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Abstract

Transformation-induced plasticity (TRIP) in steel is an important phenomenon in the complex material behaviour of steel. It may cause distortion of work-pieces during production processes, heat treatment, e.g. Hence, TRIP is intensively investigated by many researchers both experimentally and theoretically. Nevertheless, the modelling of TRIP contains open questions, in particular in case of varying loads and in case of interaction with classical plasticity (CP). It is the aim of this paper to discuss macroscopic TRIP models in the multi-phase case and to derive formulas for calculating the TRIP strain by stresses and other entities in the three-dimensional case of small deformations. Based on this, we obtain some estimates for the TRIP strain. We deal with TRIP models involving backstress generated by TRIP itself and by classical plasticity (CP). We prove that for an important class of phase transformations (PT) the final amount of TRIP can be estimated by entities which do not depend on phase evolution. Besides this, we show that the stress dependence on PT may have an essential influence on TRIP-strain evolution. We also discuss how to obtain TRIP parameters from experimental data.

Keywords

Steel, transformation-induced plasticity, modelling of 3-d case, estimates of TRIP strain, identification of parameters

1 Introduction

The complex material behaviour of steel is intensively investigated by many authors in different directions, more experimentally in order to study the phenomena (cf. [1-4, 8-10, 19, 24, 35], e.g.) and more theoretically in order to model these phenomena and their interactions (cf. [5-7, 11, 12, 14, 16, 18, 25-27, 29, 30, 33, 37, 38], e.g. and the literature cited therein). This paper is more related to the last item. We want to

- present models for transformation-induced plasticity (TRIP) in steel in the multi-phase case. These models are suitable to be incorporated into the bulk model of material behaviour,
- obtain formulas expressing the 3-d TRIP stain through the stress tensor, and the thermoelastic and classical plastic (CP) strain tensors. These formulas are applicable in mathematical investigations and numerical simulations of the bulk model of material behaviour,
- derive estimates of the TRIP strain by stresses, thermoelastic and CP strains,
- consider the influence of stress-dependent phase transformations (PT) on TRIP strain,
- present formulas for identification of TRIP parameters using experimental data,

Phase transformations (PT) occurring under non-vanishing deviatoric stress (smaller than the yield stress) lead to a permanent anisotropic deformation which cannot be described by classical plasticity (CP) at macroscopic level. This phenomenon is called transformation-induced

plasticity (TRIP). Besides this, stresses influence PT ("stress-dependent transformation behaviour" = SDTB). Thus, the phenomena of TRIP and SDTB mutually influence, and therefore, we have to study them in close connection. In order to bound the scope of this paper we focus on TRIP. For PT and SDPT we refer to [1-3, 5-8, 9, 11, 19, 24, 25, 29, 33, 43, 44, 47, 50, 52], e.g. For the same reason we do not take possible viscous effects into account. In general, one can substitute the CP behaviour by a more general inelastic one (besides TRIP). As we do not deal with concrete CP laws like flow rules, the subsequent discussion remains also valid for this extension. For thermoviscoplasticity in metals (without PT) we refer to [13, 21, 23, 28].
Finally, in this paper, we only deal with macroscopic modelling, i.e. the entities may be regarded as averages over small volumes ("reference-volume elements"). For questions concerning the micro- or mesostructure (grains, local carbon diffusion, interfaces, etc.) we refer to [14, 15, 26, 27, 30, 37].

2 Macroscopic TRIP models in multi-phase case

As already mentioned above, PT occurring under non-vanishing deviatoric stress lead to TRIP which cannot be described by classical plasticity (CP) at macroscopic level. Contrary to CP, TRIP does not own a yield stress. Thus, TRIP occurs when the (macroscopic) von Mises stress is below the yield stress of the weaker phase. At first, TRIP was modelled *for one forming phase* (pearlite from austenite). For uniaxial tension under constant stress S and constant temperature there was found by experiments (cf. [1-4, 8-10, 14, 19, 26, 27, 29, 30, 37] and the literature cited therein):

(2.1) $e_{trip}(t) = \kappa\, S\, \phi(p(t))$.

Here: $e_{trip}(t)$ – additional longitudinal strain caused by TRIP of the specimen at time t, $\kappa > 0$ – factor of proportionality called Greenwood-Johnson parameter, p(t) - (volume) fraction of the forming phase, and ϕ - so-called saturation function fulfilling

(2.2) $\phi \in C([0, 1]) \cap C^1(]0, 1[)$ (= continuous on [0, 1], continuously differentiable on]0, 1[),

(2.3) $\phi(0) = 0, \qquad \phi(1) = 1,$

(2.4) $\phi'(p) \geq 0 \qquad$ for all $p \in]0, 1[$,

where ϕ' is the derivative of ϕ. There are different proposals for ϕ in the literature in dependence of the kind of PT, ferritic or martensitic, e.g. (cf. [3, 4, 10, 14, 29] for further information and discussion). Some often used proposals for ϕ are:

(2.5) $\phi(p) := p\,(1 - \ln(p))$ (Leblond),

(2.6) $\phi(p) := p$ (Tanaka),

(2.7) $\phi(p) := p\,(2 - p)$ (Desalos, Denis),

The functions given in (2.5) and (2.7) are concave. That is why the stronger influence of PT at its begin is taken into account, at least for diffusive transformations (cf. remark 2.1 (i)). After end of a complete PT one obtains from (2.1) and (2.3)

(2.8) $e_{trip}(t_\infty) = \kappa\, S$,

with t_∞ - sufficiently large time. Equation (2.8) gives a good interpretation of κ. We refer to [9] for concrete values of κ for the pearlitic and bainitic transformation of the steel 100Cr6 at constant transformation temperatures and under constant tension stresses. In this case, κ is a linear function of temperature for moderate stresses (cf. [8-10]). For the martensitic transformation of the steel 100Cr6 there arise different values of κ for uniaxial tension and compression (cf. remark 1(ii) and [10]). For convenience we often suppress a dependence of κ on stress. For this matter we refer to remark 2.3 (iv) at the end of this point. After determining κ via (2.8), proposals for the saturation function ϕ can be tested via (2.1) (cf. point 7 and [10]).
In order to generalize the TRIP model (2.1) to the needs of modelling and simulation of work-pieces as well as to the case of more than one forming phase, we start with a differential form following from (2.1)

(2.9) $$\frac{d}{dt} e_{trip}(t) = \kappa S \frac{d}{dt} \phi(p(t)),$$

where κ and S are still assumed to be constant. (For the non-differentiability at zero of the saturation function (2.5) proposed by Leblond we refer to remark 2.2 (iii).) In (2.9) we substitute the longitudinal TRIP strain e_{trip} by an additional (linearized) strain tensor ε_{trip} due to TRIP. Furthermore, the uniaxial tension S will be substituted by the deviator σ^* of the stress tensor σ in accordance with

(2.10) $$\sigma^* := \sigma - \frac{1}{3} tr(\sigma) I \qquad (tr(\sigma) – trace of \sigma, I – unity tensor).$$

Note that *only* non-isotropic stress leads to TRIP. Hence, one obtains a tensorial extension of the TRIP model (2.9)

(2.11) $$\frac{d}{dt} \varepsilon_{trip}(x, t) = \frac{3}{2} \kappa(\theta(x, t)) \sigma^*(x, t) \frac{d}{dt} \phi(p(x, t)).$$

The model (2.11) permits variable stresses and temperatures (labelled by θ) (cf. [11, 12, 25, 29, 36, 41, 46, 48], e.g.). In the sequel x is a spatial point of the considered body (cf. remark 2.2 (ii)). The factor 3/2 is chosen in order to obtain (2.9) from (2.10) as a special case with same κ.

Remarks 2.1 (i) Using an idea of Sjöström (cf. [10] for further reference), a more general proposal for a saturation function ϕ depending on a parameter m consists in

(2.12) $$\phi(p) := \frac{p}{m-1} (m - p^{m-1}) \qquad \text{for } 0 \le p \le 1 \qquad \text{and } m > 1.$$

The proposal (2.12) covers those ones by Sjöström ($m \ge 2$), by Denis et al. ($m = 2$), by Abrassart ($m = 3/2$). As a limit case one get the proposal by Leblond (2.5) for m tending to 1, and this one by Tanaka (2.6) for m tending to infinity. Current investigations show [10], that diffusive transformations can be well described by the above proposals. Contrary to this, for martensitic transformation the graph of the saturation function may be an s-curve [10]. Hence, a general proposal containing one parameter could be:

(2.13) $$\phi(p) := \frac{1}{2} \{1 + \frac{1}{\sin(m)} \sin(m(2p - 1))\} \qquad \text{for } 0 \le p \le 1 \quad \text{and } 0 < m < \frac{\pi}{2}.$$

Clearly, the limit case for $m \to 0$ is the proposal by Tanaka (2.6). Of course, different phase transformations may have their own ϕ with specific m.

(ii) It is possible to let the parameter m in (2.12) and (2.13) depend on temperature, phase fractions or other entities. In this case we write instead of (2.11)

(2.14) $$\frac{d}{dt} \varepsilon_{trip}(x, t) = \frac{3}{2} \kappa(\theta(x, t)) \sigma^*(x, t) \frac{\partial}{\partial p} \phi(p(x, t)) \frac{d}{dt} p(x, t),$$

because TRIP should only be driven by a forming phase, and not by changing of other entities.

