TEACHERS' VIEWS ON DISTANCE LEARNING: AN EXPLORATORY STUDY

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In recent years the mathematics education community investigated and promoted different practices to enhance the student learning process: such as flipped classroom (Bergmann & Sams, 2014) and problem-solving activities (Liljedahl, 2016). These new methods may or may not involve the use of technology but focus on a student-centered approach that can favor a conceptual knowledge, in opposition to a teacher-centered method (Gamer and Gamer, 2001). On the other hand, affect-related research has provided evidence that what a teacher believes also impacts what and how they teach (Leder, Pehkonen & Törner, 2002). Teachers' beliefs are often connected one to each other, they can be seen as a system in which elements influence each other and the teaching practice (Green, 1971).

In March 2020, Italian schools were suddenly closed due to the COVID-19 pandemic: teachers were asked to move their practice online to reach students that were staying at home. We want to investigate what were the beliefs of teachers about distance learning and how their views towards mathematics and its teaching may have influenced their practice. We collected data from a structured interview with open and multiple-choice questions sent to 36 high school teachers during the first weeks of the pandemic. We asked teachers about their practice, the tools they used, their level of confidence with the online teaching and their experience overall in the first weeks. This work focuses on three questions about the tools used, the planning of the lesson, and the encountered difficulties and potentialities.

During the first week the large majority used a virtual class system (such as Google Meet, Zoom) and videos, for instance they answered "I used Google Meet", "I recorded some videos and uploaded them". Most of the teachers were concerned about their equipment and technical issues: "I didn't have a graphics tablet that I could write on and connect to the PC", "some students had weak internet connections". Most of them wanted to use a blackboard or something similar to it: "I shared my desktop and filmed the blackboard I have at home", "it is difficult to explain mathematics without a blackboard". Furthermore, a lot of teachers complained that online "it is difficult to engage with students and to understand their reaction". On the other hand, some teachers appreciated the possibility of providing personal feedback to each student, and the fact that "students can watch videos anywhere and anytime".

We observe that the potentiality of the technological tools was rarely exploited, and teachers tried to mimic the in-presence practice using web-conferences systems, home-made blackboards, and treating most of the students as an audience. We claim that the teachers' view about the math lesson is still teacher-centered, as summarized by a teacher: "I pictured myself as a youtuber".

References

Bergmann, J., & Sams, A. (2014). *Flipped learning: Gateway to student engagement*. International Society for Technology in Education.

Gamer, B.E., Gamer, L.E. (2001) Retention of concepts and skills in traditional and reformed applied calculus. *Math Ed Res J 13*, 165–184. https://doi.org/10.1007/BF03217107

Liljedahl P. (2016) Building Thinking Classrooms: Conditions for Problem-Solving. In: Felmer P., Pehkonen E., Kilpatrick J. (eds) *Posing and Solving Mathematical Problems. Research in Mathematics Education*. (pp. 361 – 386) Springer, Cham. https://doi.org/10.1007/978-3-319-28023-3_21

Leder, G. C., Pehkonen, E., & Törner, G. (Eds.). (2002). *Beliefs: A hidden variable in mathematics education?* Kluwer Academic Publishers. https://dx.doi.org/10.1007/0-306-47958-3

MATHEMATICS TEACHERS' BELIEF CENTRALITY, CONTEXT, AND PRACTICE

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Teachers' beliefs and their influence on practice continue to be of interest to those concerned with achieving lasting reforms of mathematics teaching. The field has moved from attempts to identify simple belief-practice links and claims of conflicts between beliefs and practice, to understanding beliefs as existing in systems and varying in their centrality (Green, 1971). The influence of context on their enactment, and the entailment of emotions has also been acknowledged. The problem of how the relative impacts on practice of beliefs or categories of beliefs can be predicted remains. A model derived from existing studies (Beswick, 2005, 2018) is described, along with examples and theoretical implications, as a contribution to a solution.

The model described emerged from two studies involving interviews with a total of 16 mathematics teachers and classroom observations of 6 of these (Beswick, 2005, 2018). The relative centrality and distance from a specific classroom context can be imagined as perpendicular axes in relation which a teacher's beliefs can be located. Beliefs that are highly connected to others (i.e. central) and related most closely to the specific classroom being considered are positioned nearest the origin and exert the greatest influence on practice in that context. These beliefs tend also to be associated with strong emotional responses. Impact on practice can be considered a third axis defining a 3-dimensional cartesian space in which beliefs are positioned.

Beliefs about the nature of mathematics, student capabilities, the teachers' role in relation to classroom/behaviour management, the teachers' responsibilities in relation students' learning, and engagement in ongoing professional learning have all been found to be relevant to their practice. The findings suggest that teachers' context specific, identity-related and hence central beliefs concerning their role as teacher are, at least in part, derived from primary beliefs (Green, 1971) about student capabilities. This is consistent with evidence that beliefs concerning the capabilities of students significantly impact practice and would also be positioned near to the origin of the model described. Beliefs concerning classroom/behaviour management are less context specific. That is, teachers who are significantly concerned with such issues appear to carry that concern across all of the classes that they teach. These beliefs are inherently close to the context of any given class and hence the extent of their influence on teachers' practice is primarily a function of their centrality in relation to the teacher's other beliefs in that context. Beliefs concerning how mathematics is best learned and taught are more distant from the specific classroom contexts but not as distant as beliefs concerning the nature of mathematics, or the importance of professional learning. Similarly, for those teachers who hold beliefs about school mathematics in isolation from their beliefs about mathematics as a discipline, beliefs about school mathematics are inherently closer to classroom contexts than those about the discipline. The impact of these beliefs is thus a function of their relative centrality.

The presentation will provide further illustrations along with diagrammatic representations of the model.

References

Beswick, K. (2005). The beliefs/practice connection in broadly defined contexts. *Mathematics education research journal, 17*(2), 39-68.

Beswick, K. (2018). Systems perspectives on mathematics teachers' beliefs: Illustrations from beliefs about students. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 3-18). Umeå, Sweden: PME.

Green, T. F. (1971). *The activities of teaching*. New York: McGraw-Hill.

CHARACTERISTICS OF REAL ANALYSIS AND LINEAR ALGEBRA FROM AN UNDERGRADUATE'S PERSPECTIVE

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Mathematics bachelor courses at university still come along with high dropout rates, mainly during the first year. The research for conditions that favour students' academic success and prevent dropout is multifaceted. Among other things, beliefs are analysed. Students' beliefs have an impact on mathematical learning as they influence "how one chooses to approach a problem, which techniques will be used or avoided" (Schoenfeld, 1985, p. 45) and the willingness to exert effort.

At German universities, the first year is traditionally characterised by courses in real analysis and linear algebra (Halverscheid & Pustelnik, 2013). From past experiences, the failure rate in the exams at the Ruhr University Bochum tends to be higher in real analysis than in linear algebra. Due to the impact of beliefs and the experience of the divergent results in real analysis and linear algebra, we are interested in the characteristics of both domains from an undergraduate's perspective and her related domain specific beliefs.

