Signal restoration through deconvolution applied to deep mantle seismic probes

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November 29, 2005
Introduction
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  Synthetic Data
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  Thanks and Acknowledgment
The Physical Problem

- Image Earth’s interior
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- In particular the interface layer of the liquid core and solid mantle
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- In particular the interface layer of the liquid core and solid mantle
- Earth’s interior is inaccessible
- Remote sampling is required
- Important steps include
  1. accurate characterization of seismic energy
  2. reliable estimation or measurement of seismic wave timing
The Mathematical Problem

- Topographic inversion with **extreme sparse data coverage**
The Mathematical Problem

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- Poor signal to noise ratio (**SNR**)

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The Mathematical Problem

- Topographic inversion with **extreme sparse data coverage**
- Poor signal to noise ratio (SNR)
- Extremely Ill-posed
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(Garnero, Moore, Lay, Fouch, 2004)
Evidence for simple D''

Evidence for D'' reflector

Additional signals \(\Rightarrow\) D'' reflectors

Data

73.1
74.8
78.4

Scd

S

ScS

Data

69.2
71.3
79.8

Scd

S

ScS

HRV

081994

DRLN

012397

PFO

051094

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Problem Formulation

- **Invert the blurring Effect (attenuation) of Earth’s mantle and core to ...**
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- **Invert the blurring Effect** (attenuation) of Earth’s mantle and core to ...
- (a) get **clearer evidence of the existence** of structures like the ULVS
- (b) get **timing information** to make quantitative estimates like the height of a structure.
Signal degradation is modeled as a convolution

\[ g = f \ast h + n \]

- where \( g \) is the blurred signal
- \( f \) is the unknown signal
- \( h \) is the point spread function
- \( n \) is noise
$g = f \ast h + n$

Forward Model Example
Estimation of the PSF

- Ideal goal of seismic deconvolution is to produce a spike train
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- Ideal goal of seismic deconvolution is to produce a **spike train**
- The corresponding **PSF is unknown** (if it exists)
- **Estimations** of this PSF (in seismology wavelet) come from
  - stacking traces (**problem, traces are very different**)
  - estimating Earth’s filter (**basically a low pass filter, very difficult due to inhomogeneities**)
  - use a very basic (**common**) shape, like a Gaussian (**very rough estimate**)
Gaussian Wavelet

- also called a Ricker Wavelet

\[ h(t) = \sigma \sqrt{\frac{2}{\pi}} e^{-\frac{t^2}{2\sigma^2}} \]

\( \sigma \) is a width parameter, chosen such that the wavelet approximates the phase of interest.
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Inverse Problem

- Find $f$ from $g = f \ast h + n$ given $g$ and $h$ with unknown $n$. 
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- Assuming normal distributed \( n \) yields the estimator
  \[
  \hat{f} = \arg \min_{f} \{ \| g - f \ast h \|_2^2 \} 
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  \]
- Reconstruction with \( n \) normal distr. with \( \sigma = 10^{-7} \)
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Regularization

- Add more information about the signal

\[ \hat{f} = \arg \min_f \{ \| g - f^* h \|^2_2 + \lambda R(f) \} \],

where \( R(f) \) is the penalty term and \( \lambda \) is a penalty parameter.
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- find
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  where $R(f)$ is the penalty term and $\lambda$ is a penalty parameter.
Regularization Methods

- Common methods are Tikhonov (TK)

\[ R(f) = TK(f) = \int_{-\infty}^{\infty} \| f'(t) \|_2^2 dt. \]
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- Sparse deconvolution \((L^1)\)

\[ R(f) = \| f \|_1 = \int_{-\infty}^{\infty} | f(t) | dt. \]
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Regularization Notes

\[ \hat{f} = \arg \min_f \{ \|g - f \ast h\|_2^2 + \lambda R(f) \} \]

- \( \lambda \) Governs the trade off between the fit to the data and the smoothness of the reconstruction and can be picked by the L-curve approach

\[ \lambda \] yields a piece wise constant reconstruction and preserves the edges of the signal

\[ L_1 \] yields a spike train

\[ TV \] yields a smooth reconstruction

\[ TK \] yields a spike train
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- TV yields a piece wise constant reconstruction and preserves the edges of the signal.
- TK yields a smooth reconstruction.
- \( L^1 \) yields a spike train.
- To find the minimum we use a limited memory BFGS method.
Notes on the Optimization

- All the considered objective functions (OF) are convex
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- TK is a linear least squares (LS) problem

\[
\hat{f} = \arg \min_f \left\{ \| g - Hf \|_2^2 + \lambda \| Lf \|_2^2 \right\}
\]
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  \[ \hat{f} = \arg\min_f \{ \| g - Hf \|_2^2 + \lambda \| Lf \|_2^2 \} \]
- The TV objective function is **non differentable**
  \[ J(f) = \| g - Hf \|_2^2 + \lambda \| Lf \|_1 \]
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  J(f) = \|g - Hf\|_2^2 + \lambda\|Lf\|_1
  \]
- The problems are **very large** (n order of 10000)
- **Evaluation** of the OF and its gradient is **cheap** (some FFTs and sparse matrix-vector multiplications)
use synthetic data from 1d model
▶ use synthetic data from 1d model
▶ at a critical angle of about 110 deg $SKS$ starts to diffract along the core
SKS at 112 deg deconvolved with SKS from 99 deg

- **Original SV at 112 deg**
- **TV, λ=0.01**
- **Water level decon. th=0.01**
- **Water level decon. th=0.01 convolved with Gaussian, σ=0.07**
- **L¹ with λ=0.01**
- **Wiener decon n=100, stab=0.01**
SKS at 112 deg deconvolved with a Gaussian

original SV at 112 deg

TV, $\lambda=0.01$

water level decon. th=0.01

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Wiener decon n=100, stab=0.01
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Error estimates

- Arrival time from an edge detection vs. ray theory prediction
Real Data (SV) from an earthquake in South America

(c) Original SV displacement record

(d) Deconvolved SV displacement record
Real Data (SH) from an earthquake in South America

(a) Original SH displacement record

(b) Deconvolved SH displacement record

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Evidence of the ultra low velocity zone (ULVZ)

[105, 115): 5 traces
[100, 105): 12 traces
[95, 100): 8 traces
[90, 95): 4 traces
[85, 90): 2 traces

Relative time (sec)
TV regularized deconvolution is more robust than established methods.
Conclusions

- TV regularized deconvolution is **more robust** than established methods
- Automatic travel time picking is **more accurate** than hand picking
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- TV regularized deconvolution is more robust than established methods.
- Automatic travel time picking is more accurate than hand picking.
- TV deconvolution yields usable results even for rough estimates of the wavelet.
- Better estimates of the wavelet e.g. two-sided Gaussian will improve results further.
Thanks to

- My Advisor Rosemary Renaut and Ed Garnero from Geology
- Sebastian Rost and Matthew Fouch for discussions and data
- This study was supported by the grant NSF CMG-02223