(iii) As mentioned above, the martensitic transformation of 100Cr6 has different values κ for uniaxial tension and compression, namely κ_+ and κ_-, resp. [10]. Therefore, we can propose

(2.15) $$\kappa(\sigma_m) = \frac{1}{2} (\kappa_+ + \kappa_-) + \frac{1}{2} sign(\sigma_m) (\kappa_+ - \kappa_-),$$

where $\sigma_m = 1/3\ tr(\sigma)$ is the (mean) principal stress, and sign is defined by

(2.16) $$sign(s) := -1 \quad \text{for } s < 0, \qquad sign(s) := 1 \quad \text{for } s > 0, \qquad sign(0) := 0.$$

Note, that in (2.15) tension and compression have the same weight. A more general proposal is possible. For evaluation one needs two-axial experiments (tension and torsion), keeping σ_m equal to zero (cf. [43]). Using (2.16), a possible three-dimensional ansatz for the martensitic transformation could be (cf. (2.11))

(2.17) $$\frac{d}{dt} \varepsilon_{trip}(x, t) = \frac{3}{2} \kappa(\sigma_m) \sigma^* \frac{d}{dt} \phi(p(t)).$$

(iv) As we focus on TRIP in this work, we do not discuss PT laws like Johnson-Mehl-Avrami-Kolmogorov as in (6.4). We refer to [5-7, 11, 24, 29, 33, 35, 36, 52] for discussion, open questions etc.

(v) In accordance with (2.8), the *final* longitudinal TRIP strain depends neither on the phase evolution nor on the saturation function, while the *current* value of ε_{trip} depends on p and ϕ (cf.

(2.1)). Under suitable conditions this situation will be also encountered for complex genaralizations of the ansatz (2.1) in three-dimensional case.

Now let us suppose, that in a process with $m \geq 2$ phases, k are formed ($k < m$). In this case the model in (2.11) has to be extended. One possibility to do this starts with the assumption, that every forming phase has its own contribution to TRIP strain. The addition of these deformations leads to (cf. [25])

$$\varepsilon_{trip}'(x, t) = \frac{3}{2}\sigma^*(x, t) \sum_{i=1}^{k} \kappa_i \frac{d}{dt}\phi_i(p_i(x, t)), \tag{2.18}$$

where, for convenience, we denote the time derivative by prime " ' ". Generalizing this idea for any process with growing and decreasing phase fractions, we propose (cf. [46, 48])

$$\varepsilon_{trip}'(x, t) = \frac{3}{2}\sigma^*(x, t) \sum_{i=1}^{m} \kappa_i \phi_i'(p_i(x, t)) \max\{ p_i'(x, t), 0\}, \tag{2.19}$$

where both the usual derivative and the partial one with respect to time are labelled by a prime '. In (2.12), (2.13) the p_i, $\kappa_i > 0$ and ϕ_i are respectively understood as phase fraction, Greenwood-Johnson parameter and saturation function (fulfilling (2.2) – (2.4)) of the i^{th} phase. Due to (2.13), only forming phases currently contribute to TRIP. This seems to make sense because the decrease of one phase is compensated by the growth of (at least) another one. This approach shows its advantage when investigating the bulk model (cf. remark 2.3 (i)). Let the Greenwood-Johnson parameters κ_i ($i = 1, ..., m$) depend on θ and p, i.e.

$$\kappa_i = \kappa_i(\theta, p) \tag{2.20}$$

For a possible stress dependence we refer to the remarks above. (In the sequel p stands for the vector of phase fractions $p_1, ..., p_m$.) The ansatz (2.20) takes a possible influence of phases into account, which are present, but are not currently transformed. When modelling TRIP during austenitization, it makes sense to take the phase composition of the initial material into account. By other words, there may be a difference in TRIP when forming austenite, coming from pure martensite or from pure pearlite. Thus, labelling austenite by 1 without loss of generality, a possible ansatz for the G-J parameter κ_1 corresponding to TRIP during austenitization may be

$$\kappa_1(\theta, p) = \sum_{i=2}^{m} \kappa_{1i}(\theta)\, p_i, \tag{2.21}$$

where the $\kappa_{1i} > 0$ are the G-J parameters corresponding to TRIP during austenitization coming from the i^{th} ferritic phase ($i \geq 2$).

Furthermore, it is possible to obtain a different model for TRIP in the multiphase case. For this, one regards all k ($k \geq 2$) forming phases *as one forming phase* (cf. [26, 38]). This leads to the ansatz

$$\varepsilon_{trip}'(x, t) = \frac{3}{2}\sigma^*(x, t)\, \kappa \frac{d}{dt}\phi\Big(\sum_{i=1}^{k} p_i(x, t) \Big) = \frac{3}{2}\sigma^*(x, t)\, \kappa\, \phi'\Big(\sum_{i=1}^{k} p_i(x, t) \Big) \sum_{i=1}^{k} p_i'(x, t), \tag{2.22}$$

and generalizing to the case of growing and decreasing fractions, it follows

$$\begin{aligned} \varepsilon_{trip}'(t) &= \frac{3}{2}\sigma^*(t)\, \kappa \frac{d}{dt}\phi\Big(\sum_{i=1}^{m} p_i(t)\, H(p_i'(t)) \Big) = \\ &= \frac{3}{2}\sigma^*(t)\, \kappa\, \phi'\Big(\sum_{i=1}^{m} p_i(t)\, H(p_i'(t)) \Big) \sum_{i=1}^{m} \max\{ p_i'(t), 0\} \end{aligned} \tag{2.23}$$

(suppressing the spatial dependence), where ϕ is in (2.2) – (2.4) or in (2.12), (2.13). H is the Heaviside function defined by

$$H(s) := 0 \quad \text{for } s \leq 0, \qquad H(s) := 1 \quad \text{for } s > 0. \tag{2.24}$$

In [26, 38] the authors used the ϕ defined in (2.5). The G-J parameter κ may be proposed as

$$\kappa(\theta(t), p(t)) = \sum_{i=1}^{m} \kappa_i(\theta(t), p(t))\, p_i(t)\, H(p_i'(t)) \tag{2.25}$$

Note, that for only one forming phase both the above proposals (2.19) and (2.23) coincide. Experiments must decide which proposal is better in which situation. Moreover, experiments with varying load (before end of transformation) show a decrease of TRIP strain after unloading (cf. [1-3, 16, 47, 49, 50]). This phenomenon is not predicted by the models (2.19) and (2.23), in accordance to them, ε_{trip} remains constant (s. picture 1). This observed decrease can be explained by a backstress X_{trip} associated with TRIP (cf. [39] as well as [46, 48, 51] for further discussions). In an easy way we can generalize the TRIP models (2.19) and (2.23), subtracting from the stress deviator the deviator of the backstress. Hence, instead of (2.19) we obtain

$$\varepsilon_{trip}'(x, t) = \frac{3}{2} (\sigma^*(x, t) - X_{trip}^*(x, t)) \sum_{i=1}^{m} \kappa_i \phi_i'(p_i(x, t)) \max\{ p_i'(x, t), 0\}, \tag{2.26}$$

and instead of (2.23) we obtain

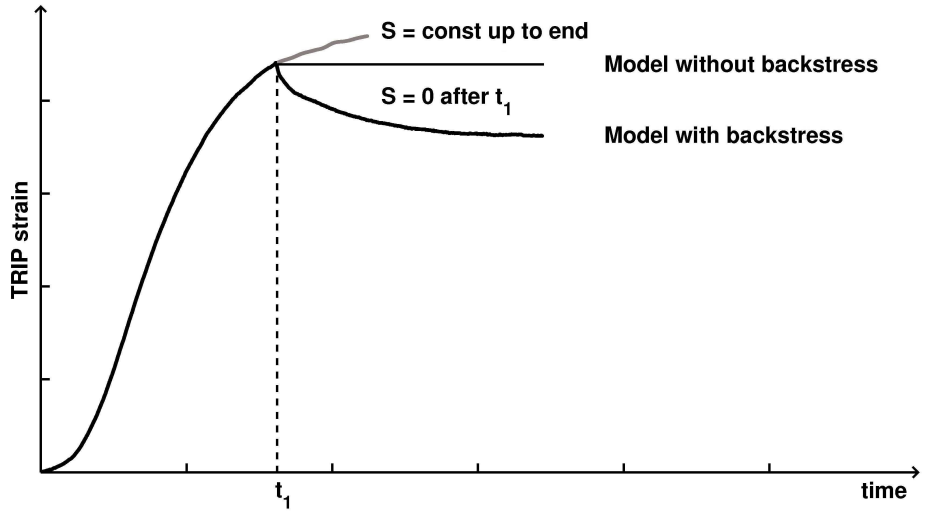
$$\varepsilon_{trip}'(x, t) = \frac{3}{2} (\sigma^*(x, t) - X_{trip}^*(x, t)) \kappa \phi'(\sum_{i=1}^{m} p_i(x, t) H(p_i'(x, t))) \sum_{i=1}^{m} \max\{ p_i'(x, t), 0\} \tag{2.27}$$

The difference $\sigma^* - X_{trip}^*$ may be called "effective TRIP stress" in analogy with CP. A simple linear ansatz for X_{trip} and X_{cp} (backstress associated with CP) taking a possible interaction of TRIP and CP into account (cf. [39, 46, 48]) reads as

$$X_{trip} = c_1 \varepsilon_{trip} + c_2 \varepsilon_{cp}, \tag{2.28}$$

$$X_{cp} = c_2 \varepsilon_{trip} + c_3 \varepsilon_{cp}, \tag{2.29}$$

where c_1, c_2, c_3 are further material parameters, and ε_{cp} is the classical-plastic part of the deformation tensor ε (cf. point 2). Generally, c_1, c_2 and c_3 may depend on temperature and on phase fractions, or, more precisely, on the kind of transformation. Therefore, this general case requires an ansatz for c_1, c_2 and c_3 like (2.20) via mixture rules (cf. [46]). Based on thermodynamic arguments, c_1 and c_3 must be non-negative, whereas c_2 may be negative. We will return to this matter in point 5, in particular in remark 5.1. A more complicated ansatz as in (2.28), (2.29) is possible (cf. [20, 21, 22, 28] for CP without PT and [51] for our case). Of course, the simpler models „TRIP without backstress" (2.19) and (2.23) are special cases of (2.26) and (2.27) with vanishing c_1, c_2 and c_3. Experiments must decide, which of the models (2.19), (2.23), (2.25) or (2.27) will better describe the reality under which conditions.



