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Higher Order Real-Time Approximations In Optimal Control of Multibody-Systems For Industrial Robots

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Abstract

The multibody system of an industrial robot leads to a mathematical modell described by ordinary differential equations. Control functions have to be determined such that a given performance index is optimized subject to additional constraints. In order to solve such optimal control problems time-consuming methods are used which have no real-time capability. Hence a robust numerical method based on the parametric sensitivity analysis of nonlinear optimization problems is suggested. Real-time control approximations of perturbed optimal solutions can be obtained by evaluating a first order Taylor expansion of the perturbed solution. Successive improvement of the constraints in direction of the optimal perturbed solution leads to an admissible solution with a higher order approximation of the objective. The proposed numerical method is illustrated by the optimal control of an industrial robot subject to deviations in the payload and initial values.

Keywords

robot control, sensitivity analysis, real-time control, nonlinear programming methods

1 Introduction

In present production lines common tasks like welding, gluing and transportation of loads are performed by industrial robots. Employing robots for dangerous, exhausting and monotonous work has become necessary in order to stay competitive on the international market: They raise production rates, bring down costs and in general improve the products' quality.
However the commonly used methods for teaching trajectories are not as advanced as one would expect. Often users rely on manually generated trajectories or on paths found by heuristic algorithms. To exploit the capacities of industrial robots improved trajectories have to be calculated.
In the last decade, this situation has motivated research to design and implement feasible controllers. The design of mathematical models complicates the numerical computations due to highly complex dynamics. In compact form the dynamics for a common industrial robot look like the well-known formula for multi-rigid-body systems

$$u = M(q)\ddot{q} + R(q, \dot{q}). \tag{1}$$

Optimal control is a powerful tool for calculating trajectories which are optimal in view of a user defined performance index, like minimizing the process time, reducing the energy consumption or even considering the wear and tear.
During robot motion one may often detect deviations from nominal parameters in the system, e.g., deviations in the load mass or in the coordinates of the trajectory. Unfortunately the comparatively high computing times for solving those perturbed optimal control problems disqualify commonly used methods for a number of applications. This motivates the development of fast and reliable *real-time control approximations* for perturbed optimal solutions.
In stability analysis, differential properties of optimal solutions with respect to perturbation parameters are studied. Sensitivity analysis is concerned with the computation of sensitivity differentials of optimal solutions. This sensitivity information enables the control engineer to estimate the changes in the modeling function and optimal solution due to small deviations of the design parameters from fixed nominal values. The purpose of this paper is to show that the property of solution differentiability is essential for designing real-time control algorithms.

In this paper approximations of perturbed solutions are obtained from non-linear programming methods that can be used in an efficient and robust way for the computation of both the nominal solution and the sensitivity differentials of perturbed solutions. Numerical results are presented for the industrial robot ABB IRB 6400 2.8. The discussion is restricted to the three main joints of the robot which are responsible for the positioning of the tool.

2 Robot equations

2.1 Dynamics

For a system with n_f degrees of freedom under holonomic constraints Lagrange's equations can be written as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j, \quad j = 1, \ldots, n_f, \tag{2}$$

where q_j describes a generalized coordinate, Q_j is the generalized work, and T is the kinetic energy.
For the dynamic system representing the manipulator of an industrial robot the generalized coordinates are chosen to be the angle positions in the joints, which connect two adjacent bodies. The generalized work depends on the gravitational potential energy V invoked by a conservative force field and on the work performed by an external control u_j. This control addresses the motor at the appropriate joint of angle q_j:

$$Q_j = -\frac{\partial V}{\partial q_j} + u_j.$$

Evaluating the derivative with respect to time in (2) yields

$$u_j = \sum_{i=1}^{n_f} \frac{\partial^2 T}{\partial \dot{q}_j \partial \dot{q}_i}\ddot{q} + \sum_{i=1}^{n_f} \frac{\partial^2 T}{\partial \dot{q}_j \partial q_i}\dot{q} - \frac{\partial (T-V)}{\partial q_j}, \quad j = 1, \ldots, n_f$$

or in compact form

$$u = T_{\dot{q}\dot{q}}\ddot{q} + T_{\dot{q}q}\dot{q} - (T-V)_q,$$

since $T = T(q, \dot{q})$ and $V = V(q)$.
Each of the n_k bodies of the manipulator accounts for the total kinetic energy

T. The kinetic energy for a single body i is divided into a translatory part $T_{i,\text{trans}}$ and a rotational part $T_{i,\text{rot}}$:

$$T = \sum_{i=1}^{n_k} T_{i,\text{trans}} + T_{i,\text{rot}}.$$

Accordingly the potential energy is given by the sum of the potential energy V_i of the individual bodies:

$$V = \sum_{i=1}^{n_k} V_i.$$

For a single body i these energy terms are given by

$$T_{i,\text{trans}} = \frac{1}{2} m_i |v_i|^2, \qquad T_{i,\text{rot}} = \frac{1}{2} \omega_i^\top J_i \omega_i, \qquad V_i = g m_i h_i.$$

The mass m_i and the moment of inertia J_i are constant for each body and can be obtained by e.g. CAD tools, measurement or by parameter estimation. As customary g is the gravitional constant.

Formulating the robot kinematics, as it is done in the next section, allows for an evaluation of the absolute coordinates S_i describing the center of gravity for each body i. The height h_i of body i is just the height of the center of gravity, i.e. $h_i := (S_i)_3$.

Since S_i depends on the generalized coordinates $q_j = q_j(t)$ the center of gravity is time dependent, too. Hence the velocity v_i is given by its first time derivative

$$v_i = \frac{dS_i}{dt} = \frac{\partial S_i}{\partial q} \dot{q}.$$

Finally ω_i denotes the angular velocity of body i.

2.2 Kinematics

To set up the formulas of the robot dynamics we require the position and orientation of the bodies of the robot manipulator with respect to a fixed base system B.

Therefore local coordinate systems K_i $(i = 1, \dots 4)$ are assigned to each body of the system. K_1 specifies the position and orientation of the first body of the robot. K_2 and K_3 are the coordinate systems of the following bodies, which form the manipulator. An additional coordinate system K_4 is considered for a counterweight.