The case study was carried out in a qualitative way, using a semi-structured interview with a narrative focus ("How would you describe real analysis and linear algebra to a secondary student?"). Additionally, a semantic differential (Osgood, Suci, & Tannenbaum, 1957) with 25 opposite pairs of adjectives, such as *theoretical-practical* or *concrete-abstract,* was used. For analysing the data, we used the documentary method.

We interviewed Hakima, a second-year student who was chosen because she attended the same courses twice within two years due to her failing the exam in the end of the first semester. Therefore, she had a more comprehensive perspective than ordinary first year students. Furthermore, being taught by two different professors in each subject, she may have benefitted from multifaceted experiences.

The findings indicate that most mentioned differences are not related to characteristics of the mathematical domains but focus on characteristics of the lecturers and their teaching style or the organizational structure of the lecture. Being asked to describe the domains to a secondary student, Hakima characterised the domains foremost among the topic *vector spaces* for linear algebra as well as *sequences* and *series* for real analysis. Based on these examples, Hakima describes the domains as rather practical in the case of linear algebra and rather theoretical in the case of real analysis. While her description and evaluation underlined the different topics, she did not refer to any mathematical technique or to any mathematical way of thinking.

Our results indicate that the lecturers' teaching style is way more important to the students than domain specific beliefs regarding the content. Therefore, it seems necessary to give more attention to the form of presentation. However, it remains unclear whether the content influences students' beliefs about real analysis and linear algebra and whether each domain calls for special supportive strategies.

References

Halverscheid, S., & Pustelnik, K. (2013). Studying Math at the University: Is Dropout predictable? In A. M. Lindmeier, & A. Heinze (Eds.), *Proc. 37th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 2, pp. 417–424). Kiel PME.

Osgood, C. E., Suci, G. J., & Tannenbaum, P. H. (1957). *The measurement of meaning*. Urbana: University of Illinois Press.

Schoenfeld, A. (1985). *Mathematical problem solving*. New York Academic Press.

CHANGING TEACHERS' IMAGES OF MATHEMATICS AND MATHEMATICS TEACHING

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Teachers' conceptions have a powerful impact on teaching through such processes as the selection of content and emphasis, styles of teaching, and modes of learning (Ernest, 1989). Green(1971) emphasized not only the content of a person's beliefs, but also the way he holds them. History could offer situations to challenge teachers' conceptions about mathematics and mathematics teaching (Fruringhetti, 2007). However, most studies were conducted with pre-service teachers.

This research project aims to examine the influence of history of mathematics on in-service teachers' conceptions about mathematics and mathematics teaching. The research questions are: 0) Are there any changes in the teachers' conception about mathematics as a discipline? If so: 1) What are these changes? 2) Are there changes in the way the teachers hold their beliefs? If so, which? 3)To what extent, if any, do these changes seem to influence the teachers' conceptions about their mathematics teaching?

The sample comprises 50 in-service lower secondary school teachers. They read, discuss and reflect the history and pedagogy of 9 topics online for one year. Data for the pre-post-test comparison was collected by identical 6-point Likert questionnaire and open-ended questionnaire. Reflection tasks were also collected during the project, as well as the follow-up interview. Two teachers are followed as individual cases.

The quantitative data indicates that there are significant changes in teachers' conceptions. Teachers tend to agree more with the problem-solving view (from 4.71 to 5.13) and less with the instrumentalist (from 4.11 to 3.61) and Platonist view (from 3.99 to 3.65). For teacher Hu, some of his conceptions about the application, the evolution and the nature of mathematics changed, while something new are added. His conceptions about mathematics as a discipline are supported by more examples, more justified, more consistent. Some unconscious beliefs becomes more conscious. From the change of both the content of beliefs/views and the way they are hold, we can know that teacher Hu' conceptions about mathematics as a discipline are more close to some aspects of Ernest's Platonist view and Problem-solving view, far away from the Instrumentalist view and some other aspects of Ernest's Platonist view. Teacher Hu' conceptions about the mathematics teaching are more close to content-focused with an emphasis on conceptual understanding view and learner-focused view. In the presentation, further results will be discussed in detail.

References

Ernest, P.(1989). The knowledge, beliefs and attitudes of the mathematics teacher: a model. *Journal of Education for Teaching*, 15 (1),13-33.

Green, T. F. (1971). The activities of teaching. New York: McGraw Hill Book Company. Hamming.

Furinghetti.F. (2007). Teacher education through the history of mathematics. *Educational Studies in Mathematics*, 66,131-143.

A COMPARATIVE STUDY ON GERMAN AND ITALIAN PERSPECTIVE TEACHERS' VIEWS OF MATHEMATICS

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International literature is increasingly disclosing the relevance of cultural aspects within the mathematics teaching and learning processes. According to Radford (2003), knowledge is inextricably linked to the activities in which the subjects engage, and this must be considered in close relationship with the socio-cultural context in which the activity takes place. In MAVI 25, Ferretti, Funghi and Blum (in press) discussed the impact of the cultural aspects on affective issues and on curricular issues. Furthermore, Bartolini Bussi, Funghi and Ramploud (2019) have highlighted how teachers' beliefs depend strongly on teachers' culture. Thus, it is of remarkable interest to investigate how and if cultural aspects influence pre-service teachers' beliefs and knowledge.

In MAVI25, Maffia and colleagues (in press) presented a clustering method to analyse answers given to multiple-answer questions about perspective teachers' view of mathematical ability. For this comparative study, we used the same method to cluster answers to the same questionnaire by future students from three different universities: University of Bologna (Italy), University of Bozen (Italy) and University of Kassel (Germany). Among the 460 respondents, 40% are from Bologna, they are students coming from different parts of Italy. 39% of our sample comes from Kassel; the students in Kassel are mostly come from places around Kassel. The remaining 21% are students from Bozen, a bilingual city situated close to the border between Italy and Austria.

First results show that students from all the three universities give less importance to natural abilities. Clusters more populated by students from Kassel are characterized by a strong attention to analytical thinking and creativity, while the percentage of students from Bologna is higher in clusters characterized by attention to flexible thinking and affective factors. Students from Bozen are almost distributed equally in all the clusters, suggesting a mix of cultural beliefs.

References

Bartolini Bussi, M. G., Funghi, S., & Ramploud, A. (2019). Mathematics Teachers' Cultural Beliefs: The Case of Lesson Study. In *International Handbook of Mathematics Teacher Education: Volume 1* (pp. 131-154). Brill Sense.

Ferretti, F., Funghi, S. & Blum, S. (in press). Issues about culture, affect and standardized assessment. In Andrà, C., Brunetto, D. & Martignone, F. (Eds). *Theorizing and Measuring Affect in Mathematics Teaching and Learning*. Springer.

Maffia, A., Rossi Tisbeni, S., Ferretti, F., & Lemmo, A. (in press). A clustering method for multiple-answer questions on pre-service primary teachers' views of mathematics. In C. Andrà, D. Brunetto, & F. Martignone (Eds.) *Theorizing and Measuring Affect in Mathematics Teaching and Learning*. Springer

Radford L. (1997). On psychology, historical epistemology and the teaching of mathematics: towards a socio-cultural history of mathematics. *For the Learning of mathematics*, 17(1), 26-33.