Picture 1: An unloading during PT may lead to a backstress effect.

Closing this point, we give some remarks.

Remarks 2.2 (Mathematical assumptions)
(i) For mathematical reasons we assume for the phase fractions

$$p_i \text{ absolutely continuous (with respect to t)} \qquad \text{for all } i = 1, ..., n \tag{2.30}$$

Hence, the derivatives with respect to t (often denoted by p') exist everywhere except for a point set with Lebesgue measure zero.
(ii) In order to model and simulate the material behaviour of a body (i.e. of a work-piece) in complex situations one normally has to deal with a coupled system of ordinary and partial differential equations (cf. [7, 25, 31, 32, 34, 36, 39, 46, 48, 51], e.g.). The reference configuration of the considered body is often identified with a so-called Lipschitz domain $\Omega \subset \mathbb{R}^n$, where n is the spatial dimension (mostly 2 or 3 in practical applications). The often used finite-element methods require that the functions looked for belong to certain function spaces (Sobolev spaces, e.g.). This is not the place to go into details, we refer to [13, 20, 31, 34]. For our purpose, it is sufficient to assume that all functions which depend on the spatial variable x are (at least) Lebesgue measurable for all $t \geq 0$.
(iii) The saturation function by Leblond in (2.5) is not differentiable at 0. If the relations (2.19) or (2.23) will be integrated in this case, one obtains improperly existing integrals for relevant PT laws [45]. In the sequel we suppose that for considered saturation functions ϕ_i and phase fractions p_i their superposition fulfils

(2.31) $\phi_i(p_i)$ absolutely continuous (with respect to t) for all i = 1, ..., n

Remarks 2.3 (i) The models (2.26) and (2.27) have the same basis structure

(2.32) $\varepsilon_{trip}'(x, t) = b(x, t)\,(\sigma^*(x, t) - X_{trip}^*(x, t))$

with a *non-negative* factor b. Thus, both models can be integrated in a thermodynamic consistent continuum-mechanic bulk model (cf. [46, 48, 51] for details and discussion).
(ii) Instead of a scalar Greenwood-Johnson parameter κ, it is also possible to consider a positively definite four-rank tensor in order to model possible effects of anisotropy like for CP (cf. [28]).
(iii) In the sequel the basis structure (2.32) allows parallel investigations of both TRIP models in a high degree. Noting the great analogy to the flow rule of CP, we emphasize again that TRIP does not have any yield stress.
(iv) Sometimes the basis model of TRIP (2.9) is corrected by a positive correction factor on the right-hand side, taking the significant influence of stresses of the weaker phase (i.e., mostly austenite) near by the yield stress into account (cf. [26, 36, 37]). Of course, such factor may be formally incorporated into the Greenwood-Johnson parameter. As mentioned above, for convenience we forego do without. Moreover, here we do not investigate the possible relations of κ with microscopic (or mesoscopic) entities (cf. [10, 14-16, 26, 27, 30, 37]).

3 Incorporation of the TRIP models into the mechanic bulk model

In this paper we suppose small deformations. (For modelling of steel behaviour in the context of finite deformations we refer to [7, 41].) Therefore, the strain tensor ε may be additively decomposed in accordance with

(3.1) $\varepsilon = \varepsilon_{tep} + \varepsilon_{cp} + \varepsilon_{trip}$,

where ε_{tep} is its thermoelastic part *including* isotropic strain caused by density changes due to PT. As already mentioned, it is possible to consider viscous effects without additional difficulties. The (generalized) thermoelastic part ε_{tep} reads as (cf. [43, 46])

(3.2) $$\varepsilon_{tep} = \frac{1+\nu}{E}\,\sigma - \frac{\nu}{E}\,\mathrm{tr}(\sigma)\,I + \alpha\,(\theta - \theta_0)\,I + \frac{1}{3}\,\frac{\rho_0 - \rho(\theta_0)}{\rho(\theta_0)}\,I$$

The first two terms in (3.2) represent the pure elastic part, the third one stands for thermal strain and the fourth one for strain caused by density changes due to PT. The new notations are: E, ν - Young's modulus and Poisson's ratio, I – unity tensor, α - linear heat-dilatation coefficient, θ_0 – initial temperature at the time t = 0, ρ_0 – density at t = 0, $\rho(\theta_0)$ - current density of the phase mixture related to θ_0. Clearly, in the absence of PT ρ_0 and $\rho(\theta_0)$ are the same. Hence, (3.2) is an

extension of the Duhamel-Neumann relations in linear thermoelasticity. Moreover, we note that the density term in (3.2) comes from a linearization in case of small density changes (cf. [42, 43, 50]). The last two terms in (3.2) may be written as a sum

$$\frac{1}{3}\frac{\rho_0 - \rho(\theta)}{\rho(\theta)} = \alpha\,(\theta - \theta_0) + \frac{1}{3}\frac{\rho_0 - \rho(\theta_0)}{\rho(\theta_0)}, \tag{3.3}$$

where $\rho(\theta)$ is the current density related to current temperature θ. The use of (3.3) is useful when processing dilatometer data (cf. [42-44, 50]). From (2.28) and (2.32) we obtain the subsequent *ordinary differential equation* for ε_{trip}:

$$\varepsilon_{trip}'(x, t) = b(x, t)\,\{\sigma^*(x, t) - c_1(x, t)\,\varepsilon_{trip}(x, t) - c_2(x, t)\,\varepsilon_{cp}(x, t)\}. \tag{3.4}$$

The dependence of c_1, c_2 on x and t is realized through the dependence on temperature or other entities. For convenience we only write $c_1(x, t)$ and $c_2(x, t)$ instead of more complicated expressions. Moreover, we use the abbreviation

$$b(x, t) := \frac{3}{2}\sum_{i=1}^{m} \kappa_i(\theta(x, t))\,\phi_i'(p_i(x, t))\,\max\{p_i'(x, t), 0\} \tag{3.5}$$

for the TRIP model (2.24) as well as

$$b(x, t) := \frac{3}{2}\,\kappa(\theta(x, t), p(x, t))\,\phi'\Big(\sum_{i=1}^{m} p_i(x, t)\Big)\,H(p_i'(x, t))\sum_{i=1}^{m}\max\{p_i'(x, t), 0\} \tag{3.6}$$

for the model given in (2.25). For obtaining (3.4) we used the subsequent relations

$$\varepsilon_{trip} = \varepsilon_{trip}^*, \qquad \varepsilon_{cp} = \varepsilon_{cp}^*, \tag{3.7}$$

expressing the observation that CP as well as TRIP are volume preserving. We note that in (3.4) the spatial point x has only the role of a parameter (cf. remark 2.2 (i)). Therefore, we often suppress it. The natural initial condition at $t = 0$ reads as

$$\varepsilon_{trip}(0) = 0. \tag{3.8}$$

Thus, the unique solution of the Cauchy problem (3.4), (3.8) (cf. remark 3.1 (i)) is given by

$$\varepsilon_{trip}(t) = \int_0^t b(s)\,(\sigma^*(s) - c_2(s)\,\varepsilon_{cp}(s))\,\exp\Big(-\int_s^t c_1(\tau)\,b(\tau)\,d\tau\Big)\,ds. \tag{3.9}$$

Alternatively, in (3.4) σ^* can be eliminated by ε_{tep}^* via (3.2), or by ε^* and ε_{cp} via (3.1) and (3.2). After this, the obtained ordinary differential equations can be resolved (for the initial condition (3.8), too). Hence, there holds

$$\varepsilon_{trip}(x, t) = \int_0^t b(x, s)\,(2\mu\,\varepsilon_{tep}^*(x, s) - c_2\,\varepsilon_{cp}(x, s))\,\exp\Big(-\int_s^t c_1\,b(x, \tau)\,d\tau\Big)\,ds \tag{3.10}$$

and

$$\varepsilon_{trip}(x, t) = \int_0^t b(s)\,\{2\mu\,\varepsilon^*(x, s) - (c_2 + 2\mu)\,\varepsilon_{cp}(x, s)\}\,\exp\Big(-\int_s^t (2\mu + c_1)\,b(x, \tau)\,d\tau\Big)\,ds, \tag{3.11}$$

respectively, where the (generally temperature and phase-dependent) shear modulus μ is defined by

$$\mu = \frac{E}{2(1 + \nu)} \tag{3.12}$$

Generally, E and ν can be expressed via linear mixture rules in accordance with

$$E(\theta, p) = \sum_{i=1}^{m} E_i(\theta)\,p_i, \qquad \nu(\theta, p) = \sum_{i=1}^{m} \nu_i(\theta)\,p_i. \tag{3.13}$$

Of course, the E_i and ν_i are related to the i^{th} phase. If the Poisson's ratios ν_i depend on the phases only weakly, we obtain approximately

$$\mu(\theta, p) = \sum_{i=1}^{m} \mu_i(\theta)\,p_i \qquad \text{with} \qquad \mu_i(\theta) := \frac{E_i}{2(1 + \nu_i)}. \tag{3.14}$$

The equations (3.9) - (3.11) are *starting points for further investigations*, analytical and numerical ones (cf. [31, 34]).

Remarks 3.1 (i) The ordinary differential equation (3.4) is linear. The spatial variable x only plays the role of a parameter. If b, σ^*, c_1, c_2 fulfil moderate conditions, then the initial-value problem (or Cauchy problem) (3.4), (3.8) has a global unique solution which is absolutely continuous with respect to t and Lebesgue measurable with respect to x (cf. [40]).
(ii) When considering the bulk model of material behaviour in order to perform numerical simulations, the full problem is often decomposed into some partial ones. In this context the equation (3.9) may be used to calculate the TRIP strain at some calculation step, knowing already the stresses, CP strains and phase fractions. Analogously, the equations (3.10) and (3.11), respectively, can be dealt with. We refer to [31, 34] for details.
(iii) The modelling of CP strain itself is often performed by a flow rule and by a yield condition (cf. [13, 20-22, 28] for CP without PT). As we focus on TRIP, we suppress this here and refer to [46, 48, 51] for modelling CP with PT.

