Dimension of Bernoulli measures for non-linear countable Markov maps

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It is well known that the Gauss map $G: [0,1) \to [0,1)$

$$G(x) = \frac{1}{x} \mod 1$$

has an absolutely continuous invariant probability measure μ_G given by

$$\mu_G(A) = \frac{1}{\log 2} \int_A \frac{1}{1+x} dx.$$

Kifer, Peres and Weiss showed that there exists a 'dimension gap' between the supremum of the Hausdorff dimensions of Bernoulli measures $\mu_{\mathbf{p}}$ for the Gauss map and the dimension of the measure of maximal dimension (which in this case is μ_G with dimension 1). In particular they showed that

$$\sup_{\mathbf{p}} \dim_{H} \mu_{\mathbf{p}} < 1 - 10^{-7}. \tag{0.1}$$

Their proof was based on considering sets of large deviations for the asymptotic frequency of certain digits from the one prescribed by μ_G .

In this talk we consider the geometric properties of T which lead to a dimension gap, and discuss an alternative proof of (0.1) and other similar results. This time the proof revolves around obtaining good lower bounds for the variance of a class of potentials, by drawing on tools from Hilbert-Birkhoff cone theory.