Next the transformations between adjacent coordinate systems can be given as 4×4-matrices by using the notation of homogeneous coordinates. Transformations from a coordinate system A to a system B will be denoted by T_A^B. Translations along a vector s are named by $\mathrm{Tr}(s)$, while rotations by an angle α e.g. about the main axis z of the local coordinate system are denoted by $\mathrm{Rot}_z(\alpha)$.

$$\begin{aligned}
T_B^{K_1} &= \mathrm{Rot}_z(q_1) = \begin{pmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \\
T_{K_1}^{K_2} &= \mathrm{Tr}(l_1)\mathrm{Rot}_y(q_2) = \begin{pmatrix} \cos q_2 & 0 & \sin q_2 & l_{11} \\ 0 & 1 & 0 & l_{12} \\ -\sin q_2 & 0 & \cos q_2 & l_{13} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \\
T_{K_2}^{K_3} &= \mathrm{Tr}(l_2)\mathrm{Rot}_y(q_3 - q_2) = \begin{pmatrix} \cos(q_3 - q_2) & 0 & \sin(q_3 - q_2) & l_{21} \\ 0 & 1 & 0 & l_{22} \\ -\sin(q_3 - q_2) & 0 & \cos(q_3 - q_2)1 & l_{23} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \\
T_{K_1}^{K_4} &= \mathrm{Tr}(l_1)\mathrm{Rot}_y(q_3) = \begin{pmatrix} \cos q_3 & 0 & \sin q_3 & l_{11} \\ 0 & 1 & 0 & l_{12} \\ -\sin q_3 & 0 & \cos q_3 & l_{13} \\ 0 & 0 & 0 & 1 \end{pmatrix}.
\end{aligned}$$

Herein the vectors $l_i = (l_{i1}, l_{i2}, l_{i3})^\top$, $i = 1, 2$ denote the position of the joints $(i+1)$ with respect to the system K_i. These transformations can be combined to express transformations with respect to the base system:

$$\begin{aligned}
T_B^{K_2} &= T_B^{K_1} T_{K_1}^{K_2} \\
T_B^{K_3} &= T_B^{K_1} T_{K_1}^{K_2} T_{K_2}^{K_3} \\
T_B^{K_4} &= T_B^{K_1} T_{K_1}^{K_4}
\end{aligned}$$

Herewith the absolute coordinates for the center of gravity S_i can be calculated from

$$S_i = T_B^{K_i} s_i, \quad i = 1, 2, 3, 4,$$

where $s_i = (s_{i1}, s_{i2}, s_{i3})^\top$ denotes the local coordinates of the center of gravity of body i.

The absolute angular velocities depend on the velocities $\dot{q}_i$ in the joints. Formulated in homogeneous coordinates we obtain

$$\begin{aligned}
\omega_1 &= \begin{pmatrix} 0 \\ 0 \\ \dot{q}_1 \\ 0 \end{pmatrix}, \\
\omega_2 &= (T_{K_1}^{K_2})^\top \omega_1 + \begin{pmatrix} 0 \\ \dot{q}_2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\sin(q_2)\dot{q}_1 \\ \dot{q}_2 \\ \cos(q_2)\dot{q}_1 \\ 0 \end{pmatrix}, \\
\omega_3 &= (T_{K_2}^{K_3})^\top \omega_2 + \begin{pmatrix} 0 \\ \dot{q}_3 - \dot{q}_2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -\sin(q_3)\dot{q}_1 \\ \dot{q}_3 \\ \cos(q_3)\dot{q}_1 \\ 0 \end{pmatrix}.
\end{aligned}$$

The absolute position of the tool center point TCP is given by

$$S_{\mathrm{TCP}} = T_B^{K_3} l_3.$$

Herein l_3 denotes the relative coordinate of the TCP with respect to K_3. By derivating we get the vector of velocity

$$v_{\mathrm{TCP}} = \frac{d}{dt} S_{\mathrm{TCP}} = \frac{\partial S_{\mathrm{TCP}}}{\partial q} \dot{q}.$$

2.3 Implementation of the control equation

Next we present the mass matrix $M = (M)_{i,j},\ i,j = 1,2,3$ and the right hand side vector $R = (R_1, R_2, R_3)^\top$ of (1). With the abbreviations

$$\begin{aligned}
a_1 &= s_{31}\, \cos q_3 + s_{33}\, \sin q_3 \\
a_2 &= s_{33}\, \cos q_3 - s_{31}\, \sin q_3 \\
a_3 &= s_{41}\, \cos q_3 + s_{43}\, \sin q_3 \\
a_4 &= s_{43}\, \cos q_3 - s_{41}\, \sin q_3 \\
a_5 &= s_{21}\, \cos q_2 + s_{23}\, \sin q_2 \\
a_6 &= s_{23}\, \cos q_2 - s_{21}\, \sin q_2 \\
b_1 &= l_{23}\, \cos q_2
\end{aligned}$$

$$
\begin{aligned}
b_2 &= l_{23} \sin q_2 \\
c_1 &= a_1 b_1 - a_2 b_2 \\
c_2 &= a_2 b_1 + a_1 b_2
\end{aligned}
$$

the mass matrix M is given by

$$
\begin{aligned}
M_{1,1} &= m_1 s_{11}^2 + m_2((a_5 + l_{11})^2 + s_{22}^2) + \\
&\quad m_3((a_1 + b_2 + l_{11})^2 + s_{32}^2) + m_4(a_3 + l_{11})^2 + \\
&\quad i_{1z} + i_{2x} \sin^2 q_2 + i_{2z} \cos^2 q_2 + i_{3x} \sin^2 q_3 + i_{3z} \cos^2 q_3 \\
M_{1,2} = M_{2,1} &= -m_2 a_6 s_{22} - m_3 b_1 s_{32} \\
M_{1,3} = M_{3,1} &= -m_3 a_2 s_{32} \\
M_{2,2} &= m_2(s_{23}^2 + s_{21}^2) + m_3 l_{23}^2 + i_{2y} \\
M_{2,3} = M_{3,2} &= m_3 c_2 \\
M_{3,3} &= m_3(s_{33}^2 + s_{31}^2) + m_4(s_{43}^2 + s_{41}^2) + i_{3y}.
\end{aligned}
$$

Defining

$$
\begin{aligned}
d_1 &= m_2(a_5 + l_{11})a_6 + m_3(a_1 + b_2 + l_{11})b_1 + (i_{2x} - i_{2z}) \sin q_2 \cos q_2 \\
d_2 &= m_4(a_3 + l_{11})a_4 + m_3(a_1 + b_2 + l_{11})a_2 + (i_{3x} - i_{3z}) \sin q_3 \cos q_3
\end{aligned}
$$

yields

$$
\begin{aligned}
R_1 &= 2\dot{q}_1 \dot{q}_2\, d_1 + 2\dot{q}_1 \dot{q}_3\, d_2 + \dot{q}_2^2\,(m_2 a_5 s_{22} + m_3 b_2 s_{32}) + \dot{q}_3^2\, m_3 a_1 s_{32} \\
R_2 &= -\dot{q}_1^2\, d_1 - \dot{q}_3^2\, m_3 c_1 - g(m_2 a_5 + m_3 b_2) \\
R_3 &= -\dot{q}_1^2\, d_2 + \dot{q}_2^2\, m_3 c_1 - g(m_3 a_1 + m_4 a_3).
\end{aligned}
$$

Moreover the velocity of the TCP is given by

$$
\begin{aligned}
a_7 &= \cos q_3 l_{31} + \sin q_3 l_{33} \\
a_8 &= \cos q_3 l_{33} - \sin q_3 l_{31}
\end{aligned}
$$

$$
\begin{aligned}
\|v_{\mathrm{TCP}}\|^2 &= (b_2 \dot{q}_2 + a_7 \dot{q}_3)^2 + ((a_7 + b_2 + l_{11})\dot{q}_1)^2 \\
&\quad + (-l_{32}\dot{q}_1 + b_1 \dot{q}_2 + a_8 \dot{q}_3)^2.
\end{aligned}
$$

