Nonnegative Matrix Factorization (NMF)

Classical NMF Problem
- For a given matrix $X \in \mathbb{R}_{\geq 0}^{N \times T}$, find matrices $B \in \mathbb{R}_{\geq 0}^{N \times K}$ and $C \in \mathbb{R}_{\geq 0}^{K \times T}$ with $K \ll \min\{N, T\}, \text{s.t. } X \approx BC$.

Task Areas
- Used e.g. for dimension reduction, data compression, basis learning and source separation.

Optimization Theory
- Review and extension of the majorize-minimization principle for generalized NMF models.\(^1\)
  \[
  \min_{B, C \geq 0} D_{\text{NMF}}(X, BC) + \sum_{\ell=1}^{L} \gamma(P_{\ell}(B, C))
  \]

Applications
- MALDI Imaging
  - Development of supervised NMF models for extracting tumor-specific spectral patterns.\(^2\)
- Dynamic Computerized Tomography
  - Reconstruction and low-rank decomposition for dynamic inverse problems via NMF.\(^3\)
- Clustering
  - Development of an orthogonal NMF model with total variation regularization to enforce spatial coherence.
  - Find connections between regularized orthogonal NMF and generalized K-means models.

Audio Source Separation

We want to identify and separate the contributions of different instruments in a music recording.

We use an algorithm to represent a discrete mixture via shifted continuous patterns from a parametric model.

Sparse Pursuit

With this, we can recover the spectra of the individual instruments.