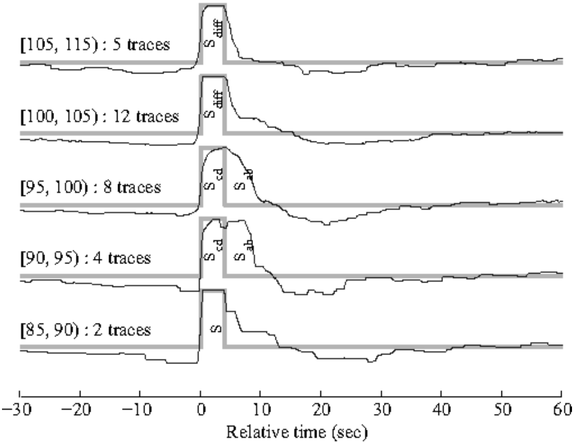


Real Data (SH) from an earthquake in South America





Evidence of the ultra low velocity zone (ULVZ)



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- Automatic travel time picking is **more accurate** then hand picking
- TV deconvolution yields usable results even for rough estimates of the wavelet
- Better estimates of the wavelet e.g. two-sided Gaussian will improve results further

Thanks to

- My Advisor Rosemary Renaut and Ed Garnero from Geology
- Sebastian Rost and Matthew Fouch for discussions and data
- This study was supported by the grant NSF CMG-02223