2.4 Frictional and restoring forces

Frictional and restoring forces go beyond the scope of the Lagrangian Mechanics. However, these forces are considered by the following modification of (1):

$$
\ddot{q} = M(q)^{-1}(Du - R(q,\dot{q}) - \tau_{\mathrm{fric}}(\dot{q}) - \tau_{\mathrm{rest}}(q)) \tag{3}
$$

The Coulomb friction in the joint angles is included by the additional force τ_{fric}:

$$\tau_{\text{fric}}(\dot{q}) = \begin{pmatrix} 380 \tanh(3\dot{q}_1) \\ 345 \tanh(3\dot{q}_2) \\ 337 \tanh(3\dot{q}_3) \end{pmatrix}.$$

The body of the robot is stabilized by pneumatic cylinders. Hence an additional restoring force τ_{rest} similar to Hooke's Law is considered:

$$\tau_{\text{rest}}(q) = \begin{pmatrix} 0 \\ 3783.4297 \cdot q_2 \\ 0 \end{pmatrix}.$$

The control vector u is normalized by the diagonal matrix

$$D = diag(3412.256, 3465.5725, 3465.5725)^\top.$$

2.5 Technical data

All computations presented hereafter are applied to the industrial robot ABB IRB 6400 2.8 without any tool mounted to the end effector. In detail we have the moments of inertia

$$J_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 28.0 \end{pmatrix}, \quad J_2 = \begin{pmatrix} 33.63 & 0 & 0 \\ 0 & 28.43 & 0 \\ 0 & 0 & 9.4 \end{pmatrix},$$

$$J_3 = \begin{pmatrix} 8.6607 & 0 & 0 \\ 0 & 181.8004 & 0 \\ 0 & 0 & 195.2742 \end{pmatrix},$$

while the masses are given by

$$m_1 = 510.0, \quad m_2 = 240.0, \quad m_3 = 294.9 + p_4, \quad m_4 = 465.0.$$

Herein $p_4 \in \mathbb{R}$ provides a perturbation parameter in the mass of the payload and will be used later.

Moreover we have

$$l_1 = \begin{pmatrix} 0.188 \\ 0.0 \\ 0.900 \end{pmatrix}, \quad l_2 = \begin{pmatrix} 0.0 \\ 0.0 \\ 0.950 \end{pmatrix}, \quad l_3 = \begin{pmatrix} 1.900 \\ 0.0 \\ 0. \end{pmatrix},$$

and the centers of gravity

$$s_1 = \begin{pmatrix} 0.130 \\ 0.0 \\ 0.783 \end{pmatrix}, \quad s_2 = \begin{pmatrix} -0.010 \\ 0.007 \\ 0.430 \end{pmatrix},$$

$$s_3 = \begin{pmatrix} \frac{161.92959+1.6445p_4}{294.9+p_4} \\ -0.0233 \\ 0.2171 \end{pmatrix}, \quad s_4 = \begin{pmatrix} -0.665 \\ 0.0 \\ -0.015 \end{pmatrix}.$$

2.6 Constraints

We consider the following 20 control and state constraints:

- Control constraints defined by the torque voltages

$$-1 \leq u_i(t) \leq 1, \quad i = 1, 2, 3. \tag{4}$$

- State constraints of first order imposed for the angular velocities

$$100°/s \;\leq\; \dot{q}_i(t) \;\leq\; 100°/s, \quad i = 1, 2, 3. \tag{5}$$

- State constraints of second order for the angles

$$\begin{array}{rcccl} -180° & \leq & q_1(t) & \leq & 180°, \\ -70° & \leq & q_2(t) & \leq & 70°, \\ -28° & \leq & q_3(t) & \leq & 105°, \\ -65° & \leq & q_2(t) - q_3(t) & \leq & 65°. \end{array} \tag{6}$$

For the definition of the *order* of a state constraint, cf. Hartl et al. [7].

In this paper we will determine point-to-point trajectories with the following initial and terminal conditions:

$$\begin{array}{rclcrcl} q(t_0) & = & (\frac{\pi}{2} - p_1, -p_2, -p_3)^\top, & \quad & q(t_f) & = & (-\frac{\pi}{4}, 0, 0)^\top, \\ \dot{q}(t_0) & = & 0, & & \dot{q}(t_f) & = & 0. \end{array} \tag{7}$$

Hereafter we assume, that there might exist perturbations in the initial position of the robot, which are modelled by parameters $p_1, \ldots, p_3 \in \mathbb{R}$ in (7). It is worth to mention, that formulation (7) includes also the more general case of trajectory perturbations, this means deviations from the nominal trajectory that occure during the motion of the robot.
In summary we examine perturbations in the payload and the initial values $p = (p_1, p_2, p_3, p_4)^\top \in \mathbb{R}^4$.

3 Optimal Control Problem

Equations (3)–(7) define a parametric control problem which allows us to simulate the robot on the computer by fixing a special control vector function $u(t)$ and integrate the robot dynamics for a given initial value and a given time intervall $[0, t_f]$.
Nevertheless this is still a demanding problem since on the one hand the fixed control will in general not fit the constraints in (4)–(7) and on the other hand we can assume that there exist infinitely many control functions satisfying the point-to-point conditions in (7).
Contrariwise we can take advantage of the second problem by picking an 'expediently' control function out of the infinitely many possibilities. Therefore we require the following objective functional as a convex combination of energy and power to be minimized:

$$J[u, t_f, p] := \int_0^{t_f} \alpha \|u(t)\|_2^2 + (1-\alpha)\|\dot{q}(t)\|_2^2 \, dt, \tag{8}$$

with weight factor $0 \le \alpha \le 1$.
Beside the objective in (8) other functionals are conceivable like e.g. minimizing the time or the wear and tear. For a wider choice of possible objectives see, e.g. Knauer [8]. In summary, the optimal control problem is to determine control functions $u_i : [0, t_f] \to \mathbb{R}$, $i = 1, 2, 3$, that minimize the functional (8) subject to the control problem (3)–(7).
Let $x(t) \in \mathbb{R}^n$ denote the state of a system and $u(t) \in \mathbb{R}^m$ the control function in a time interval $[0, t_f]$. We consider the following simplified optimal control problem subject to control and state constraints.

$$\begin{array}{ll} \text{Minimize} & F(x,u,p) = g(x(t_f), t_f, p) + \displaystyle\int_{t_0}^{t_f} f_0(x(t), u(t), p)\, dt \\ \text{subject to} & \dot{x}(t) = f(x(t), u(t), p) \qquad \text{for all } \ t \in [0, t_f], \\ & x(0) = \varphi(p), \qquad \psi(x(t_f), p) = 0, \\ & C(x(t), u(t), p) \le 0 \qquad \text{for all } \ t \in [0, t_f]. \end{array} \tag{9}$$

Herein the problem depends on the parameter $p \in P := \mathbb{R}^{N_p}$ which denotes data perturbations in the system as described before. The parametric control problem will be referred to as problem OCP(p). Obviously the path planning problem of a robot in section 2 combined with the objective (8) is included in formulation (9) as a special case.

