Online Parameter Identification

Model identification of complex nonlinear systems is difficult and can result in:
- Models depending on unknown or time-varying parameters,
- Unexpected system behavior,
- Deviations in predicted costs.

The aim of the PhD project is to provide online parameter updates to stay closer to offline computed optimal trajectories or predicted costs. An enlargement of the controller’s area of convergence may be achieved.

Combine Parameter Identification and Control Methods

- PI + Real-time adaptive LQR controller
- Repeated PI + nonlinear OC (Sequential loop)¹
- PI + nonlinear model predictive control (MPC)

Parameter identification (PI)

Find states and parameters for measurements \( x_{\text{meas},i} \) in \( \mathbb{R}^n \) so that:

\[
\min_{x,p} \sum_{i=1}^{N} \| x(t_i, p) - x_{\text{meas},i} \|^2,
\]

subject to:

\[
x(t_i) = F(x(t_i), u(t_i), p).
\]

Optimal control (OC)

Find controls \( u(t) \) that optimize control effort and distance to final point:

\[
\min_{x,u} \int_{0}^{T} L(t, x(t), u(t)) dt + \Phi(x(0), x(T))
\]

subject to:

\[
x(t_i) = F(x(t_i), u(t_i), p),
\]

\[
\psi(x_0, x_\text{a}) = 0.
\]

Real-time adapted Riccati controller

Solve parameter dependent LQR problem as optimization problem with feedback control law \( u(t) = -K(p)x(t) \):

\[
\min_{K(p)} x^T Q(x) x + x^T R(p) x dt
\]

subject to:

\[
x(t) = A(p)x(t) - B(p)Kx(t),
\]

and update solution \( K(p_k) \) with offline computed parametric sensitivities \( \psi_{x,p}(x_\text{0}) \);

\[
\dot{K}(p) = K(p_k) + \psi_{x,p}(x_\text{0}) \frac{dK(p_k)}{dp}.
\]

Applications

- Benchmark example: inverted pendulum on a cart.
- Nonlinear adaptive MPC for an idealized robot manipulator with time-varying payloads.²
- Examination of parameter identification methods: Real-world application on industrial robot.³

Decomposition in Nonlinear Programming

Consider problems with intricate sub-systems \( \psi \), implicitly given via \( G(x, p) = 0 \). Equivalent NLP formulations:

\[
\min_p F(\psi(p), p)
\]

Reduced

\[
\min_{x,p} F(x, p)
\]

Decomposed

Decomposition offers several advantages, e.g.

- Robustness, Stability
- Beneficial algorithmic behavior

Following examples: Blocks for \( \psi \)

Parameter Identification: \( \psi \) is ODE solution

Fit a dynamic model to given data by optimizing parameters:

\[
\min_{p, x_\text{a}} J = \frac{1}{2} \sum_{i=1}^{N} \| x(t_i, p) - z(t_i) \|^2
\]

subject to:

\[
x(t) = f(t, x(t), p), \quad t \in [t_\text{a}, t_\text{f}], \quad x(t_\text{a}) = x_\text{a}
\]

Algorithmic behavior of WORHP for a simple pendulum

Numerical comparisons of single/multiple shooting and full discretization w.r.t. finding desired solutions.

- Applications to robotic system³,⁴
- Development of homotopy-optimization approach⁵

Bilevel Programming: \( \psi \) is NLP solution

NLP with nested, parameterized lower-level problem (LL):

\[
\min_{x, y, \lambda_k} F(x, y)
\]

subject to:

\[
G(x, y) \leq 0 \quad \text{with} \quad y \in \arg \min \{ f(x, s) \mid g(x, s) \leq 0 \}
\]

Embed process of numerically solving LL into NLP⁶:

\[
\min_{x, y_k, \lambda_k} F_N
\]

subject to:

\[
\nabla f_k(y_{k+1} - y_k) + \nabla f_k + \nabla g_k \lambda_k = 0, \quad \lambda_k \geq 0,
\]

\[
y_k - g_{2k}(y_k - y_{k-1}) \leq 0, \quad G_N \leq 0
\]

- Original solutions can be recovered
- Compete against KKT approach on test library
- Initialization & adaptivity strategies

⁶ Sch., Fliss, Flasskamp, & Bäckens. A Reformulation Technique for Nonlinear Bilevel Programs Based on Sequential Quadratic Programming. (in preparation)