NATURE OF MATHEMATICS, META-SCIENTIFIC REFLECTION AND SCIENTIFIC PROPAEDEUTICS

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In the German secondary school ("Gymnasium"), students shall become acquainted with the fundamentals of scientific ways of working, they shall develop a scientific attitude and be able to think about the nature and the limits of scientific findings. This first contact with scientific ways of working and thinking is called scientific propaedeutics ("Wissenschaftspropädeutik"). One aim of this first contact is that students are better prepared for studying at university. Since scientific propaedeutics is primarily a concept of general education and has been rarely discussed in mathematics education yet, we adapt this concept to the subject mathematics. Theoretical approaches to conceptualize scientific propaedeutics propose a model consisting of three hierarchical levels: (1) meta-scientific knowledge, (2) methodological awareness and (3) meta-scientific reflection (Müsche, 2009). To specify this model to mathematics, we used the concepts "nature of science", discussed in science education, and domain-specific "beliefs" (Törner, 2002), well-known in mathematics education. In this presentation, we focus on the third dimension of this model.

The presented study aims to describe the concept meta-scientific reflection on mathematics and to investigate how this concept is related to prevalent concepts of mathematics education such as nature of mathematics and epistemological beliefs. Accordingly, the research questions are:

- What is meta-scientific reflection on mathematics as one aspect of science propaedeutics?
- How meta-scientific reflection on mathematics can be assessed?

To answer the first question, we consider the specific character of mathematics with its deductive structure, built on definitions, theorems and proofs. The third level of scientific propaedeutics, namely meta-scientific reflection on Mathematics as an academic discipline, includes thinking and discussing about the certainty of mathematical knowledge and about the specific characteristics of mathematics in comparison to other scientific disciplines. Students who achieve the third level of scientific propaedeutics are able to recall and apply knowledge about mathematics, to systemize academic disciplines in categories and categorize specific scientific disciplines (e.g. mathematics as a formal science) into the wide field of disciplines. One interesting question in this regard is whether students refer to mathematics as a natural science (like physics) or not and how they justify their decision.

To answer the second question, we developed a questionnaire and conducted a pilot study, implementing in an online survey, with pre-service teacher students. Since the survey is currently in progress, we cannot present any results yet. We expect that more than 50 students will participate in the survey. In the presentation, first results will be discussed in detail.

References

Müsche, H. (2009). Wissenschaftspropädeutik aus psychologischer Perspektive – Zur Dimensionierung und Konkretisierung eines bildungstheoretischen Konzeptes. *TriOS*, *4*(2), 61-109.

Törner, G. (2002). Mathematical beliefs—A search for a common ground: Some theoretical considerations on structuring beliefs, some research questions, and some phenomenological observations. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 73–94). Dordrecht, The Netherlands: Kluwer Academic Publishers.

THEORETICAL IMPLICATIONS FROM MULTIPLE MOBILE EYE-TRACKING STUDIES ON TEACHER-STUDENT INTERACTION

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My presentation will examine theoretical implications from three eye-tracking studies on teacher-student nonverbal interaction during collaborative mathematical problem solving. The studies ware a part of MathTrack research project (University of Helsinki). The data collection took place on three 9th-grade mathematics lessons, where the teachers scaffolded the students' geometry problem solving (Steiner tree problem for four points). For the analyses, I used the collaborative phases of the lessons, when the students solved the problems in groups of four. These students and the teachers wore gaze-tracking glasses during the session. Additionally, stationary video cameras and microphones recorded their actions and conversations.

I coded the teacher gazes according to their gaze targets and durations. The coding unit I used was a dwell: single visit to a researcher-defined gaze target from entry to exit (Holmqvist & Andersson, 2017). For the third study, I also added student gazes directed at their teacher to the analyses. The stationary video recordings complemented the gaze recordings. For the first two studies, I coded the teacher intentions of scaffolding interaction. These intentions were cognitive, affective, and metacognitive scaffolding (Van de Pol et al., 2010), and nonverbal moments of monitoring and fading. For the third study, we coded teacher behaviors using Interpersonal Theory (Leary, 1957). The studies used mixed-method approach.

Some general characteristics of teacher gaze have already been charted by recent research (McIntyre et al., 2019), but the continuous changes and especially the reciprocal eye contact communication in teacher-student interaction have remained unexplored. The approach I used in these studies, examined classroom interaction from momentary process perspective. Traditionally, the continuous measurement data is compared to data on rather static aspects, such as teacher interpersonal style (Pennings et al., 2018), cultural background, or expertise (McIntyre et al., 2019). My aim was to explore the effects on teacher-student eye contact and teacher visual attention that are relative to momentary changes in the classroom interaction, that is, pedagogical and personal states.

An interview with one of the teachers affirmed my interpretation that his momentary gaze behavior was directed by his pedagogical vision but affected by the momentary changes in teacher-student interaction. Due to this, he was sometimes not able to provide the nonverbal support he would have wanted to. This finding was similar with other teachers, too. Hence, I may conclude that the teachers' gaze behavior is relative to their momentary scaffolding and interpersonal behaviors, and through them are building block of teacher-student relationships (see Pennings et al., 2018). Interestingly, also the students' tendency to look at their teachers was in relation to teachers' interpersonal behaviors. This underlines the importance of using multiple eye trackers simultaneously and zooming into the momentary micro-level interactions when interpreting the data.

Holmqvist, K., & Andersson, R. (2017). Eye tracking: A comprehensive guide to methods, paradigms and measures. Lund, Sweden: Lund Eye-Tracking Research Institute.

Leary, T. (1957). Interpersonal diagnosis of personality; a functional theory and methodology for personality evaluation. Eugene, OR: Resource Publications.

McIntyre, N.A., Jarodzka, H., & Klassen, R.M. (2019).Capturing teacher priorities: Using real-world eye-tracking to investigate expert teacher priorities across two cultures. Learning and Instruction, 60, 215-224. doi: 10.1016/j.learninstruc.2017.12.003.

Pennings, H. J. M., Brekelmans, M., Sadler, P., Claessens, L. C. A., van der Want, A. C., & van Tartwijk, J. (2018). Interpersonal adaptation in teacher-student interaction. Learning and Instruction, 55, 41-57. doi: 10.1016/j.learninstruc.2017.09.005

Van de Pol, J., Volman, M., & Beishuizen, J. (2010). Scaffolding in teacher-student interaction: a decade of research. Educational Psychology Review, 22, 271-297. doi: 10.1007/s10648-010-9127-6

THEORY-DRIVEN DETERMINANTS OF SCHOOL STUDENTS' STEM CAREER GOALS: A PRELIMINARY INVESTIGATION*

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The aims of this study were to investigate Turkish school students' attitudes towards STEM disciplines and careers and explore determinants of students' STEM career goals. In total, 120 lower secondary school students (aged 11 to 14) completed the STEM Semantic Survey including an open-ended question about their career intention after high school and the reasons for their choice. Using the conceptualisation of the influences of behavioural, personal, and contextual variables in career choice decisions, the students' descriptions of career choice reasons (Lent, Brown, & Hackett, 1994; 2000) were presented to elaborate on the variables that influence their STEM career goals. Attitudes towards individual STEM disciplines were from moderate to high and towards STEM careers were high. The gender difference was negligible. One of the key determinants of students' career choice intentions was interests, involving interest in a particular career (e.g. architect) and career-relevant activities (e.g. planning, drawing, and designing) or subjects (e.g. mathematics). Larger, societal influences (altruism and patriotism) were among the motives of students' career goals. Implications for research, practice, and policy-making were presented.