The functions $g : \mathbb{R}^n \times P \to \mathbb{R}$, $f_0 : \mathbb{R}^n \times \mathbb{R}^m \times P \to \mathbb{R}$, $f : \mathbb{R}^n \times \mathbb{R}^m \times P \to \mathbb{R}^n$, $\varphi : P \to \mathbb{R}^n$, $\psi : \mathbb{R}^n \times P \to \mathbb{R}^r$, $0 \leq r \leq n$, and $C : \mathbb{R}^n \times \mathbb{R}^m \times P \to \mathbb{R}^k$ are assumed to be sufficiently smooth on appropriate open sets. The final time t_f is either fixed or free.

Let us choose a *reference* or *nominal* parameter p_0 and consider problem OCP(p_0) as the *unperturbed* or *nominal* problem. Pontryagin's minimum principle applied to OCP(p_0) leads to a multipoint *boundary value problem.* Its solution allows for a calculation of $x_0(t)$, $u_0(t)$ and the associated adjoint function $\lambda_0(t)$, $0 \leq t \leq t_f$. This method is not real-time capable in general but strong C^1-stability enables us to embed the unperturbed solution $x_0(t)$, $u_0(t)$, $\lambda_0(t)$ into a family of optimal solutions $x(t,p), \lambda(t,p), u(t,p)$ to the perturbed problem OCP(p). For more details see Malanowski, Maurer [10], [11], Maurer, Pesch [13], [14]. This type of strong C^1-stability is crucial for designing real-time approximations of perturbed solutions. Namely, it allows to calculate sensitivity differentials

$$\frac{\partial y}{\partial p}(t, p_0)$$

which are evaluated along the nominal solution and which satisfy a (linear) boundary value problem, too.

These sensitivity differentials permit an approximation of the perturbed solution $y(t,p) := (x(t,p), \lambda(t,p), u(t,p))$ by its first order Taylor expansion:

$$y(t,p) \approx y_0(t) + \frac{\partial y}{\partial p}(t, p_0)(p - p_0). \tag{10}$$

The quantities $y(t,p_0)$ and $\frac{\partial y}{\partial p}(t,p_0)$ are computed *off-line.* Then the benefit of (10) is that only a matrix-vector multiplication and a vector-vector addition have to be performed *on-line* to approximate $y(t, p_0 + \Delta p)$ very fast. Consequently, (10) is particularly suitable for time critical processes and hence can be used as a *real-time approximation.* For special perturbations, this approach has also been investigated by Bock, Krämer-Eis [1], Krämer-Eis [9], and Pesch [15], [16].

There exist two well-known drawbacks for real-time approximations of type (10): First of all the complete boundary value problem (nominal solution and sensitivity differentials) has to be solved including the adjoint equations. Moreover one should know precisely the structure of the optimal solution, i.e., the number of active time intervals for the control and state constraints

(4)–(6). Usually, it is rather difficult to determine the structure of the optimal control and to find appropriate estimations for adjoint variables. On the other hand the open-loop expression (10) does not lead to an admissible solution due to violations e.g. in $\psi(x(t_f), p) = 0$. For this reason, the approximation (10) can only be used for perturbed problems, if the violations are sufficiently small in view of practical requirements. Especially the exponential error growth in initial value problems for ODE systems caused by parameter deviations explains, why the real-time approximation (10) for perturbed optimal control problems cannot be used in general.
Hence the following section is concerned with the reduction of those violations in the constraints by a method which dispenses with adjoint variables. But the adjoint variables will not be lost, they can be recovered a posteriori from Lagrange multipliers obtained via the optimization approach presented hereafter.

4 Nonlinear Optimization

The numerical solution of (9) by nonlinear programming (NLP) techniques is well developed and there exists a number of excellent methods. These methods use a suitable *discretization* of the the control problem (9) by which it is transcribed into an NLP problem. We reflect the main idea for the simple Euler method subsequently. Moreover, for notational simplicity and for an integer $N_t > 1$ we choose equidistant mesh points $\tau_i := (i-1)h$, $i = 1, \ldots, N_t$, $h := \frac{t_f}{N_t - 1}$. Let $u^i \in \mathbb{R}^m$ denote approximations for $u(\tau_i)$, then for given $z := (u^1, \ldots, u^{N_t}) \in \mathbb{R}^{m \cdot N_t}$ state approximations $x^i \in \mathbb{R}^n$ of the values $x(\tau_i)$ can be achieved recursively as functions of the control variables:

$$\begin{aligned} x^1(z) &:= \varphi(p), \\ x^{i+1}(z) &:= x^i(z) + hf(x^i(z), u^i, p), \quad i = 1, \ldots, N_t - 1. \end{aligned} \tag{11}$$

By this means the control problem (9) is replaced by:

$$\begin{array}{lrcl} \min\limits_{z} & g(x^{N_t}(z), p) + \sum\limits_{i=0}^{N_t - 1} hf_0(x^i(z), u^i, p) & & \\ \text{subject to} & \psi(x^{N_t}(z), p) & = & 0, \\ & C(x^i(z), u^i, p) & \leq & 0, \quad i = 1, \ldots, N_t. \end{array} \tag{12}$$

Note that a free final time t_f can be handled as an additional variable in z. Problem (12) defines an NLP problem of form

$$\begin{array}{lll} \min\limits_{z} & H(z,p), & \\ \text{subject to} & G_i(z,p) = 0, & i = 1,\dots,N_e, \\ & G_i(z,p) \le 0, & i = N_e+1,\dots,N_c, \end{array} \tag{13}$$

which can be solved efficiently for suitable N_e, N_c and functions H and G_i by standard techniques, e.g. SQP methods.
All calculations described hereafter were performed by the code NUDOCCCS of Büskens [2, 3] which has implemented also various higher order approximations for state and control variables. The treatment of stiff ODEs, grid refinement techniques or numerical check of second order sufficient optimality conditions can also be found in [3]. The convergence of solutions discretized via Euler's method to solutions of the continuous control problem has been proved in Malanowski, Büskens and Maurer [12].
By solving the NLP problem (13) we obtain an estimate of the *continuous* control and state variables (x,u) of (9) depending on the applied discretization. Likewise, all other variables and functions of the continuous problem (9) can be determined approximately. Unfortunately this method is still not real-time capable.