4 Estimates of the TRIP strain

Now it is the aim, to derive estimates of the TRIP strain through stresses and other entities, using the relations (3.9) - (3.11). We formulate first results concerning upper estimates of ε_{trip} for both TRIP models (2.24), (2.25).

Theorem 4.1 (i) Let the material parameter c_1 in (2.26) fulfil

(4.1) $\quad c_1 \geq 0.$

Then there holds the subsequent estimate for both TRIP models (2.24), (2.25) for all indices i, j $\in \{1, ..., n\}$ and for (almost) all $x \in \Omega$ (this repeating formulation will be suppressed in future)

$$(4.2) \quad \forall\, t \geq 0 \quad : \quad |\varepsilon_{tripij}(x, t)| \leq \max_{s\in[0, t]} \{|\sigma_{ij}^*(x, s)| + |c_2(x, s)|\, |\varepsilon_{cpij}(x, s)|\} \int_0^t b(s)\, ds.$$

(ii) Besides (4.1) we assume

$$(4.3) \quad \forall\, i \in \{1, ..., k\} \quad \forall\, t \geq 0 \quad : \quad p_i'(t) \geq 0$$

$$(4.4) \quad \forall\, i \in \{k + 1, ..., m\} \quad \forall\, t \geq 0 \quad : \quad p_i'(t) \leq 0,$$

(with $1 \leq k < m$). Then there holds the subsequent estimates for the TRIP model (2.24)

$$(4.5) \quad \forall\, t \geq 0 \quad : \quad |\varepsilon_{tripij}(x, t)| \leq \frac{3}{2} \max_{s\in[0, t]} \{|\sigma_{ij}^*(x, s)| + |c_2(x, s)|\, |\varepsilon_{cpij}(x, s)|\} \bullet$$
$$\bullet \sum_{r=1}^{k} \max_{s\in[0, t]} \{|\kappa_r(x, s)|\}(\phi_r(p_r(x, t)) - \phi_r(p_r(x, 0))),$$

$$(4.6) \quad \forall\, t \geq 0 : |\varepsilon_{tripij}(x, t)| \leq \frac{3}{2} \max_{s\in[0, t]} \{|\sigma_{ij}^*(x, s)| + |c_2(x, s)|\, |\varepsilon_{cpij}(x, s)|\} \sum_{r=1}^{k} \max_{s\in[0, t]} \{|\kappa_r(x, s)|\},$$

whereas we analogously obtain for the TRIP model (2.25)

$$(4.7) \quad \forall\, t \geq 0 \quad : \quad |\varepsilon_{tripij}(x, t)| \leq \frac{3}{2} \max_{s\in[0, t]} \{|\sigma_{ij}^*(x, s)| + |c_2(x, s)|\, |\varepsilon_{cpij}(x, s)|\} \bullet$$
$$\bullet \max_{s\in[0, t]} \{|\kappa(x, s)|\} \sum_{r=1}^{k} (\phi(p_r(x, t)) - \phi(p_r(x, 0))),$$

$$(4.8) \quad \forall\, t \geq 0 : \quad |\varepsilon_{tripij}(x, t)| \leq \frac{3}{2} \max_{s\in[0, t]} \{|\sigma_{ij}^*(x, s)| + |c_2(x, s)|\, |\varepsilon_{cpij}(x, s)|\} \max_{s\in[0, t]} \{|\kappa(x, s)|\}.$$

(iii) Assume that there exist two positive numbers c_{10}, c_{11} such that

(4.9) $\quad 0 < c_{10} \leq c_1(x, s) \leq c_{11} < \infty \quad$ for all possible arguments.

Then there holds the following estimate for both TRIP models (2.24), (2.25)

$$(4.10)\ \forall\, t \ge 0 : |\varepsilon_{tripij}(x, t)| \le \frac{1}{c_{10}} \max_{s\in[0,\,t]} \{|\sigma_{ij}^*(x, s)| + |c_2(x, s)|\, |\varepsilon_{cpij}(x, s)|\}\ \{1 - \exp(-c_{11} \int_0^t b(s)\, ds\}.$$

(iv) Under (4.3), (4.4) and (4.9) there hold for the TRIP models (2.24)

$$(4.11)\quad \forall\, t \ge 0 \quad : \quad |\varepsilon_{tripij}(x, t)| \le \frac{1}{c_{10}} \max_{s\in[0,\,t]} \{|\sigma_{ij}^*(x, s)| + |c_2(x, s)|\, |\varepsilon_{cpij}(x, s)|\} \bullet$$

$$\bullet \{1 - \exp(-c_{11} \frac{3}{2} \sum_{r=1}^{k} \max_{s\in[0,\,t]} \{|\kappa_r(x, s)|\}(\phi_r(p_r(x, t)) - \phi_r(p_r(x, 0)))\},$$

$$(4.12)\quad \forall\, t \ge 0 \quad : \quad |\varepsilon_{tripij}(x, t)| \le \frac{1}{c_{10}} \max_{s\in[0,\,t]} \{|\sigma_{ij}^*(x, s)| + |c_2(x, s)|\, |\varepsilon_{cpij}(x, s)|\} \bullet$$

$$\bullet \{1 - \exp(-c_{11} \frac{3}{2} \sum_{r=1}^{k} \max_{s\in[0,\,t]} \{|\kappa_r(x, s)|\})\}.$$

Finally, for the TRIP models (2.25) the analogous estimates read as

$$(4.13)\quad \forall\, t \ge 0 \quad : \quad |\varepsilon_{tripij}(x, t)| \le \frac{1}{c_{10}} \max_{s\in[0,\,t]} \{|\sigma_{ij}^*(x, s)| + |c_2(x, s)|\, |\varepsilon_{cpij}(x, s)|\} \bullet$$

$$\bullet \{1 - \exp(-c_{11} \max_{s\in[0,\,t]} \{|\kappa(x, s)|\} \frac{3}{2} \sum_{r=1}^{k} (\phi(p_r(x, t)) - \phi(p_r(x, 0)))\},$$

$$(4.14)\quad \forall\, t \ge 0 \quad : \quad |\varepsilon_{tripij}(x, t)| \le \frac{1}{c_{10}} \max_{s\in[0,\,t]} \{|\sigma_{ij}^*(x, s)| + |c_2(x, s)|\, |\varepsilon_{cpij}(x, s)|\} \bullet$$

$$\bullet \{1 - \exp(-\frac{3}{2} c_{11} \max_{s\in[0,\,t]} \{|\kappa(x, s)|\})\}.$$

Proof : (i) The estimate (4.2) easily follows from (3.9), taking (4.1) into account.
(ii) If additionally (4.3), (4.4) are given, then the integral in (4.2) can be integrated. Using the properties (2.3), (2.4), (2.28) of the saturation functions, (4.5) – (4.8) follow immediately.
(iii) In case of (4.9) we obtain from (3.9) (suppressing x)

$$(4.15)\quad \forall\, t \ge 0 : |\varepsilon_{tripij}(t)| \le \max_{s\in[0,\,t]} \{|\sigma_{ij}^*(s)| + |c_2(s)|\, |\varepsilon_{cpij}(s)|\} \int_0^t b(s) \exp(-\int_s^t c_1(\tau)\, b(\tau)\, d\tau)\, ds \le$$

$$\le \frac{1}{c_{10}} \max_{s\in[0,\,t]} \{|\sigma_{ij}^*(s)| + |c_2(s)|\, |\varepsilon_{cpij}(s)|\} \int_0^t c_1(s)\, b(s) \exp(-\int_s^t c_1(\tau)\, b(\tau)\, d\tau)\, ds =$$

$$= \frac{1}{c_{10}} \max_{s\in[0,\,t]} \{|\sigma_{ij}^*(s)| + |c_2(s)|\, |\varepsilon_{cpij}(s)|\}\ \{1 - \exp(-c_{11} \int_0^t b(\tau)\, d\tau)\}.$$

(iv) Under (4.3), (4.4) one easily deduces (4.11) - (4.14), respectively, from (4.15).

The von Mises stress is defined by

$$(4.16)\qquad \sigma_v(x, t) := (\frac{2}{3} \sum_{i,\,j=1}^{n} \sigma_{ij}^*(x, t)\, \sigma_{ij}^*(x, t))^{\frac{1}{2}}.$$

Hence, for all indices $i, j \in \{1, ..., n\}$ and for (almost) all $x \in \Omega$ one obtains from (4.16)

$$(4.17)\qquad |\sigma_{ij}^*(x, t)| \le \sqrt{\frac{3}{2}}\, \sigma_v(x, t).$$

Thus, using (4.17), we can derive further estimates from those in theorem 4.1. For example we formulate:

Theorem 4.2 Under (4.1) the subsequent estimate is valid for both TRIP models (2.24), (2.25)

$$(4.18)\quad \forall\, t \ge 0 \quad : \quad |\varepsilon_{tripij}(x, t)| \le \max_{s\in[0,\,t]} \{\sqrt{\frac{3}{2}}\, \sigma_v(x, t) + |c_2(x, s)|\, |\varepsilon_{cpij}(x, s)|\} \int_0^t b(s)\, ds.$$

Analogously, similar estimates can be derived from (3.10) or (3.11). In the case

(4.19) $c_2 = 0$,

i.e., if there is no influence of CP on TRIP, then the TRIP strain can be evaluated by the thermoelastic one in the following manner.