References

Lent, R. W., Brown, S. D., & Hackett, G. (1994). Toward a unifying social cognitive theory of career and academic interest, choice, and performance [Monograph]. *Journal of Vocational Behavior, 45,* 79–122.

Lent, R. W., Brown, S. D., & Hackett, G. (2000). Contextual supports and barriers to career choice: A social cognitive analysis. *Journal of Counseling Psychology, 47,* 36–49.

Hatisaru, V. (submitted). Theory-driven determinants of school students' stem career goals: a preliminary investigation.

EXPLORING EVIDENCE OF MATHEMATICAL TASKS IN THE DRAWINGS OF MIDDLE SCHOOL STUDENTS*

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As part of a larger research project exploring a group of 120 Turkish middle school students' (grades 6 to 8, aged 11 to 14) perceptions of their mathematics classroom experiences, this study presents an analysis of the nature of mathematical tasks and the forms of mathematical representations depicted in students' drawings. An analysis of the data obtained from the students' drawing task (Draw a Mathematics Classroom Test) revealed little to no variety in students' classroom experiences in relation to the types of mathematical tasks or mathematical representations. The most common mathematical tasks were found to be tasks that focus on procedural skills, while the most common way students represented the mathematics was through symbolic representations. None of the student drawings involved physical or contextual representations. Findings raise concerns about whether Turkish students are well prepared for the demands of the 21st century.

Hatisaru, V. (in press). Exploring evidence of mathematical tasks in the drawings of middle school students. *International Electronic Journal of Mathematics Education*.

ADULTS' BELIEFS REGARDING ADULT INTERVENTION IN DEVELOPING YOUNG CHILDREN'S GEOMETRICAL THINKING

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The importance of engaging young children with geometry and spatial activities has been acknowledge by several preschool curricula. However, for children to take advantage of the academic opportunities provided at preschool, some level of support from the home environment is necessary (Anders et al., 2012). Beliefs can impact on actions (Maas, 2011). Thus, if we wish to increase children's opportunities to engage with geometric activities at home, a first step is to investigate adults' beliefs. In this presentation we focus specifically on adults' beliefs regarding their intervention in children's learning geometrical concepts. (1) Do adults believe that adult involvement is important to children's geometrical concept development? (2) Do adults believe that they need guidance to help foster this development?

Participants were 42 adults, none of whom were preschool teachers. They answered two open questions: (1) In your opinion, is it important for an adult to be involved in developing preschool (ages 3-6) children's geometric reasoning? Explain. (2) In your opinion, is it important for an adult to receive guidance so that he/she can help foster geometric reasoning among young children (ages 3-6)? Explain. A first step in data analysis was to assess the frequency of positive and negative responses to the two questions. Content analysis was conducted to analyse reasons participants offered for their responses.

Thirty-seven adults (88%) believed that adult intervention is important. Phrases taken from participants' reasons were categorized according to adults' actions (e.g., raising awareness, naming), geometrical concepts/activities mentioned (e.g., triangles, shape composition), importance for the child (e.g., it will raise the child's interest about geometry). Among the five who indicated that intervention was not important some reasoned that children will come across geometric concepts in preschool. Of those who claimed adult intervention is important, 78% stated that guidance for adults is also important, noting both cognitive reasons and affective reasons. For example, one adult stated that guidance was important "for acquiring tools with which to promote this knowledge in a positive manner." Four adults stated that it is not important for adults to receive guidance and four stated that it depends on the type of guidance offered. For example, one adult stated, "Even without guidance, adults expose children to shapes."

In general, for various reasons, adults believe they can play an important part in children's early geometry development. Understanding why some adults are or are not interested in guidance, and why, might help educators attract more adults to participate in workshops to help them interact playfully with children and geometry.

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References

Anders, Y., Rossbach, H. G., Weinert, S., Ebert, S., Kuger, S., Lehrl, S., & Von Maurice, J. (2012). Home and preschool learning environments and their relations to the development of early numeracy skills. *Early Childhood Research Quarterly, 27*(2), 231-244. https://doi.org/10.1016/j.ecresq.2011.08.003

Maass, K. (2011). How can teachers' beliefs affect their professional development?. *ZDM Mathematics Education, 43*(4), 573-586. https://doi.org/10.1007/s11858-011-0319-4

BELIEFS CRISIS DURING THE PANDEMIC: VOICES FROM MATHEMATICS TEACHERS IN ITALY AND HONG KONG

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In this contribution we report part of a wider study concerning teachers' beliefs on mathematics teaching and learning during the COVID-19 pandemic.

Following Phillip (2007), we see beliefs as "lenses that affect one's view of some aspect of the world or as dispositions toward action" (p. 259). An interesting trend of research concerns belief change. Liljedahl (2010) argues that change may happen in a rapid and profound way when an existing belief starts to be questioned or even rejected by the teacher, typically during or after an experience of professional development. Freeman (1989) argues that change does not necessarily mean a change in practice, rather a change in awareness. Our working hypothesis is that the "forced" shift to distance teaching, an experience far stronger than a professional development course, may have fostered a teacher change, especially in terms of awareness. We introduce the concept of "belief crisis", conceptualized as a questioning of existing beliefs and practice. The present study is aimed at presenting and discussing instances of "belief crisis" through the voices of the teachers.

The larger study was conducted in two regions, Italy and Hong Kong, that were early and deeply touched by the pandemic. In these two regions, all teachers have expressed distance teaching for several months. The study was organized in two steps: a first online questionnaire (administered to 193 teachers, from grade 1 to grade 13) and a subsequent semi-structured interview, performed on a convenient sample of teachers (29 teachers). For this contribution we confine ourselves to the analysis and comparison of the exemplary "voices" of two teachers, Marilu (from Italy) and Anita (from Hong Kong). For both teachers the strong experience was the starting point for a reflection on what is really "non-negotiable" in their teaching. Both teachers recognized also that the distance teaching experience will influence their future teaching since they had the opportunity to try and succeed in some teaching methods that they would like to keep also in present teaching. Here we see the first hints of teacher change, as a result of the documented belief crisis.

References

Freeman, D. (1989). Teacher training, development, and decision making: A model of teaching and related strategies for language teacher education. *TESOL Quarterly*, 23(1), 27-45.

Liljedahl, P. (2010). Noticing rapid and profound mathematics teacher change. *Journal of Mathematics Teacher Education, 13*(5), 411–423.

Philipp, R.A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257-315). Charlotte, NC: Information Age.

METHOD OF ANALYSIS AND SYNTHESIS IN EDUCATION OF MATHEMATICS

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The method of analysis and synthesis has long history which started from Ancient geometry and it is "a method Greek geometers used in looking for proofs of theorems (theoretical analysis) and for constructions to solve problems (problematical analysis)" (Hintikka & Remes 1974, 1). It is not clear how to specify the method, but generally it is accepted that the analysis starts from what is eventually sought for and that it is followed by synthesis which is formulated in strict logical form. Usually textbooks follow their own logical form which is, of course, part and parcel of mathematical practise. However, the approach hides the constructive aspects of geometrical and, more generally, mathematical reasoning (Mumma 2012).