5 Parametric sensitivity analysis

So far we were able to transform a perturbed control problem into a perturbed NLP problem. The results presented hereafter do not depend on the discretization technique used. After solving (13) we know the set and the number N_a of active constraints. Since inactive constraints have no impact on the optimal solution, the solution of (13) is the same as the solution of

$$\begin{array}{ll} \min\limits_{z} & H(z,p) \\ \text{subject to} & G^a(z,p) = 0, \end{array} \tag{14}$$

if $G^a = (G_1^a,\dots,G_{N_a}^a)^\top$ denotes the collection of the active constraints. We restrict the discussion to formulation (14). Let $\eta = (\eta_1,\dots,\eta_{N_a})^\top$ denote the Lagrange multiplier for the Lagrangian

$$L(z,\eta,p) = H(z,p) + \eta^\top G^a(z,p),$$

then sufficient conditions for the differentiability of an optimal solution $z(p)$ w.r.t. p are given by

Theorem 1 *Let H and G^a be twice continuously differentiable w.r.t. z and p. Let z_0 be a strong regular local solution of (14) for a fixed parameter p_0 with Lagrange multiplier η_0, i.e. $G^a(z_0, p_0) = 0$ and*

1. *$rg\,(\nabla_z G^a(z_0, p_0)) = N_a$ (z_0 is regular),*
2. *$\nabla_z L(z_0, \eta_0, p_0) = 0, \eta_0^\top G^a(z_0, p_0) = 0$ (necessary optimality conditions),*
3. *$(\eta_0)_i > 0$ for $i = 1, \ldots, N_a$ (strict complementarity)*
4. *$v^\top \nabla^2_{zz} L(z_0, \eta_0, p_0) v > 0$, $\forall v \in ker(\nabla_z G^a(z_0, p_0)(z_0, p_0))$, $v \neq 0$ (second order sufficient conditions).*

Then there exists a neighborhood $\mathcal{P}(p_0)$ such that (14) possesses a unique strong regular local solution $z(p)$ and $\eta(p)$ for all $p \in \mathcal{P}(p_0)$. Furthermore, $z(p)$ and $\eta(p)$ are continuously differentiable functions of p in $\mathcal{P}(p_0)$ and it holds

$$\begin{pmatrix} \nabla^2_{zz} L(z_0, \eta_0, p_0) & \nabla_z G^a(z_0, p_0)^\top \\ \nabla_z G^a(z_0, p_0) & 0 \end{pmatrix} \begin{pmatrix} \frac{dz}{dp}(p_0) \\ \frac{d\eta}{dp}(p_0) \end{pmatrix} = - \begin{pmatrix} \nabla^2_{zp} L(z_0, \eta_0, p_0) \\ \nabla_p G^a(z_0, p_0) \end{pmatrix}. \tag{15}$$

Herein $\nabla^2_{zz} L$ denotes the Hessian of the Lagrangian. Notice, that the left matrix in (15) is non-singular under the assumptions of Theorem 1. Hence the sensitivity differentials dz/dp and $d\eta/dp$ at p_0 can be calculated explicitly by solving the linear equation system. The proof of the theorem is based on the implicit function theorem and can be found in Fiacco [6] or Büskens [3]. The assumptions in Theorem 1 can be checked numerically by use of the projected or reduced Hessian, compare Büskens and Maurer [4] or Büskens [3]. As described before for optimal control problems a first order Taylor approximation for $z(p_0 + \Delta p)$ can be calculated extremly fast by

$$z(p) := z(p_0 + \Delta p) \approx \tilde{z}(p) := z(p_0) + \frac{dz}{dp}(p_0) \Delta p \tag{16}$$

for deviations Δp in p.
Although formula (16) yields acceptable real-time approximations for small

perturbations Δp, especially for larger deviations and in case of active constraints (16) leads to a non admissible solution, e.g.

$$G^a(\tilde{z}(p), p) = \varepsilon_1 \neq 0. \tag{17}$$

Additionally formula (16) might be worse in view of optimality, as the following theorem shows, cf. Büskens [5]:

Theorem 2 *Let the assumptions of Theorem 1 hold and let the functions H and G^a in (14) be three times continuously differentiable w.r.t. to z and p. Then there exists a neighborhood $U(p_0)$ of p_0 with*

$$\begin{aligned} \|z(p) - \tilde{z}(p)\| &= \mathcal{O}(\|\Delta p\|^2), & (18)\\ \|H(z(p), p) - H(\tilde{z}(p), p)\| &= \mathcal{O}(\|\Delta p\|^2), & (19)\\ \|G^a(\tilde{z}(p), p)\| &= \mathcal{O}(\|\Delta p\|^2). & (20) \end{aligned}$$

In the unconstrained case, i.e., $N_a = 0$, we have

$$\|H(z(p), p) - H(\tilde{z}(p), p)\| = \mathcal{O}(\|\Delta p\|^3). \tag{21}$$

Note, that the order of optimality of the objective is higher in the unconstrained case.
Introducing an artifical perturbation q in (14) enables us to improve the real-time approximation (16). We treat the problem

$$\begin{array}{ll} \min\limits_{z} & H(z, p) \\ \text{subject to} & G^a(z, p) - q = 0. \end{array} \tag{22}$$

If the nominal perturbation is chosen to $q = q_0 = 0$, problem (22) is equivalent to (16). Moreover we are able to calculate the sensitivities $\frac{dz}{dq}(q_0) = \frac{dz}{dq}(0)$ and $\frac{d\eta}{dq}(q_0) = \frac{d\eta}{dq}(0)$ similar to (15), since problem (22) fulfills the assumptions of Theorem 1, if (14) does.
Now we have the munition to calculate higher order admissible real-time approximations.

6 Higher Order Real-Time Approximations

We suggest the following corrector iteration method to achieve admissibility for the active constraints without loss of its optimality. If an actual deviation of the form $(p, q_0) = (p, 0)^\top \in \mathbb{R}^{N_p+N_a}$ from the nominal parameter $(p_0, 0)^\top$ is detected, equation (16) provides a very fast *open-loop approximation* for the perturbed solution. It was shown in (17), that this approximation causes an error ε_1 in the active constraints $G^a(\tilde{z}(p), p)$. Note, that this error is of the form of the new perturbation parameter q in (22). Hence we can hope, that a better approximation in view of optimality and especially admissibility can be found by

$$\begin{aligned} z(p) \approx \tilde{z}^{[2]}(p) \quad &:= \quad \underbrace{z(p_0) + \frac{dz}{dp}(p_0)\triangle p}_{=:\tilde{z}^{[1]}(p)} - \tfrac{dz}{dq^a}(0)\,\varepsilon_1 \\ &= \quad \tilde{z}^{[1]}(p) - \tfrac{dz}{dq^a}(0)\; G^a(\tilde{z}^{[1]}(p), p), \end{aligned} \tag{23}$$

with $\tilde{z}^{[1]}(p)$ from (16) and $\frac{dz}{dp}(p_0)$ respectively $\frac{dz}{dq^a}(0)$ from equation (15). Since the nominal solution $z(p_0)$ as well as the sensitivity differentials $\frac{dz}{dp}(p_0)$ and $\frac{dz}{dq^a}(0)$ can be calculated off-line, equation (23) provides also a fast computation of the real-time approximation, since no gradient calculation is needed. The additional term $\frac{dz}{dq^a}(0)G^a(\tilde{z}^{[1]}(p), p)$ in equation (23) can be understood as a correcting *feedback step* for the error caused by equation (17). The following theorem holds, cf. Büskens [5]:

Theorem 3 *Let the assumptions of Theorem 1 hold and let the functions H and G^a in (14) be three times continuously differentiable w.r.t. to z and p. Then there exists a neighborhood $U(p_0)$ of p_0 and a vector $v \in \mathbb{R}^{N_z}$, $v \in ker(\nabla_z G^a(z_0, p_0))$ and $\|v\| = \mathcal{O}(\|\Delta p\|^2)$ with*

$$\begin{aligned} \|z(p) - \tilde{z}^{[2]}(p)\| &= \|v\| + \mathcal{O}(\|\Delta p\|^3), & (24)\\ \|H(z(p), p) - H(\tilde{z}^{[2]}(p), p)\| &= \mathcal{O}(\|\Delta p\|^3), & (25)\\ \|G^a(\tilde{z}^{[2]}(p), p)\| &= \mathcal{O}(\|\Delta p\|^3). & (26) \end{aligned}$$

Note that the admissibility in (26) is improved as well as the optimality (25) of the objective although the variables $\tilde{z}^{[2]}(p)$ are still of order $\mathcal{O}(\|\Delta p\|^2)$. Approximation (23) is quite good, especially in comparison to the first order approximation (16), but we can do better since (23) causes an error in the active constraints again of form of the artifical perturbation q:

$$G^a(\tilde{z}^{[2]}(p), p) = \varepsilon_2 \neq 0. \tag{27}$$

Hence an additional improvement of (23) is given by

$$z(p) \approx \tilde{z}^{[3]}(p) := \tilde{z}^{[2]}(p) - \frac{dz}{dq^a}(0)\, G^a(\tilde{z}^{[2]}(p), p). \tag{28}$$

Obviously the correcting feedback steps in (23) and (28) form an iterative process which can be described as follows:

1. Choose $\varepsilon^\infty \in \mathbb{R}_+$ and initialize $\tilde{z}^{[1]}(p)$ by (16), set $k := 1$.
2. If $\|G^a(\tilde{z}^{[k]}(p), p)\|_2 < \varepsilon^\infty$ then STOP.
3. Calculate
$$\tilde{z}^{[k+1]}(p) := \tilde{z}^{[k]}(p) - \frac{dz}{dq^a}(0)\, G^a(\tilde{z}^{[k]}(p), p), \tag{29}$$
and set $k := k + 1$.
4. Goto 2.

The algorithm can be enlarged to an approximation of the Lagrangian multipliers, but will not be discussed here. Although the main request of the correcting feedback steps is to find an admissible solution, the improved order of optimality is not lost. In enhancement of Theorem 3 we obtain, cf. Büskens [5].

Theorem 4 *Let the assumptions of Theorem 1 hold and let the functions H and G^a in (14) be three times continuously differentiable w.r.t. to z and p. Then there exists a neighborhood $U(p_0)$ of p_0 and a vector $v \in \mathbb{R}^{N_z}$ with $v \in ker(\nabla_z G^a(z_0, p_0))$ and $\|v\| = \mathcal{O}(\|\Delta p\|^2)$ such that for all $p \in U(p_0)$ the sequence $\tilde{z}^{[k]}(p)$ in (29) converges to a fixed point $\tilde{z}^{[\infty]}(p)$ with*

$$\begin{aligned} \|z(p) - \tilde{z}^{[\infty]}(p)\| &= \|v\| + \mathcal{O}(\|\Delta p\|^3), && (30)\\ \|H(z(p), p) - H(\tilde{z}^{[\infty]}(p), p)\| &= \mathcal{O}(\|\Delta p\|^3), && (31)\\ \|G^a(\tilde{z}^{[\infty]}(p), p)\| &= 0. && (32) \end{aligned}$$

Note, that the fixed point in Theorem 4 is not unique, cf. Büskens [5]. Nevertheless, any fixed point of iteration (29) fulfills (30)-(32). Sensitivity-Theorem 1 predicts the existence of a neighbourhood where the active constraints remain unchanged. This guarantees the existence of a fixpoint. In more detail one can show the relations $\|G^a(\tilde{z}^{[k]}(p), p)\| = \mathcal{O}(\|\Delta p\|^{k+1})$ and $\|H(z(p), p) - H(\tilde{z}^{[k]}(p), p)\| = \mathcal{O}(\|\Delta p\|^3)$, $k = 2, 3, 4, \ldots$, cf. Büskens [5]. Hence the algorithm can be terminated at any time without loss of optimality and admissibility.

7 Numerical Results

The purpose of this section is to illustrate the theoretical results presented in the sections 3–6 for the mathematical model of the robot ABB IRB 6400 2.8 introduced in section 2. We consider the perturbed optimal control problem defined by (3)–(8). The nominal perturbation is set to $p_0 = (0,0,0,0)^\top$ in $OCP(p_0)$.
We choose a fixed final time $t_f = 2.0$, weight factor $\alpha = 0.9$ and $N_t = 101$. This leads to $N_t \cdot m = 101 \cdot 3 = 303$ control variables.
In the first step the optimal nominal solution is calculated by solving the optimal control problem (3)–(8) with a 4th order approximation for the state variables and a linear interpolation of the control. All computations use the initial estimates $u^i = 0$, $i = 0, \ldots, N_t$, for the control functions. For the nominal parameter p_0 we obtain $J[u, p_0] = 0.7238392$ after about 6 seconds of computational time on a 3GHz PC. The optimal nominal controls are given in figure 1, while the optimal nominal trajectory can be found in figure 3 as a three dimensional plot.

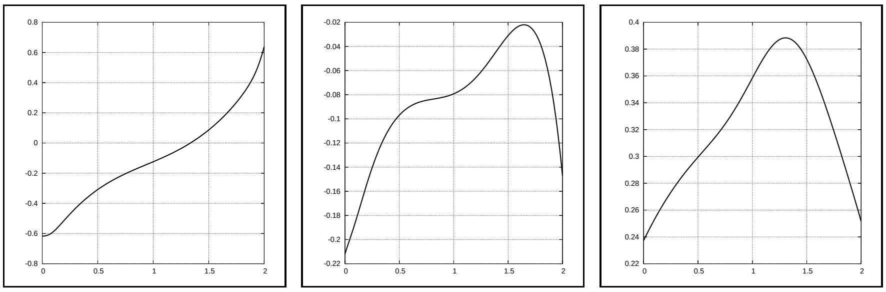

Figure 1: Optimal nominal controls $u_0(t)$.