Theorem 4.3 (i) Under (4.1), (4.19) there holds for both TRIP models (2.24), (2.25)

$$(4.20)\quad \forall\, t \ge 0 \quad : \quad |\varepsilon_{tripij}(x, t)| \le 2 \max_{s\in[0,\,t]} \{\mu(x, s)\} \max_{s\in[0,\,t]} \{|\varepsilon_{tepij}{}^*(x, s)|\} \int_0^t b(s)\, ds.$$

(ii) Under (4.1), (4.3), (4.4), (4.19) there hold for the TRIP model (2.24)

$$(4.21)\quad \forall\, t \ge 0 \quad : \quad |\varepsilon_{tripij}(x, t)| \le 3 \max_{s\in[0,\,t]} \{\mu(x, s)\} \max_{s\in[0,\,t]} \{|\varepsilon_{tepij}{}^*(x, s)|\} \bullet$$
$$\bullet \sum_{r=1}^{k} \max_{s\in[0,\,t]} \{|\kappa_r(x, s)|\} (\phi_r(p_r(x, t)) - \phi_r(p_r(x, 0))),$$

$$(4.22)\quad \forall\, t \ge 0 : \quad |\varepsilon_{tripij}(x, t)| \le 3 \max_{s\in[0,\,t]} \{\mu(x, s)\} \max_{s\in[0,\,t]} \{|\varepsilon_{tepij}{}^*(x, s)|\} \sum_{r=1}^{k} \max_{s\in[0,\,t]} \{|\kappa_r(x, s)|\}$$

Of course, the assertions of theorem 4.3 also hold, if there is no CP. This situation is given when quenching thin rings obeying a varying martensite-start temperature. The occurring distortion is only explainable by TRIP, because the thermal stresses are not sufficiently large (cf. [17]).

The theorems 4.2 and 4.3 allow to withdraw some useful assertions about TRIP strain behaviour. In particular, under certain circumstances the TRIP strain can be regarded as small. We illustrate this with an example.

Example 4.4 We consider the pearlitic transformation of the steel 100Cr6 at 700°C. The yield stress (0.02%) of austenite is about 70 MPa at this temperature. The Greenwood-Johnson parameter is about $1.4 \bullet 10^{-4}$ $(\text{MPa})^{-1}$ for the TRIP model without backstress (2.9) and about $2.0 \bullet 10^{-4}$ $(\text{MPa})^{-1}$ for the TRIP model with backstress (2.24) (cf. [9, 10, 49]).

(i) Assuming that classical plasticity does not occur, we obtain from (4.18)

$$(4.23)\quad \forall\, t \ge 0 : \quad |\varepsilon_{tripij}(x, t)| \le 0.017.$$

(ii) If there is no influence of classical plasticity on TRIP, i.e., if (4.19) is given, then the bound in (4.23) may be something higher due to possible hardening.

Therefore, in these two cases the TRIP strain can be regarded as small.

Remarks 4.5 (i) The estimates (4.6), (4.8), (4.12), (4.14) and (4.22) do not explicitly depend on the evolution of the phase fractions. This may be useful for mathematical and numerical investigations.

(ii) Assuming (4.1), (4.3), (4.4) and (4.19), the TRIP strain can be estimated from above by upper bounds on the deviatoric stress and of Greenwood-Johnson parameters.

(iii) Due to (4.1) the TRIP backstress has no direct influence on the estimates in theorem 4.1. Experiments show that the parameter κ is larger when taking a backstress into account (cf. [47]). We note that the assumption (4.1) is thermodynamically requested, whereas c_2 may be negative (cf. [39, 46, 48] and remark 5.1 below).

(iv) The assumptions (4.3), (4.4) include the dissolution of austenite into ferritic phases as well as the austenitization (in each case without subsequent further transformation). If there is a transformation process consisting of finitely many parts for which the assumptions (4.3), (4.4) are valid (with an individual number k), then the parts (ii) and (iv) of theorem 4.1 have to be modified. We will return to this matter in point 5.

(v) If (4.3), (4.4) are not given and if the situation in remark 4.2 (iv) is not attainable, then the functions $\phi'(p(t))\max\{p'(t), 0\}$ in (3.5) and (3.6) are *not* time derivatives on $[0, \infty]$. In this rather academic case one has to estimate integrals of kind

(4.20) $\int_0^t \phi_r'(p_r(x, s)) \max\{p_r'(s), 0\}\, ds.$

Therefore, in this case the estimates of TRIP strain may *depend* on phase evolution and saturation functions.

5 Estimates of the TRIP strain for uniaxially applied stress

In the case of an applied uniaxial (tension or compression) stress we can derive more specific estimates for the TRIP strain including estimates from below. The general setting of such uniaxial experiments is that small cylindrical steel samples are exposed to tension or compression along their axis under controlled temperature (cf. [42-44, 47, 49, 50] for details). Let the tension or compression be $S = S(t)$. Denoting the longitudinal TRIP strain by e_{trip} (= ε_{trip11} in a suited coordinate system) and the classical plastic strain by e_{cp} (= ε_{cp11} in the same coordinate system) and taking

(5.1) $\sigma_{11}^* = \frac{2}{3} S$

into account, we obtain from (3.5) and (3.9) for the TRIP models (2.26) and (2.27) with *only one forming* phase p (i.e., $p' \geq 0$)

(5.2) $e_{trip}(t) = \int_0^t \kappa(s)\, \phi'(p(s))\, p'(s)\, (S(s) - \frac{3}{2} c_2(s)\, e_{cp}(s)) \bullet$

$$\bullet \exp(-\frac{3}{2} \int_s^t c_1(\tau)\, \kappa(\tau)\, \phi'(p(\tau))\, p'(\tau) d\tau)\, ds.$$

As spatial homogeneity is assumed, in (5.2) the spatial variable x does not occur. In some special cases (constant κ, c_1, step-wise load F, $c_2 = 0$) the integrals in (5.2) can be integrated analytically. After this, one obtains formulas for model evaluation (cf. point 7 for details). Moreover, the formula (5.2) shows an interesting consequence of the models developed above.

Remark 5.1 Based on thermodynamic considerations, c_1 in (2.28) and c_3 in (2.29) must be positive, whereas c_2 may be negative (cf. [48]). Let us consider a stress-free phase transformation *after* a CP pre-deformation of the mother phase. As (5.2) shows, for positive c_2 the direction of the TRIP strain is contrary to the direction of the CP strain. But for negative c_2 the directions of CP and TRIP strains coincide. There are experiments showing both cases of behaviour [3, 38]. An interesting question is whether this phenomenon can be explained by investigations at the microscopic or mesoscopic level (cf. [37]).

Now we want to derive some formulas from (5.2).

Theorem 5.2 (i) Under

(5.3) $0 < c_{10} \leq c_2(s) \leq c_{11} < \infty, \qquad p(0) = 0$

one obtains from (5.1)

(5.4) $\forall\, t \geq 0 : |e_{trip}(t)| \leq \frac{2}{3c_{10}} \max_{s\in[0,t]} \{|S(s) - \frac{3}{2} c_2(s)\, e_{cp}(s)|\}\, (1 - \exp(-\frac{3}{2} c_{11}\, \phi(p(t)) \max_{s\in[0,t]} \{\kappa(s)\})) \leq$

$$\leq \frac{2}{3c_{10}} \max_{s\in[0,t]} \{|S(s) - \frac{3}{2} c_2(s)\, e_{cp}(s)|\}\, (1 - \exp(-\frac{3}{2} c_{11} \max_{s\in[0,t]} \{\kappa(s)\})\ ).$$

(ii) Under (5.3), (2.12) and

(5.5) $c_2 = 0,$

(5.6) $S(s) \geq 0$

there holds

(5.7) $\forall\, t \geq 0 : |e_{trip}(t)| \geq \frac{2}{3c_{11}} \min\limits_{s\in[0,\,t]} \{S(s)\}\, (1 - \exp(-\frac{3}{2} c_{10}\, \phi(p(t))\, \min\limits_{s\in[0,\,t]} \{\kappa(s)\})) \geq$

$$\geq \frac{2}{3c_{11}} \min\limits_{s\in[0,\,t]} \{S(s)\}\, (1 - \exp(-\frac{3}{2} c_{10}\, p(t)\, \min\limits_{s\in[0,\,t]} \{\kappa(s)\})).$$

Obviously, (5.4) is a special case of (4.12), (4.14).

Remark 5.3 (i) When performing TRIP experiments with small samples, it could be useful to start with a stress-free initial state (see point 7 for details). In other words, one has $S(0) = 0$ and (5.4) becomes trivial. In practice the load F starts with zero and will be quickly brought up to a positive level before the transformation starts. This small time interval can be neglected.
(ii) Analogously, we neglect the time interval of a quick jump of S. For example, we model a quick initial loading from zero to S_1 followed by a quick jump to S_2 at the time t_1 with the help of a step-wise function

(5.8) $S(s) := S_1$, for $0 \leq t \leq t_1$, and $S(s) := S_2$, for $t > t_1$.

Clearly, S given by (5.7) only approximates the reality. A choice of t_1 before the end transformation allows to study backstress effects. We will return to this in point 7. In the case of (5.6), (5.8) and only one forming phase we easily obtain from (5.2)

$$(5.9)\ \forall\, t \geq t_1 \quad : e_{trip}(t) = \exp(-\frac{3}{2}\int_{t_1}^{t} c_1(\tau)\, \kappa(\tau)\, \phi'(p(\tau))\, p'(\tau) d\tau\,) \int_{0}^{t_1} \kappa(s)\, \phi'(p(s))\, p'(s)\, S_1 \bullet$$

$$\bullet \exp(-\frac{3}{2}\int_{s}^{t_1} c_1(\tau)\, \kappa(\tau)\, \phi'(p(\tau))\, p'(\tau) d\tau\,)\, ds +$$

$$+ \int_{t_1}^{t} \kappa(s)\, \phi'(p(s))\, p'(s)\, S_2 \exp(-\frac{3}{2}\int_{s}^{t} c_1(\tau)\, \kappa(\tau)\, \phi'(p(\tau))\, p'(\tau) d\tau\,)\, ds.$$

Analogously as in theorem 5.2 we can derive from (5.9) upper and lower estimates.

The approach developed above can also be applied to changing phase transformations and thus to cycles. For convenience we assume the subsequent situation

(5.10) phase 1 $\rightarrow$ phase 2 for $0 \leq t \leq t_1$, and phase 2 $\rightarrow$ phase 1 for $t \geq t_1$.