More technically, the logical form adduces "the analysis-of-proof view" (Hintikka & Remes 1974, 31) which analyses the steps of deduction. Unfortunately, this does not explicate the essential constructive aspects of mathematical and geometrical reasoning. The idea of analysis is to proceed "as an 'upward' movement, that is, as a search of premises rather than as a sequence of conclusions" (Hintikka & Remes 1974, xiv) which is expressed by "the analysis-of-figures view" (Hintikka & Remes 1974, 32). The idea is to search for the auxiliary constructions needed in the formal proofs which makes the analysis constructive (Hintikka & Remes 1974; Niiniluoto 2018).

The idea of constructions is to bring new information into the reasoning process. The additional information can be expressed in linguistic form which takes place especially in algebraic contexts or in pictorial form which takes place especially in geometry which emphasizes visual expressions in reasoning (Priest, De Toffoli & Findlen 2018, 49). At the same, this shows why the logical way to express mathematics entails that "visual representation remains a second-class citizen in both the theory and practice of mathematics" (Barwise & Etchemendy 1991, 9). Moreover, the systematic use of the method of analysis and synthesis shows the methodological role of mathematics in different fields of science: mathematics is not merely a formal tool but, at the same, methodological model of constructive knowledge acquisition. (Hintikka & Remes 1974; Priest, De Toffoli & Findlen 2018; NIiniluoto 2018).

The role of reasoning in mathematics education needs further research since constructions are important factors for mathematics learning. In the presentation, methodology issues will be connected to the methodology of analysis and synthesis.

References

Barwise, J., & Etchemendy, J. (1991). Visual Information and Valid Reasoning, in Zimmermann, W. & Cunningham, S. (Eds.). (1991). *Visualization in Teaching and Learning Mathematics*. Washington (D.C.), Mathematical Association of America; 9-24.

Hintikka, J., & Remes, U. (1974). *The Method of Analysis: Its Geometrical Origin and Its General Significance*, Dordrecht, D. Reidel Publishing Company

Mumma, J. (2012). Constructive geometrical reasoning and diagrams. *Synthese, 186*(1), 103-119.

Niiniluoto, I. (2018). *Truth-Seeking by Abduction*, Cham: Springer

Priest, G., De Toffoli, S., & Findlen, P. (2018). Tools of Reason: The Practice of Scientific Diagramming from Antiquity to the Present. *Endeavour, 42*, 49-59.

DEVELOPMENT INTO A MATHEMATICALLY HIGH-ACHIEVING STUDENT DURING BASIC EDUCATION

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Several studies show that student's self-concept predicts better achievement in school (e.g. Bryan, Glynn & Kittleson, 2011; Jiang, Song, Lee & Bong, 2014). Hiltunen and Nissinen (2018) found that mathematically high-achieving Finnish students in PISA 2015 had a higher motivation than other students. However, also other factors have been shown to be important for student's mathematics achievement.

In this study, we investigate the development of mathematically high-achieving students from basic education to the end of compulsory school. We focus on the best achievers in the grade nine evaluation and examine the development of their mathematical performance and attitudes from the grade three. The research questions are:

1. How did the mathematical competence of high-achieving students develop during basic education?

2. Which factors distinguished the mathematically high-achieving students from others?

3. What characterizes students who succeed lower than average level in the third grade and yet develop into high-achieving students in the ninth grade?

In this study, we use Finnish national evaluation data for mathematics that EDUFI and FINEEC have collected (e.g. Metsämuuronen, 2013). The longitudinal evaluation followed one age group's development of mathematical competence during 2005-2015 from the third grade of basic education to the twelfth grade. In our study we focused on mathematically high-achieving students of the grade nine to examine how their skills had evolved during basic education. Moreover, we analysed what kind of factors distinguish them from others less well achieving students.

The longitudinal data consists of 2051 students, who have taken part in mathematics examinations in the third, sixth and ninth grades. We defined high-achieving students based on success in the examination of the ninth grade. Total number of high-achieving students was 256 (12,5 %).

In this study, we used the comparison of means and the Regression analysis for analysing the data. We used different forms of t-test and the Analysis of Variance (ANOVA) to compare between group means.

The results indicated that high-achieving students are distinguished from others in the grade three and the difference with other students is clearly visible in the grade six. Students' previous mathematical competence, self-concept and the level of parents' education were explanatory factors for better competence in the grade nine. We found that the level of high-achieving students can be attained from a lower than average competence level.

References

Bryan, R. R., Glynn, S. M. & Kittleson, J. M. (2011). Motivation, achievement and advanced placement intent of high school students learning science. *Science Education*, 95(6), 1049–1065. https://doi.org/10.1002/sce.20462.

Hiltunen, J. & Nissinen, K. (2018). Erinomaiset matematiikan osaajat. Teoksessa J. Rautopuro & K. Juuti (eds.), *PISA pintaa syvemmältä. PISA 2015 Suomen pääraportti*. Kasvatusalan tutkimuksia 77. Jyväskylä: Suomen kasvatustieteellinen seura, 213–234.

Jiang, Y., Song, J., Lee, M. & Bong, M. (2014). Self-efficacy and achievement goals as motivational links between perceived contexts and achievement. *Educational Psychology*, 34(1), 92–117. https://doi.org/10.1080/01443410.2013.863831.

Metsämuuronen, J. (ed.) (2013). *Perusopetuksen matematiikan oppimistulosten pitkittäisarviointi vuosina 2005-2012*. Koulutuksen seurantaraportit 2013:4. Helsinki: Opetushallitus.

THE MAKING OF THE ASSESSEE: A CRITICAL REVIEW ON STUDENT POSITIONING IN RECENT MATHEMATICS ASSESSMENT RESEARCH

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There have been numerous calls from researchers to revisit the purpose and practices of mathematics classroom assessment. Not much has changed; students' role in mathematics assessment is still the one of *receiver*. Traditionally, the division of roles in the classroom assessment of mathematics has been straightforward: Teachers conduct assessment by taking the position of an assessor, while students are the ones being assessed. This 'natural' position for students as the assessee has been discussed, questioned and connected with reduced student agency in mathematics (Nieminen, 2020; Nieminen & Tuohilampi, 2020) and more broadly in education (Boud & Falchikov, 2006).

In this study, we critically approach classroom assessment of mathematics by investigating how students are positioned in research concerning classroom mathematics assessment through a systematic literature review. We aimed to create a snapshot of contemporary research on the topic; thus, we reviewed 77 journal articles from 2015-2020. Drawing on the Foucauldian conceptualisations of power, agency and subject positioning in a way they have been operationalised in the field of mathematics assessment (cf. Nieminen, 2020), we problematised and deconstructed the 'natural' opposition of assessee/assessor. The deconstructive analytical process sought for discourses that maintained and disrupted the subject position of 'the assessee in the research articles.