Note that the control and state constraints in (4)–(6) do not become active, hence we find $G^a(z_0, p_0) = (q(t_f) - (-\frac{\pi}{4}, 0, 0)^\top, \dot{q}(t_f))^\top = 0$, $N_a = 6$. All assumptions in Theorem 1 are satisfied for the nominal problem. Especially the Hessian in Theorem 1 is positive definite on the kernal of the Jacobian of the active constraints G^a with smallest eigenvalue $\nu = 0.13 \cdot h$, $h = \frac{t_f}{N_t - 1} = \frac{2}{100}$. Thus the nominal solution is a strong local minimum and we can apply equation (15) to calculate $\frac{dz}{dp}(p_0)$ and $\frac{\partial u}{\partial p}(t_i, p_0) \approx \frac{du^i}{dp}(p_0)$ respectively. Figure 2 displays the sensitivity differentials of the controls. Note that the sensitivities of the controls with respect to perturbations in the payload are much smaller than the others. This is due to the fact that the counterweight was

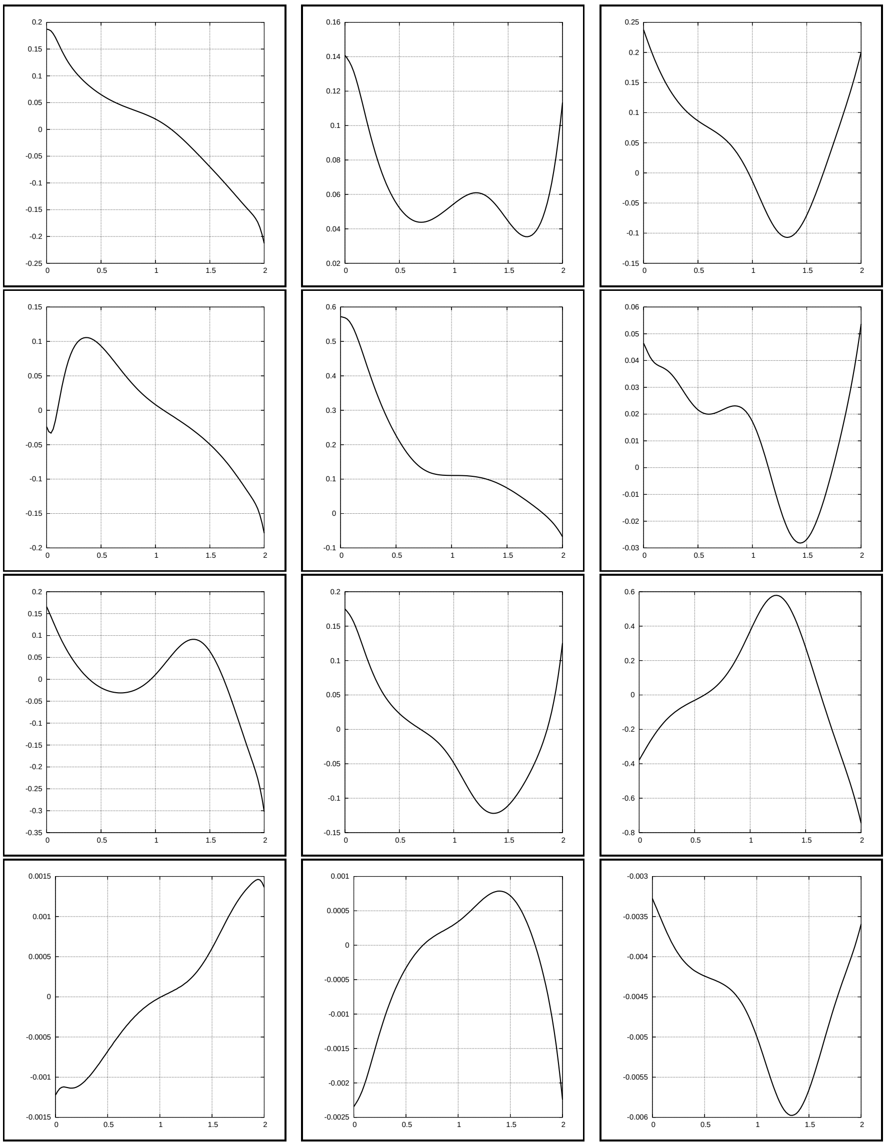

Figure 2: Sensitivities $\frac{\partial u}{\partial p_j}(t_i, p_0) \approx \frac{du^i}{dp_j}(p_0)$, $j = 1, 2, 3, 4$.

especially designed to make the robot insensitive in terms of different payloads. Therefore as a spin-off we can notice the good job the robot engineers did by constructing the special geometry of the robot.
According to relation (16), the sensitivity differentials are necessary to evaluate a first order Taylor expansion of the perturbed solution. However equations (23), (28) and (29) remind us that the sensitivity differentials with respect to perturbations in the terminal conditions are needed, too. For lack of space we dispense with a depiction of these sensitivities.

In order to judge the quality of the real-time approximations for the robot problem, we set up the following Table 1 which lists the relative errors

$$\zeta_H^{[k]}(p) := \frac{H(\tilde{z}^{[k]}(p), p) - H(z(p), p)}{H(z(p), p)}, \qquad k = 0, 1, 2, \dots \tag{33}$$

of the objective for different perturbations p. Herein and in the following $\zeta_H^{[0]}$ denotes the relative error of the objective obtained after an integration of the perturbed system using the nominal control variables. The error $\zeta_G^{[k]}(p)$ in the nonlinear constraints is defined by

$$\zeta_G^{[k]}(p) := \max_i G_i^a(\tilde{z}^{[k]}(p), p), \qquad k = 0, 1, 2, \dots . \tag{34}$$

The first eight iterations and the solution of the fixed point predicted in Theorem 4 for different perturbations $p_a = (2°, 2°, 2°, 3kg)$, $p_b = (-4°, 2°, -1°, 1kg)$ and $p_c = (20°, 20°, 20°, 30kg)$ are listed in Table 1.

	$p = p_a$		$p = p_b$		$p = p_c$	
k	$\zeta_G^{[k]}(p)$	$\zeta_F^{[k]}(p)$	$\zeta_G^{[k]}(p)$	$\zeta_F^{[k]}(p)$	$\zeta_G^{[k]}(p)$	$\zeta_F^{[k]}(p)$
0	$1.36 \cdot 10^{-01}$	$6.83 \cdot 10^{-02}$	$7.07 \cdot 10^{-02}$	$1.88 \cdot 10^{-03}$	$1.87 \cdot 10^{\pm 00}$	$1.14 \cdot 10^{\pm 00}$
1	$4.76 \cdot 10^{-03}$	$2.38 \cdot 10^{-03}$	$1.91 \cdot 10^{-03}$	$7.66 \cdot 10^{-04}$	$3.90 \cdot 10^{-01}$	$2.26 \cdot 10^{-01}$
2	$2.22 \cdot 10^{-04}$	$3.71 \cdot 10^{-06}$	$3.05 \cdot 10^{-05}$	$1.08 \cdot 10^{-05}$	$2.17 \cdot 10^{-01}$	$3.87 \cdot 10^{-02}$
3	$5.75 \cdot 10^{-06}$	$2.10 \cdot 10^{-06}$	$7.61 \cdot 10^{-07}$	$1.92 \cdot 10^{-07}$	$7.70 \cdot 10^{-02}$	$1.39 \cdot 10^{-02}$
4	$2.59 \cdot 10^{-07}$	$6.89 \cdot 10^{-07}$	$2.20 \cdot 10^{-08}$	$7.85 \cdot 10^{-08}$	$2.14 \cdot 10^{-02}$	$5.12 \cdot 10^{-03}$
5	$9.20 \cdot 10^{-09}$	$6.34 \cdot 10^{-07}$	$6.56 \cdot 10^{-10}$	$8.10 \cdot 10^{-08}$	$4.18 \cdot 10^{-03}$	$7.93 \cdot 10^{-03}$
6	$3.44 \cdot 10^{-10}$	$6.32 \cdot 10^{-07}$	$1.97 \cdot 10^{-11}$	$8.09 \cdot 10^{-08}$	$1.14 \cdot 10^{-03}$	$6.85 \cdot 10^{-03}$
7	$1.39 \cdot 10^{-11}$	$6.32 \cdot 10^{-07}$	$5.94 \cdot 10^{-13}$	$8.09 \cdot 10^{-08}$	$4.07 \cdot 10^{-03}$	$7.21 \cdot 10^{-03}$
8	$5.60 \cdot 10^{-13}$	$6.32 \cdot 10^{-07}$	$1.80 \cdot 10^{-14}$	$8.09 \cdot 10^{-08}$	$1.40 \cdot 10^{-03}$	$7.10 \cdot 10^{-03}$
∞	0	$6.32 \cdot 10^{-07}$	0	$8.09 \cdot 10^{-08}$	0	$7.13 \cdot 10^{-03}$

Table 1: Admissible real-time approximations for different perturbations.