In the case of (5.6) we obtain from (3.9) (cf. (5.2))

$$(5.11)\ \forall\, t \geq t_1 \quad : e_{trip}(t) = \exp(-\frac{3}{2}\int_{t_1}^{t} c_{11}(\tau)\, \kappa_1(\tau)\, \phi_1'(p_1(\tau))\, p_1'(\tau) d\tau\,) \int_{0}^{t_1} \kappa_2(s)\, \phi_2'(p_2(s))\, p_2'(s)\, S(s) \bullet$$

$$\bullet \exp(-\frac{3}{2}\int_{s}^{t_1} c_{12}(\tau)\, \kappa_2(\tau)\, \phi_2'(p_2(\tau))\, p_2'(\tau) d\tau\,)\, ds +$$

$$+ \int_{t_1}^{t} \kappa_1(s)\, \phi_1'(p_1(s))\, p_1'(s)\, S(s) \exp(-\frac{3}{2}\int_{s}^{t} c_{11}(\tau)\, \kappa_1(\tau)\, \phi_1'(p_1(\tau))\, p_1'(\tau) d\tau\,)\, ds.$$

The parameter c_{11}, κ_1 and the saturation function ϕ_1 are referred to the PT $2 \rightarrow 1$, whereas c_{12}, κ_2 and ϕ_2 are referred to the PT $1 \rightarrow 2$. As above, from formula (5.11) one can derive further estimates under different conditions (S = const., e.g.). Finally, we give some remarks.

Remarks 5.4 (i) As an example for the situation described in (5.10) one can regard the PT austenite into ferrite and backwards in an hypoeutectoid steel. The PT pearlite to austenite in a (slightly) hypereutectoid steel (100Cr6, e.g.) is actually a superposition of two PT, namely of the decomposition of Ferrite and of carbide. One has to decide whether the forming of austenite can be approximated as a single PT (cf. [35], e.g.).

(ii) The question arises whether there is an influence of the first PT $1 \to 2$ on the subsequent one $2 \to 1$. This influence may be realised via the so-called accumulated TRIP strain

$$(5.12) \qquad s_{trip}(x, t) := \int_0^t \left(\frac{2}{3} \varepsilon_{trip}'(x, \tau) : \varepsilon_{trip}'(x, \tau) \right)^{\frac{1}{2}} d\tau,$$

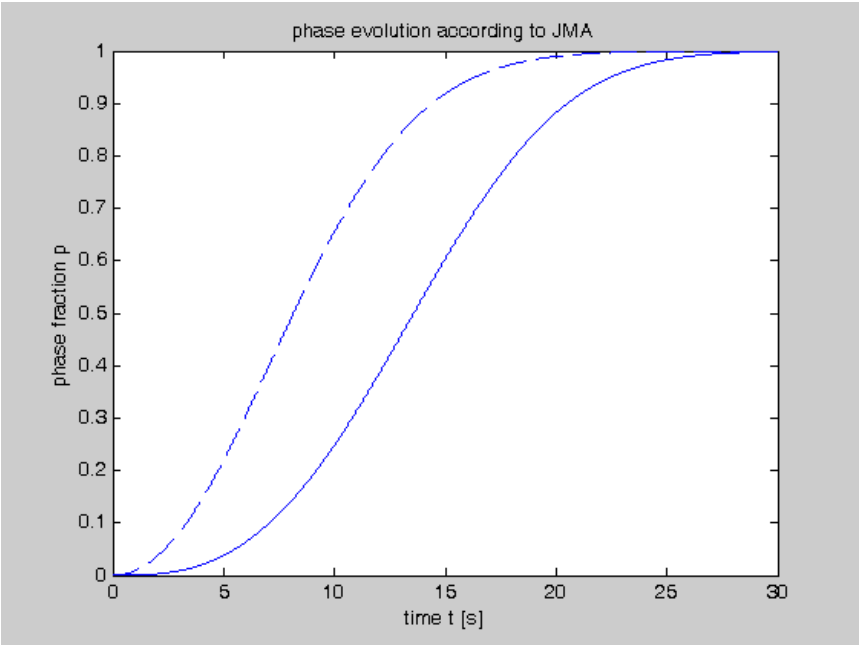
which the entities κ and c_1 may depend on (cf. [48]). Firstly performing separate experiments of the sort $1 \to 2$ and $2 \to 1$, respectively, in order to obtain parameters for separate PT, and after this PT like (5.10), this influence can be studied with the help of (5.11).
(iii) If no backstress is assumed, (5.11) shows a formal additivity, i.e. the TRIP strains of the single PTs are summed up. Unfortunately, also in this case, there may be an influence of the first PT on the second one via (5.12) (cf. [38]).

6 Influence of stress-dependent transformation behaviour on TRIP

It is well-known, that the evolution of phase fractions is influenced by the occurring stresses (cf. [1-3, 8, 9, 11, 12, 25, 29, 33], e.g.). This phenomenon is called stress-dependent transformation behaviour (SDTB). In accordance with (2.24) and (2.25), respectively, the evolution of the TRIP strain is influenced by the evolution of the phase fractions. Besides this, stress and TRIP strain are coupled via the bulk model (cf. [46, 48, 51], e.g.). Therefore, TRIP and PT are coupled, too. Fortunately, these two phenomena can by studied separately. In point 7 we will explain this, giving a general scheme for parameter identification and evaluating of models.

Now we want to investigate how the TRIP strain depends on SDTB. We consider a small cylindrical steel specimen and assume spatial homogeneity of temperature, stresses and phase fractions at all times. For convenience let us deal with the simple case without backstress. For constant temperature and (tension) stress S the longitudinal TRIP strain e_{trip} reads in the case of one forming phase (cf. (1) and point 7)

$$(6.1) \qquad e_{trip}(t) = \kappa \, S \, \phi(p(t)).$$



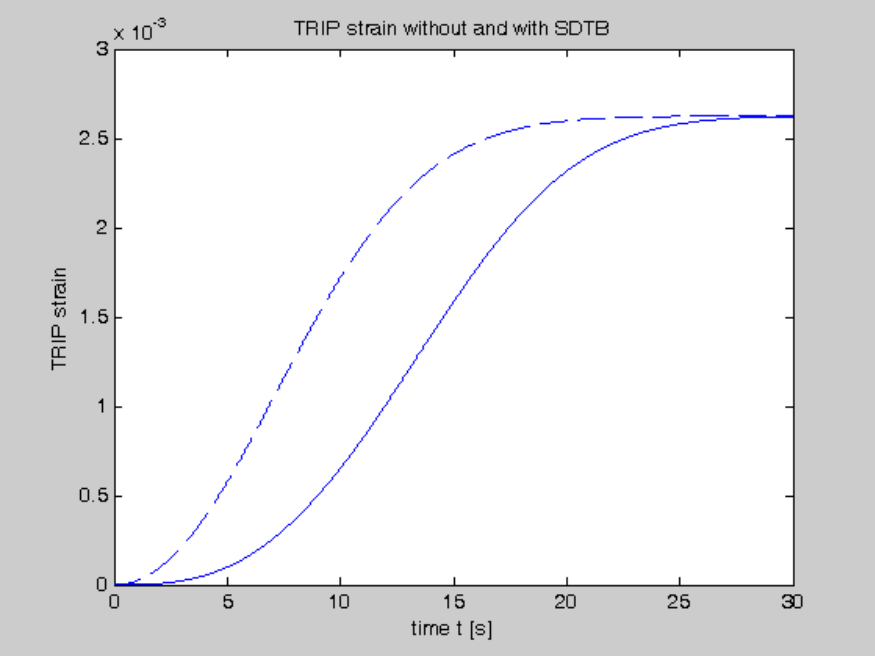
Picture 2: Phase evolution p^0 under zero stress (-) and p under 25 MPa (--) at 650°C for the steel 100Cr6.

Additionally, performing a *stress-free* parallel experiment for the *same constant* temperature, we obtain the phase evolution $p^0 = p^0(t)$. In picture 2 we see the essential influence of stress on PT. The question is now, how e_{trip} will change, if the phase evolution under stress p is substituted by the stress-free one p^0. Thus, the new longitudinal TRIP strain $e_{trip,0}$ corresponding to p^0 reads as

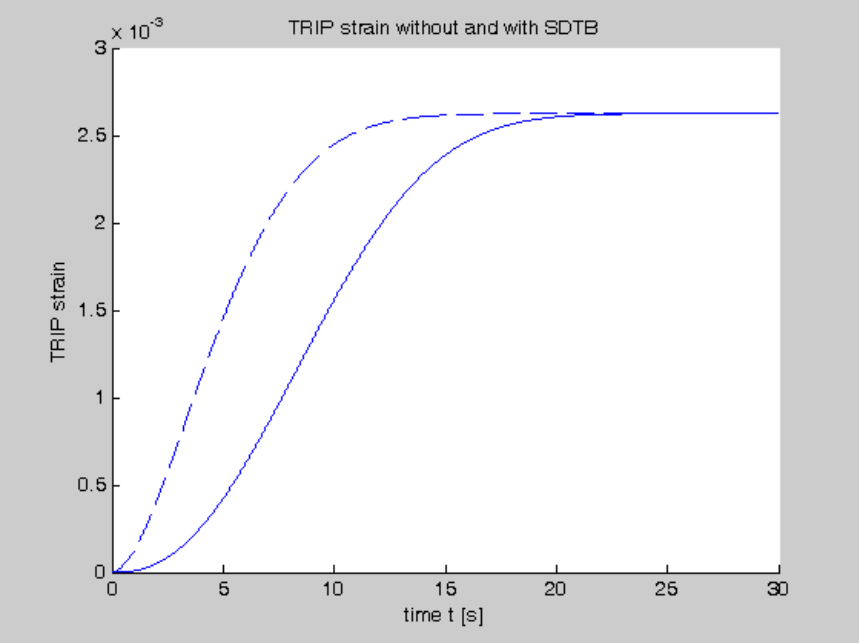
(6.2) $e_{trip,0}(t) = \kappa\, S\, \phi(p^0(t))$.