What was identified in our analysis were the four overarching and prevalent discourses identified as part of the assessee's position deconstruction; these discourses maintained the position of the assessee, and were all prevalent across the data segments. *The measurement discourse* positioned the assessee as the receiver of assessment practices, seeing the assessee as a source for data collection, analysis and comparison. *The medical discourse* blurred the boundaries between medical and pedagogical through psychological and cognitive language, emphasising the psychometric premises of mathematics assessment. *The performance discourse* framed the assessee not as a learner but as a performer whose achievements needed to be rendered visible and promoted through effective assessment practices. Finally, the *monitoring discourse* highlighted how mathematics assessment was expected to steer the assessee towards certain kinds of learning and studying processes, positioning the assessee as the object of monitoring.

We argue that all these four discourses limit the agency of the 'assessee' (cf. Nieminen, 2020). Furthermore, our findings indicate that 'the assessee' is, above all, an individual, not a communal learner. We call for re-thinking of student positioning in assessment, not as mere receivers but as resourceful and communal thinkers and doers of mathematics.

References

Boud, D., & Falchikov, N. (2006). Aligning assessment with long-term learning. *Assessment & Evaluation in Higher Education*, *31*(4), 399–413.

Nieminen, J. H. (2020). The lunatics have taken over the assessment: Utilising summative self-assessment to theorise – and disrupt – the interplay of power and agency in undergraduate mathematics education. *Doctoral dissertation*.

Nieminen, J. H., & Tuohilampi, L. (2020). 'Finally studying for myself'–examining student agency in summative and formative self-assessment models. *Assessment & Evaluation in Higher Education*, 1-15.

AGENTIC IDENTITY CONSTRUCTION AS A PART OF STUDENTS' FEEDBACK LITERACY IN MATHEMATICS ASSESSMENT

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Recent years have seen a vast amount of assessment literature on 'the new paradigm of feedback' that emphasises students' active engagement with making sense and acting upon feedback information (Carless & Boud, 2018). This 'new paradigm' contrasts with the 'old paradigm', which frames feedback mainly as delivery of information. A similar shift can be identified in the field of mathematics assessment research as well, especially in relation to formative assessment. The notion of 'feedback literacy' has been used to highlight students' active role in feedback processes, and in particular their trainable feedback-related skills. Managing one's affect has been named as a central skill for feedback literate students. In this study I observe identity construction as a part of mathematical feedback practices especially in relation to affect, while bringing a much needed disciplinary knowledge to the general models for feedback literacy.

Earlier literature on the impact of emotions on feedback behavior (e.g. Molloy, Borrell-Carrió, & Epstein, 2013) and on managing one's emotions as a part of students' feedback literacy (Carless & Boud, 2018) has largely conceptualised emotions as a *barrier* for successful feedback practices. This notion might ring true in teacher- and examination-driven assessment cultures such as mathematics. Yet, taking into consideration the earlier works on mathematics-related affect with its complexity and embodiment (e.g., Hannula, 2012), I argue that 'emotional skills' as a part of feedback literacies reach beyond maintaining one's objectivity while receiving critical feedback (cf. Carless & Boud, 2018). Assessment type (e.g., summative/formative, low/high stakes) and feedback practices frame the situations in which students make sense and act upon their emotions in relation to feedback; thus, emotions play their part in how feedback moderates mathematical knowledge and ideas in mathematics classrooms.

Yet only some forms of feedback affect us as a person. Take, for example, a situation where a grade in mathematics examination strengthens one's perception of oneself as a mathematics learner. Elsewhere, a written comment by the teacher on a student's mathematical solution might not provoke any emotions or actions at all. In this theoretical work, I combine the theoretical frameworks of *feedback literacy* and *identity* to conceptualise how mathematical feedback and emotions not only shape but construct mathematical learners. Drawing on Sfard's and Prusak's (2005) analytical tool for identifying processes, I propose a theoretical model for understanding agentic identity construction mechanisms as a crucial part of feedback literacy in mathematics assessment. It is argued that feedback literate students can reflect on their emotions in mathematical feedback processes, while agentically taking part in their own identifying processes.

References

Carless, D., & Boud, D. (2018). The development of student feedback literacy: enabling uptake of feedback. *Assessment & Evaluation in Higher Education*, *43*(8), 1315-1325.

Hannula, M. S. (2012). Exploring new dimensions of mathematics-related affect: Embodied and social theories. *Research in Mathematics Education*, *14*(2), 137-161.

Molloy, E., Borrell-Carrió, F., & Epstein, R. (2013). The impact of emotions in feedback. *Feedback in higher and professional education: Understanding it and doing it well*, *5071*, 50-71.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, *34*(4), 14-22.

ASSESSING STUDENTS' BELIEFS ABOUT MATHEMATICS AS A DISCIPLINE

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Changing students' beliefs is a complicated matter, and so is assessing any potential changes in students' beliefs. If we wish for students to become critical and reflective thinkers, their beliefs need to be evidentially held (Green, 1971), which means that they must be formed through experience and reflection upon these experiences. However, when it comes to students' beliefs about mathematics as a discipline, experiences are not easily obtained. This dimension of the belief system includes beliefs about the role of mathematics in the world, the nature, application, history, philosophy and aesthetics of mathematics, and thus exceeds mathematics as a school subject (Jankvist, 2015). According to the Danish competency framework (Niss & Højgaard, 2019), which is widely implemented in Danish mathematics education, mathematical competence consists of two main pillars: eight action-oriented competencies and three forms of overview and judgment. The latter concern 1) the actual application of mathematics, 2) the historical development of mathematics, and 3) the nature of mathematics as a subject area. These three forms of overview and judgment are based on both knowledge and beliefs about mathematics as a discipline. Thus, they constitute an applicable framework for assessing such beliefs, as shown in this study addressing the following research question:

- What characterizes middle school students' beliefs about mathematics as a discipline?

As part of a longitudinal intervention study of middle school students' beliefs about mathematics as a discipline, three 6th grade classes (a total of 70 students) completed a 20-item questionnaire designed to outline their baseline beliefs. Subsequently, three students with diverse views about and attitudes towards mathematics were selected for further interviews. The data from the questionnaires and the interviews of the three students have been analysed using the three forms of mathematical overview and judgment as coding categories as well as Green's notion of evidentially and non-evidentially held beliefs. The analysis shows that even though the three students differ a lot in their mathematical abilities, level of motivation, attitude towards mathematics and learning behaviour, all of them are struggling with the questions concerning mathematics as a discipline. Their answers are often hesitant, uncertain, shallow or incoherent. All three students mainly associates mathematics with numbers and calculations, and – apart from grocery shopping – they all seem to have difficulties providing examples of the use of mathematics in everyday life.

These results indicate that the students' beliefs about mathematics as a discipline are, if not non-existent, then limited or unstable, and thereby peripheral (Green, 1971). Combined with the lack of examples by the students, this fact points toward non-evidentially held beliefs. One of the aims in the longitudinal project is to provide the students with opportunities to develop evidentially held beliefs by presenting concrete examples, both of the application and the historical development of mathematics as well as mathematical methods and discussions. Followed by room for reflection, these experiences can potentially lead to central and influential beliefs about mathematics as a discipline.

References

Green, T. F. (1971). *The Activities of Teaching*. New York: Mcgraw-Hill Book Company.
Jankvist, U. T. (2015). Changing students' images of "mathematics as a discipline". *Journal of Mathematical Behavior, 38*, 41-56.
Niss, M. & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics, 102*(1), 9-28.