Even in the case of very large perturbations p_c the method converges. The zeros in the last line of Table 1 have to be understood as zeros in the sense of machine precision. Hence all real-time approximations lead to admissible solutions within machine precision. The results indicate that the precision

obtained via the proposed method is by far higher than the one calculated by (10) and (16) respectively. Computing time for iteration 1 is about $2.6 \cdot 10^{-6}$ seconds, while each of the other iterates needs about $5.0 \cdot 10^{-4}$ seconds. It should be noted that, in a practical implementation, the computational times for the iterations $1, 2, 3, \ldots,$ can be reduced by an additional factor of 101 (number of gridpoints), if the time during the motion of the robot is used for computing the needed approximations.
Figure 3 shows the solutions of the robot trajectory for perturbation p_c. Note that perturbation p_c is much larger then deviations appearing in practice.

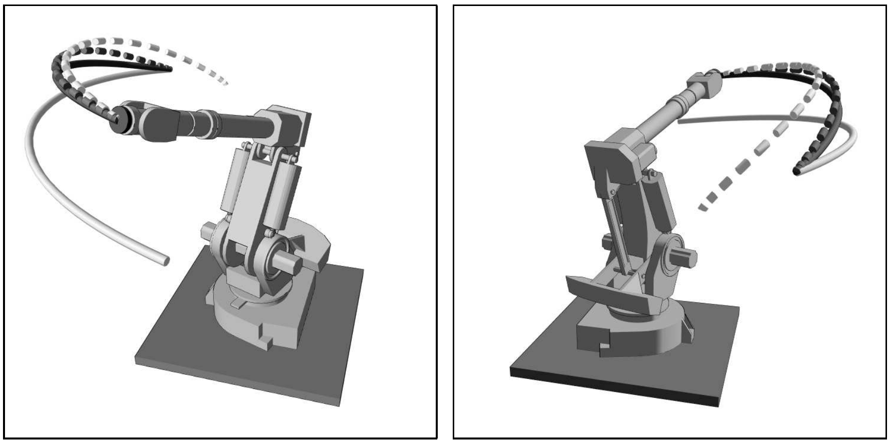

Figure 3: Optimal trajectories: nominal (solid, bright), perturbed (solid, dark), first Taylor approximation (dashed, bright), admissible real-time approximation (dashed, dark).

The numerical results clearly indicate that the real-time optimal control approximations exhibit a favorable and robust quality, since the nonlinear constraints are satisfied exactly, the objective is achieved with sufficiently high precision and the computational time for the approximation is much smaller than the operation time of the robot.

References

[1] Bock, H.G. and Krämer-Eis, P., 'An efficient algorithm for approximate computation of feedback control laws in nonlinear processes', Zeitschrift f. Angewandte Mathematik und Mechanik 61, 1981, T330–T332.

[2] Büskens, C., 'Direkte Optimierungsmethoden zur numerischen Berechnung optimaler Steuerungen', Diploma thesis, Institut für Numerische Mathematik, Universität Münster, Münster, Germany, 1993.

[3] Büskens, C., 'Optimierungsmethoden und Sensitivitätsanalyse für optimale Steuerprozesse mit Steuer- und Zustands-Beschränkungen', Dissertation, Institut für Numerische Mathematik, Universität Münster, Münster, Germany, 1998.

[4] Büskens, C. and Maurer, H., 'SQP-methods for solving optimal control problems with control and state constraints: adjoint variables, sensitivity analysis and real-time control.' Journal of Computational and Applied Mathematics, 120, 2000, 85–108.

[5] Büskens, C., 'Echtzeitoptimierung und Echtzeitoptimalsteuerung parametergestörter Probleme'. Habilitation, Fachbereich Mathematik und Physik, Universität Bayreuth, Bayreuth, Germany, 2002.

[6] Fiacco, A.V., 'Introduction to Sensitivity and Stability Analysis in Nonlinear Programming', Mathematics in Science and Engineering, 165, Academic Press, New York, 1983.

[7] Hartl, R.F., Sethi, S.P. and Vickson, R.G., 'A survey of the maximum principles for optimal control problems with state constraints', SIAM Review, 37, 1995, 181–218.

[8] Knauer, M., 'Sensitivitätsanalyse verschiedener Gütekriterien bei der optimalen Bahnplanung von Industrierobotern', Diploma thesis, Fachbereich Mathematik und Physik, Universität Bayreuth, Bayreuth, Germany, 2001.

[9] Krämer-Eis, P., 'Ein Mehrzielverfahren zur numerischen Berechnung optimaler Feedback-Steuerungen bei beschränkten nichtlinearen Steuerungsproblemen', Bonner Mathematische Schriften, 164, University of Bonn, Germany, 1985,

[10] Malanowski, K. and Maurer, H., 'Sensitivity analysis for parametric control problems with control-state constraints', Computational Optimization and Applications, 5, 1996, 253–283.

[11] Malanowski, K. and Maurer, H., 'Sensitivity analysis for state constrained optimal control problems', Discrete and Continuous Dynamical Systems, 4, 1998, 241–272.

[12] Malanowski, K., Büskens, C. and Maurer, H., 'Convergence of approximations to nonlinear optimal control problems', In A.V. Fiacco, editor, Mathematical Programming with Data Perturbations, Lecture Notes in Pure and Applied Mathematics, 195, Marcel Dekker Inc., 1998, 253–284.

[13] Maurer, H. and Pesch, H.J., 'Solution differentiability for parametric nonlinear control problems', SIAM Journal on Control and Optimization, 32, 1994, 1542-1554.

[14] Maurer, H. and Pesch, H.J., 'Solution differentiability for parametric nonlinear control problems with control-state constraints', J. of Optimization Theory and Applications, 23, 1995, 285-309.

[15] Pesch, H.J., 'Real-time computation of feedback controls for constrained optimal control problems, Part 1: Neighbouring extremals', Optimal Control Applications & Methods, 10, 1989, 129-145.

[16] Pesch, H.J., 'Real-time computation of feedback controls for constrained optimal control problems, Part 2: A correction method based on multiple shooting', Optimal Control Applications & Methods, 10, 1989, 147-171.

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