Clearly, due to (2.3) the values of e_{trip} and $e_{trip,0}$ coincide at the begin and the end of the (complete) transformation. Comparing $e_{trip,0}$ with e_{trip}, we obtain the subsequent relative error at t

(6.3) $$err_{rel}(t) := \frac{|e_{TRIP}(t) - e_{TRIP,0}(t)|}{e_{TRIP}(t)} = \frac{|\phi(p(t)) - \phi(p^0(t))|}{\phi(p(t))}.$$



Picture 3: TRIP strain without (-) and with (--) SDTB using Tanaka's proposal (2.6).



Picture 4: TRIP strain without (-) and with (--) SDTB using Leblond's proposal (2.5).

This is not the place to model SDPT in detail. And so we only shortly deal with it. One possibility to describe the phase-fraction evolution under (uniaxial) constant stress F in case of constant transformation temperature θ is given by the Johnson-Mehl-Avrami equation with temperature- and stress-dependent parameters like

(6.4) $$p(t) = 1 - \exp\left(-\left(\frac{t}{\tau(\theta, S)}\right)^{n(\theta, S)}\right),$$

where the PT with the only forming phase p is assumed to be complete. We refer to [9] for concrete values of τ and n for the pearlitic and bainitic transformation of the steel 100Cr6. We present an example with real data. From [9] we have for the steel 100Cr6 for the pearlitic transformation at 650°C

(6.5) $\kappa = 10.5 \bullet 10^{-5}\ (\mathrm{MPa})^{-1}$,

(6.6) $\tau(650, 0) = 15.4$ s $\quad n(650, 0) = 2.9$, (stress in MPa),

(6.7) $\tau(650, 25) = 9.7$ s $\quad n(650, 25) = 2.1$, (stress in MPa).

From (6.4) – (6.6) we obtain two curves for the phase evolution (see picture 2). Clearly, the TRIP strain essentially depends on the saturation function ϕ, in particular at the beginning of transformation. As extreme cases we take the proposals due to Leblond (2.5) and the one due to Tanaka (2.6). So we obtain for these two cases the corresponding curves for e_{trip} and $e_{trip,0}$, respectively (s. pictures 3 and 4). Finally, we calculate the relative error after 5 and 10 seconds after beginning of PT by formula (6.3):

(6.8) $err_{rel}(5) = 0{,}83$, $\quad err_{rel}(10) = 0{,}62$ for Tanaka's proposal (2.6),

(6.9) $err_{rel}(5) = 0{,}71$, $\quad err_{rel}(10) = 0{,}36$ for Leblond's proposal (2.5).

It seems that there are significant errors when neglecting stress-dependent transformation behaviour. Therefore, the evaluation of TRIP models by experiments with small probes requires more measured data as in stress-free dilatometry. Besides the longitudinal strain one needs the transversal strain (s. the next point 7). Only in exceptional cases with a small stress dependence on phase evolution, TRIP can be investigated using an additional stress-free dilatometer experiment without measuring the transversal strain of the probe.

7 Parameter identification

In order to study TRIP and stress-dependent phase transformations (SDPT) in steel one usually performs tests with small cylindrical specimen in special devices like dilatometers, which can measure temperature θ, length l, diameter d, and applied (uniaxial) stress S as functions of time. Assuming spatial homogeneity, we write down the relations for the (bulk) longitudinal strain ε_L and the (bulk) transversal strain ε_D:

(7.1) $$\varepsilon_L(t) = \frac{l(t) - l}{l} = \frac{1}{E(\theta(t))}\, S(t) + \left(\sqrt[3]{\frac{\rho_0}{\rho(\theta(t))}} - 1\right) + e_{trip}(t) + e_{cp}(t),$$

(7.2) $$\varepsilon_D(t) = \frac{d(t) - d}{d} = \frac{-\nu(\theta(t))}{E(\theta(t))}\, S(t) + \left(\sqrt[3]{\frac{\rho_0}{\rho(\theta(t))}} - 1\right) - \frac{1}{2}\, e_{trip}(t) - \frac{1}{2}\, e_{cp}(t),$$

where l and d are the initial length and diameter, respectively, corresponding to $t = 0$, ρ_0, $\rho(\theta(t))$ - densities at the beginning (of austenite, e.g.) and of the phase mixture, respectively. E and ν are Young modulus and Poisson's ratio at $\theta(t)$, respectively. The first terms in (7.1) and (7.2) are elastic strains, the second ones include the thermal strains and the strains due to density changes coming from PT. Furthermore, (7.1) and (7.2) take the volume preservation of TRIP and CP into account. In the absence of CP, i.e. for stresses S smaller than the yield stress of the weaker phase, we obtain formulas allowing to calculate the TRIP strain and the evolution of the forming phase (pearlite from austenite, e.g.) via the measured data (cf. [50]), namely

(7.3) $$\varepsilon_V(t) = \varepsilon_L(t) + 2\,\varepsilon_D(t) = \frac{1}{3\,K(\theta(t))}\, S(t) + 3\left(\sqrt[3]{\frac{\rho_0}{\rho(\theta(t))}} - 1\right)$$ "volume strain",

(7.4) $$\varepsilon_L(t) - \varepsilon_D(t) = \frac{1}{2\,\mu(\theta(t))}\, S(t) + \frac{3}{2}\, e_{trip}(t)$$ "strain difference",

where ε_V is the volume strain, and K is the compression modulus defined by

$$K = \frac{E}{3\,(1 - 2\nu)}, \tag{7.5}$$

(cf. (3.12)). In contrast to [42-44, 47, 49], here we do not linearize the density term in (7.3). Clearly, from (7.4) we obtain the evolution of the TRIP strain e_{trip} without assuming any special law for TRIP like discussed above (cf. (2.26), (2.27)). Sometimes, elastic effects are neglected. This gives the approximated formula (cf. [1-3, 8-10])

$$e_{trip}(t) = \frac{2}{3}\,(\varepsilon_L(t) - \varepsilon_D(t)). \tag{7.6}$$

Furthermore, from (7.3) we obtain the evolution of the phase fraction of the forming phase. At first let us assume, that there are only two phases, the parent phase labelled by "1", such as austenite, e.g., and the forming one labelled by "2", pearlite, e.g. The entities K and μ generally depend on the phase fractions. We assume the following mixture rules

$$\frac{1}{K(\theta(t))} = \frac{1}{K_1(\theta(t))}\,p_1(t) + \frac{1}{K_2(\theta(t))}\,p_2(t), \qquad \frac{1}{\mu(\theta(t))} = \frac{1}{\mu_1(\theta(t))}\,p_1(t) + \frac{1}{\mu_2(\theta(t))}\,p_2(t), \tag{7.7}$$

where p_1 and p_2 are the (volume) fractions of austenite and pearlite, respectively. We note the mixture rule for the bulk density (cf. [42] for details, e.g.):

$$\rho = \rho_1\,p_1 + \rho_2\,p_2, \tag{7.8}$$

where ρ, ρ_1 and ρ_2 are the densities of the phase mixture, of austenite und pearlite, respectively. Clearly, for the phase fractions we have

$$p_1(t) + p_2(t) = 1 \qquad \text{for } t \geq 0. \tag{7.9}$$

Combining (7.3), (7.7), (7.8), and (7.9), there arises a relation for determining the evolution of the forming phase:

$$\varepsilon_V(t) = \frac{S(t)}{3}\,\{\,\frac{1}{K_1(\theta(t))} + (\,\frac{1}{K_2(\theta(t))} - \frac{1}{K_1(\theta(t))}\,)\,p_2(t)\,\} + \tag{7.10}$$

$$+ 3\,(\sqrt[3]{\frac{\rho_0}{\rho_1(\theta(t)) + (\rho_2(\theta(t)) - \rho_1(\theta(t)))\,p_2(t)}} - 1) \qquad \text{for } t \geq 0.$$

Unfortunately, (7.10) is a non-linear equation for the determination of p_2 which has to be solved numerically. We remark that one can approximately use elastic parameters K and μ independent of the phase composition (cf. [47, 49]). In order to employ (7.10) one needs (besides the measured data θ, ε_L, ε_D, S) the densities of the phases 1 and 2 as functions of the temperature $\theta = \theta(t)$. The initial density ρ_0 is usually the density of phase 1 at the beginning, $t = 0$, corresponding to the start temperature θ_0. These densities can be obtained by separate (stress-free) dilatometer tests with steel probes of the same charge. Due to small variations in the densities (caused by (slightly) inhomogeneous chemical composition) the formula (7.10) may give values for p not having the limit 1 for $t \to \infty$ in the case of complete transformations. In this case one has to divide the values of p by the limit value $p(t_\infty)$, where t_∞ is a sufficiently large time after which the transformation can be considered finished. Sometimes the formula (7.10) will be simplified. Linearising the root in (7.1) and the forthcoming formulas via

$$3\,(\sqrt[3]{\frac{\rho_0}{\rho(\theta(t))}} - 1) \approx \frac{\rho_0 - \rho(\theta(t))}{\rho(\theta(t))}, \tag{7.11}$$

we obtain instead of (7.10) a simpler formula

$$(7.12)\quad p_2(t) = \frac{1 + \varepsilon_V(t) - \frac{S(t)}{3\,K_1(\theta(t))} - \frac{\rho_0}{\rho_1(\theta(t))}}{\frac{S(t)}{3}\left(\frac{1}{K_2(\theta(t))} - \frac{1}{K_1(\theta(t))}\right) + \rho_0\left(\frac{1}{\rho_2(\theta(t))} - \frac{1}{\rho_1(\theta(t))}\right)} \qquad \text{for } t \geq 0.$$

We note that the above formulas are valid for tests with varying temperature and load as well as for incomplete transformations.