FOSTERING ENGAGEMENT IN LEARNING MATHEMATICS: THE DIGITAL MATH ESCAPE ROOM “LEONARDO DA VINCI”

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Basing on the self-determination theory of Deci and Ryan (2002), it is the aim of this article to discuss a proposal of a math learning environment using the conception of digital gamification. „Gamification is the application of game features, mainly video game elements, into non-game context for the purpose of promoting motivation and engagement in learning.”(Alsawaier, 2018, p. 56) How does the gamification situation affect engagement of younger students towards math (lessons)? Does a problem solving situation embedded in a digital math escape room help to develop and/or support orientations like interest and intrinsic motivation? To give a first answer, the project “Digital math escape room Leonardo da Vinci” was created. Within a mathematical-historical context, students solve self-determined mathematical problems using their knowledge on mathematics. It is interesting to accompany students using their mathematical beliefs to find and to solve problems in such an open learning situation (Kloosterman, 1996). It is already shown that gamification and especially escape rooms have a positive effect on mathematical learning engagement. Due to the period of homeschooling, we assume that digital escape rooms may also influence students’ motivation. The research questions are:

- Does the digital escape room provide a meaningful context for students?
- In how far do students experience success playing the digital game?
- In how far do students experience individual autonomy playing the digital game?
- In how far do students experience social inclusion playing the digital game?

The theory of basic cognitive needs, as described by Deci and Ryan (2002), form the basis of analysing the digital math escape room. Experiences of competence, of autonomy and of relatedness are needs that determine the learner’s interest, motivation and handling during the digital escape room game. In August 2020, these needs are measured by accompanying the activities in this special learning environment of German students of grade 4 to 8 of different school types and by using post-questionnaires with closed as well as open questions. The survey will be provided in a digital form by the web application LimeSurvey. Qualitative data will be analysed with support of the software MAXQDA, whereas quantitative data with SPSS 25.

We aim to derive first indications regarding possible trends with respect to the discussed special conception of digital gamification.

References

Alsawaier, R. S. (2018). The effect of gamification on motivation and engagement. *The International Journal of Information and Learning Technology 35*(1), 56-79. https://doi.org/10.1108/IJILT-02-2017-0009

Deci, E. L., & Ryan, R. M. (2002). Overview of self-determination theory: An organismic dialectical perspective. In E. L. Deci, & R. M. Ryan (Eds.), *Handbook of self-determination research* (pp. 3–33). University of Rochester Press.

Kloosterman, P. (1996). Students’ beliefs about knowing and learning mathematics: Implications for motivation. In M. Carr (Ed.), *Motivation in mathematics* (pp. 131–156). Cresskill, NJ: Hampton.

THE CHALLENGE OF LOGARITMS IN TECHNOLOGY EDUCATION

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Restricted only to the starting level mathematical exercises for radio communication we may still study principles of several technical systems because detection of radio waves is tied to technology. Learning these phenomena requires formal thinking of physics, basic concepts of signal and skills in mathematical procedures. Today, the phenomenon of wirelessly transmitted data with its associated systems is an everyday fact. In school education, phenomena are described as concepts of physics, in which quantities and units follow the SI concept (e.g. Kurki-Suonio, 2013).

Yet practical radio engineering may unpleasantly surprise an inexperienced and one solution-oriented student. This study examines the differences between two didactical approaches (formalisms): subject-didactic physics and pragmatism-oriented engineering (solution-oriented calculation based practises). Main issue for learning is how well logarithmic calculation and ready-made formulas are related to each problem under consideration. We will include empirical observations on how to solve aims in the curriculum.

Our research questions are: 1) How to work with the “free space attenuation” concept? 2) How to work with the two competing didactical approaches (formalisms) in calculation and how to teach based on the findings?

We present reasons for existence of traditional and partly forgotten calculation methods. The early innovations utilizing radio waves date back to an era when distance was measured with the practical units and concepts were just named. Correspondingly, the calculation aids were simple: a slide rule (en.wikipedia.org) and various spreadsheet books (e.g. Kivelä, 2005). Thus, for calculations, it was natural to utilize various shortcuts and features or even memory rules. Most of radio engineering calculation problems were transformed into logarithmic thinking so the result was evaluated by addition and by subtraction. Unfortunately, the result thus obtained is related to the chosen model of units. Today numeric value is easily achieved, but the data might still be mixed with tradition in engineering or in SI-units. On the other hand in modern acoustics, where the logarithmic model for perception and hearing works well, there seems to be no problems as long as the traditions of the field are still followed. (e.g. Hänninen, 2015).

For better learning it is necessary refresh both the physical phenomena of wave propagation and the basic calculation rules on logarithm. It is also essential to show in practise, how any logarithmic value is formed or to be returned to the linear (Physics type) value. An example, the directivity for antenna is always in decibels. This value cannot be entered into a formula with the SI notation, a transformation must be made. But it is just fine for any logarithmic based engineering formula. In order to obtain a more accurate result detailed models as well as ready-made formulas or even applications are available. For rough estimate utilization and understanding the free space attenuation model is still an invaluable tool. It may also clarify the principles of calculations. Therefore, in education both didactical approaches are relevant.

References

Hänninen, A. (2015). Äänekkyyden mittaaminen ja äänen jälkityöt televisioon.
Kivelä, S. (2005). Matematiikan opetus ja tietokoneaikakausi. *Arkhimedes,* 1.
Kurki-Suonio, K. (2013) Uusi SI-opas. *Dimenssio 5, 61-66.*
Slide rule, https://en.wikipedia.org/wiki/Slide_rule

BELIEFS ABOUT SCHOOL MATHEMATICS VS. UNIVERSITY MATHEMATICS AND BELIEFS ABOUT TEACHING AND LEARNING IN DIFFERENT CONTEXTS OF STUDENTS' ACHIEVEMENTS

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This study with 206 pre-service teachers (PSTs) from the Mulawarman University, Indonesia, aims to investigate the relationship of PSTs' beliefs about school mathematics (SM) and university mathematics (UM) with their beliefs about teaching and learning in different contexts of students' achievements. The two contexts in this study are classes dominated by high-achieving (HA class) vs. low-achieving (LA class) students. Further, we use concepts from Ernest (1989) to interpret PSTs' beliefs: the instrumentalist view, the Platonist view, and the problem-solving view. The instrument used is the TBTP (Safrudiannur & Rott, 2020) which, for this study, was modified: (1) differing between SM and UM and (2) adding items related to mathematics teaching and learning. The Cronbach's alpha coefficients of the modified instrument are 0.78 for the instrumentalist, 0.74 for the Platonist, and 0.81 for the problem-solving view.

In line with Geisler and Rolka (2020), the results of our study indicate that PSTs differ their beliefs about SM and UM. However, the results seem to be not in line with the assumption that teachers' beliefs about SM are likely more influential than their beliefs about UM in the classroom contexts (Beswick, 2012). The correlations are shown in Table 1; our data analyses indicate that beliefs about UM are stronger correlated with beliefs about teaching and learning than beliefs about SM—in both contexts, HA and LA classes. The significant correlations imply the necessity to design math lectures for teacher education at universities carefully.

