Dimension of Bernoulli measures
for non-linear countable Markov maps

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It is well known that the Gauss map $G : [0, 1) \to [0, 1)$

$$G(x) = \frac{1}{x} \mod 1$$

has an absolutely continuous invariant probability measure $\mu_G$ given by

$$\mu_G(A) = \frac{1}{\log 2} \int_A \frac{1}{1+x} \, dx.$$  

Kifer, Peres and Weiss showed that there exists a ‘dimension gap’ between the supremum of the Hausdorff dimensions of Bernoulli measures $\mu_p$ for the Gauss map and the dimension of the measure of maximal dimension (which in this case is $\mu_G$ with dimension 1). In particular they showed that

$$\sup_p \dim_H \mu_p < 1 - 10^{-7}. \quad (0.1)$$

Their proof was based on considering sets of large deviations for the asymptotic frequency of certain digits from the one prescribed by $\mu_G$.

In this talk we consider the geometric properties of $T$ which lead to a dimension gap, and discuss an alternative proof of (0.1) and other similar results. This time the proof revolves around obtaining good lower bounds for the variance of a class of potentials, by drawing on tools from Hilbert-Birkhoff cone theory.