A further essential simplification of the above formulas can be achieved, if the compression modulus K does not depend on the phase composition, i.e. if there holds

$$(7.13)\qquad K(\theta, p) = K(\theta).$$

Assuming (7.13) and combining (7.3), (7.8) and (7.9), we obtain a simpler formula for p_2:

$$(7.14)\qquad p_2(t) = \frac{\rho_1(\theta(t))}{\rho_1(\theta(t)) - \rho_2(\theta(t))}\left\{1 - \frac{\rho_0}{\rho_1(\theta(t))}\left[1 + \frac{\varepsilon_V(t)}{3} - \frac{S(t)}{9\,K(\theta(t))}\right]^{-3}\right\} \qquad \text{for } t \geq 0.$$

In the case of *constant* transformation temperature, i.e. for

$$(7.15)\qquad \theta(t) = \theta_0 \qquad \text{for } t \geq 0,$$

we have

$$(7.16)\qquad \rho_0 = \rho_1(\theta_0).$$

Therefore, under (7.15) the formula (7.14) can be reduced to

$$(7.17)\qquad p_2(t) = \frac{\rho_1(\theta_0)}{\rho_1(\theta_0) - \rho_2(\theta_0)}\left\{1 - \left[1 + \frac{\varepsilon_V(t)}{3} - \frac{S(t)}{9\,K(\theta_0)}\right]^{-3}\right\} \qquad \text{for } t \geq 0,$$

A further simplification of (7.17) is possible for complete transformations. In this case for a sufficiently large time t_∞ there will be only phase 2, in other words, (7.17) reads as

$$(7.18)\qquad 1 = p_2(t_\infty) = \frac{\rho_1(\theta_0)}{\rho_1(\theta_0) - \rho_2(\theta_0)}\left\{1 - \left[1 + \frac{\varepsilon_V(t_\infty)}{3} - \frac{S(t_\infty)}{9\,K(\theta_0)}\right]^{-3}\right\}.$$

Hence, (7.17) and (7.18) lead to a formula *not containing the densities*

$$(7.19)\quad p_2(t) = \left\{1 - \left[1 + \frac{\varepsilon_V(t)}{3} - \frac{S(t)}{9\,K(\theta_0)}\right]^{-3}\right\} \bullet \left\{1 - \left[1 + \frac{\varepsilon_V(t_\infty)}{3} - \frac{S(t_\infty)}{9\,K(\theta_0)}\right]^{-3}\right\}^{-1} \qquad \text{for } t \geq 0.$$

Clearly, after linearization we obtain from (7.19)

$$(7.20)\quad p_2(t) = \frac{\varepsilon_V(t) - \frac{S(t)}{3\,K(\theta_0)}}{\varepsilon_V(t_\infty) - \frac{S(t_\infty)}{3\,K(\theta_0)}}.$$

Sometimes the influence of the elasticity is neglected. This leads to the simple formula used for instance in [1-3, 8-10]

$$(7.21)\quad p_2(t) \approx \frac{\varepsilon_V(t)}{\varepsilon_V(t_\infty)},$$

which can be regarded as a transformation degree expressed by the volume strain.

Now, let us return to TRIP. Using the phase fraction p_2 calculated by (7.10) (or by its subsequent simplifications), we get the evolution of e_{trip} from (7.4), (7.7), (7.9):

$$(7.22)\qquad e_{trip}(t) = \frac{2}{3}\left(\varepsilon_L(t) - \varepsilon_D(t)\right) - \left\{\frac{1}{\mu_1(\theta(t))} + \left(\frac{1}{\mu_2(\theta(t))} - \frac{1}{\mu_1(\theta(t))}\right) p_2(t)\right\}\frac{S(t)}{3}.$$

We emphasize that (7.10), (7.22) allow to calculate the evolution of the forming phase and of the TRIP strain without assuming specific models for TRIP as well as for SDPT. Therefore, the

formulas (7.10) and (7.12) are the starting points for parameter identification and for evaluation of these models.

The *general scheme* is as follows: Assuming special laws for TRIP, for instance (2.9), as well as for SDPT, for instance a Johnson-Mehl-Avrami ansatz with stress-dependent parameters as in (6.4) (cf. [8, 9]), one can compare the values of e_{trip} and p_2 obtained via data from (7.10), (7.22) with those ones calculated by the proposed special laws.

In the case of a TRIP model without backstress one usually performs tension (and/or compression) tests at constant temperature with constant loads up to the end of a complete PT. Using (2.1), κ easily follows from the final extent of TRIP strain divided by S. After this, proposals for the saturation function ϕ can be tested, using the phase evolution obtained by (7.10) or its simplifications and an optimising procedure (cf. [10]).

In the case of a TRIP model *with* backstress a separate determination of κ is impossible. Therefore, one performs not only tension (or compression) tests with constant load up to the end, but also with step-wise loads (cf. [47, 49, 50]). We assume only temperature dependence of κ and c_1, i.e.

(7.23) $\qquad \kappa = \kappa(\theta), \qquad c_1 = c_1(\theta).$

Then we obtain for constant temperature θ_0 and constant load S (> 0) from (5.2) (with $c_2 = 0$)

$$e_{trip}(t) = \frac{2\,S}{3c_1(\theta_0)}\,(1 - \exp(-\tfrac{3}{2}\,c_1(\theta_0)\,\kappa(\theta_0)\,\Phi(p(t)))\,) \qquad \text{for } t \geq 0. \tag{7.24}$$

Hence, we have

$$e_{trip}(t_\infty) = \frac{2\,S}{3c_1(\theta_0)}\,(1 - \exp(-\tfrac{3}{2}\,c_1(\theta_0)\,\kappa(\theta_0))\,). \tag{7.25}$$

Additionally, we perform a test with a step-wise load similar as in (5.8) at the same constant temperature θ_0, i.e.

(7.26) $\qquad S(s) := S, \qquad \text{for } 0 \leq t \leq t_1, \quad \text{and} \quad S(s) := 0, \qquad \text{for } t > t_1,$

where at the time t_1 the transformation is not yet finished. In this case the integration of (5.2) yields (7.24) for $0 \leq t \leq t_1$, and

$$e_{trip}(t) = \frac{2\,S}{3c_1(\theta_0)}\exp(-\tfrac{3}{2}\,c_1(\theta_0)\,\kappa(\theta_0)\,\Phi(p(t)))\,\{\exp(\tfrac{3}{2}\,c_1(\theta_0)\,\kappa(\theta_0)\,\Phi(p(t_1))) - 1\} \quad \text{for } t \geq t_1. \tag{7.27}$$

In particular we obtain from (7.27)

$$e_{trip}(t_1) = \frac{2\,S}{3c_1(\theta_0)}\,\{1 - \exp(-\tfrac{3}{2}\,c_1(\theta_0)\,\kappa(\theta_0)\,\Phi(p(t_1)))\,\}, \tag{7.28}$$

as well as

$$e^{I}_{trip}(t_\infty) = \frac{2\,S}{3c_1(\theta_0)}\exp(-\tfrac{3}{2}\,c_1(\theta_0)\,\kappa(\theta_0))\,\{\exp(\tfrac{3}{2}\,c_1(\theta_0)\,\kappa(\theta_0)\,\Phi(p(t_1))) - 1\}, \tag{7.29}$$

where $e^{I}_{trip}(t_\infty)$ is the final TRIP strain for the loading path given by (7.26). Hence, from (7.25), (7.28) and (7.29) we get the following formulas for determining $c_1(\theta_0)$, $\kappa(\theta_0)$ and $\Phi(p(t_1))$

$$c_1(\theta_0) = \frac{2\,S}{3\,e_{trip}(t_1)}\,\frac{e_{trip}(t_1) - e^{I}_{trip}(t_\infty)}{e_{trip}(t_\infty) - e^{I}_{trip}(t_\infty)}, \tag{7.30}$$

$$\kappa(\theta_0) = -\frac{2}{3c_1(\theta_0)}\ln(1 - \frac{3\,c_1(\theta_0)}{2\,S}\,e_{trip}(t_\infty)), \tag{7.31}$$

$$\Phi(p(t_1)) = -\frac{2}{3c_1(\theta_0)\,\kappa(\theta_0)}\,\ln(1 - \frac{3\,c_1(\theta_0)}{2\,S}\,e_{trip}(t_1)). \tag{7.32}$$

Experiments suggest that at least c_1 does not only depend on temperature, but possibly on the stress history too (cf. [50] for discussion). The value $\Phi(p(t_1))$ may give a hint which proposal for ϕ (cf. (2.5) – (2.7), (2.12), (2.13)) could be suitable. Generally, using the phase evolution (7.10) and the values for κ and c_1 obtained by (7.30), (7.31), one has to determine ϕ by an optimisation procedure.

We conclude with the subsequent remarks.

Remark 7.1 (i) The approach developed above allows to test models for TRIP and STPT independently of each other. Although this is very convenient in application, in principle these models must be evaluated simultaneously. That means, the values for p and e_{trip} coming from measured data via (7.10) and (7.22) must be simultaneously compared with values for p and e_{trip} predicted by proposed models for TRIP and SDPT. If one proposes TRIP with backstress and (6.4), then the parameters and functions κ, c_1, ϕ, τ and n have to be simultaneously determined by an optimisation procedure. In future the authors will present results in this direction.

(ii) Here we focus on uniaxial tension-compression tests. But generally the dependence on stress is realised through the invariants of the stress tensor, through the von Mises stress (4.16) and through the mean stress $\sigma_m = 1/3\ tr(\sigma)$ e.g. (cf. remark 2.1 (ii)). Therefore, for a sufficient modelling of 3d behaviour one needs two-axial tests, i.e. tension-compression-torsion (cf. [43]).

(iii) In the absence of PT and TRIP one can study the CP behaviour (or viscous behaviour), using the formula (7.4) with ε_{cp} instead of ε_{trip}. The advantage is, that the isotropic strain due to temperature changes does not occur in (7.4).

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