Table 1. The results of the Pearson correlations between beliefs

Associated views	Beliefs about mathematics	Beliefs about mathematics teaching and learning			
		HA class		LA class	
		Teaching	Learning	Teaching	Learning
Instrumentalist view	School Mathematics (SM)	0.194**	0.145*	0.044	0.099
	University Mathematics (UM)	0.199**	0.178*	0.235***	0.164*
Platonist view	School Mathematics (SM)	0.192**	0.311***	0.033	-0.003
	University Mathematics (UM)	0.097	0.213**	0.031	0.021
Problem-solving view	School Mathematics (SM)	0.210**	0.105	0.170*	0.214**
	University Mathematics (UM)	0.329***	0.157*	0.168*	0.298***

* $p < .05$, ** $p < .01$, *** $p < .001$

References

Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics*, *79*(1), 127-147.

Ernest, P. (1989). The impact of beliefs on the teaching of mathematics. In P. Ernest (Ed.), *Mathematics teaching: The state of the art* (pp. 249–254). London: Falmer Press.

Geisler, S. & Rolka, K. (2020). "That wasn't the math I wanted to do!"—students' beliefs during the transition from school to university mathematics. *International Journal of Science and Mathematics Education*, online first, https://doi.org/10.1007/s10763-020-10072-y

Safrudiannur & Rott, B. (2020). Offering an approach to measure beliefs quantitatively: Capturing the influence of students' abilities on teachers' beliefs. *International Journal of Science and Mathematics Education*, online first, https://doi.org/10.1007/s10763-020-10063-z

THE DEVELOPMENT OF EPISTEMOLOGICAL BELIEFS IN THE COURSE OF MATHEMATICAL STUDIES

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In modern societies, epistemological beliefs—that is psychologically held individual, supposedly domain-specific, understandings or propositions about knowledge and knowing (Hofer & Pintrich, 1997)—play an important role. For us, the impact of educational processes (with a focus on mathematics) at universities on the development and shaping of individual epistemological beliefs is of particular research interest. Generally, in psychological research, this development is described as starting with fixed absolutistic beliefs (such as "mathematical knowledge is certain"), reaching flexible, cross-linked, evaluative beliefs. However, findings of Rott and Leuders (2015, 2017) suggest that at least in mathematics the common classification of beliefs like "mathematical knowledge is certain" as naïve and "uncertain" as sophisticated is not applicable without analysing the arguments that back these belief positions.

The project at hand examines the development of university students' belief positions and argumentations regarding the dimensions "certainty of knowledge" and "justification of knowledge" in the course of their mathematical studies. 1875 students were surveyed in October 2017, October 2018, and October 2019 at the University of Cologne in a longitudinal study. Considering the methodical findings and procedure of Rott and Leuders (2015, 2017), 586 students in 2017 (median semester 1), 433 students in 2018 (median semester 3) and 856 students in 2019 (median semester 4) of diverse mathematical university courses responded with written arguments (15-20 minutes per belief dimension) to answer open questionnaire items about epistemological beliefs during mathematical lectures.

The results show positional shifts from "mathematical knowledge is uncertain" and "inductively justified" towards "mathematical knowledge is certain" and "deductively justified". This is surprising as the latter belief positions are linked to naïve beliefs in the literature. Especially students from mathematically challenging study programmes shift towards the afore-mentioned positions. The results also show that belief argumentation (i.e. reasoning supporting the positions) significantly increases during the course of the study, from ca. 5 % in 2017 to ca. 13% of the respondents who argue sophisticatedly in 2019. Looking at both results jointly raises questions concerning the widespread conceptualization of epistemological beliefs in psychological research, which is mostly tied to belief position (see above). The shift in positions and belief argumentation can probably be traced back to the common formal structure of university lectures, seminars and tutorials in mathematics. The actual reasons and liable education processes for belief shaping should be studied in greater detail in the future.

References

Hofer, B. K. & Pintrich, P. R. (1997). The development of epistemological theories: Beliefs about knowledge and knowing and their relation to learning. *Review of Educational Research 1997*, 67(1), 88–140.

Rott, B., Leuders, T. & Stahl, E. (2015). Assessment of mathematical competencies and epistemic cognition of preservice teachers. *Zeitschrift Für Psychologie, 223(1),* 39–46. https://doi.org/10.1027/2151-2604/a000198

Rott, B. & Leuders, T. (2017). Mathematical competencies in higher education: Epistemological beliefs and critical thinking in different strands of pre-service teacher education. In: *Journal for educational research online 9, 2*, 115-136.

WORLD VIEWS OF LINEAR ALGEBRA UNIVERSITY COURSES

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In research articles, at least since the 80s, the term *'world views'* has often been used (e.g., Schoenfeld, 1985, Frank 1988). It had become apparent that in California the introduction of a new curriculum for implementing problem solving failed because of the incompatible world views of the teachers. With 'worldviews' the basic professional and epistemological attitudes of people towards 'knowledge objects' were understood. In this respect, these are generalized supply systems that have been discussed since 1996. The lecture will describe further situations in which world views have been discussed in the literature as objects of research over the last three decades (e.g. Törner & Grigutsch, 1994, Bulmer & Rolka, 2005, Cobb, 2002).

Worldviews also help to prototype technical and cognitive views of sub-areas of mathematics. Here we are interested in university textbooks on *linear algebra*. Even if such books, if one compares the index lists of technical terms, do not differ significantly, they do differ considerably with regard to the intended *world view of linear algebra*. We are able to work out at least *five* different world views.

- *The strict algebraic prototype*
- *The algebraic-geometric hybrid prototype*
- *The theory-application prototype:*
- *Applied-Linear-Algebra-prototype*
- *Learning of linear algebra using integrated CAS-tools*

World views are important for teaching and learning linear algebra because they are framing the elements of course-specific knowledge. There are clues that they also shape motivations or in contrary may aggravate the access to a specific content. Thus, it is important to be always aware of the underlying (subjective) world view of some field of content or even globally the mathematics. We will make explicit various world views about a course of linear algebra which shape often implicitly our understanding of this knowledge field.

It should not be overlooked as a fact of experience that the first encounter with a knowledge context of a term is the most lasting.

References (Style: MAVI References_title)

Bulmer, M. & Rolka, K. (2005). The "a4-Project'-statistical world views expressed through pictures. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference for the International Group for the Psychology of Mathematics Education,* Vol.2, (pp. 193—200). Melbourne: Psychology of Mathematics Education.

Cobb, P. (2002). Epistemological world views, subject matter contexts, and the institutional setting of teaching. *Issues in Education: Contributions from Educational Psychology,* 8 (2), 149--157.

Frank, M.L. (1988). Problem solving and mathematical beliefs. *The Arithmetic Teacher,* 35 (5), 32--34.

Schoenfeld, A.H. (1985). *Mathematical problem solving.* Orlando (FL): Academic Press.

Törner, G. & Grigutsch, S. (1994). Mathematische Weltbilder bei Studienanfängern - eine Erhebung. (German) (Mathematical world views on first-year-students - a study). *Journal für Mathematik-Didaktik,* 15 (3/4), 211--252.

Törner, G. & Pehkonen, E. (1996). On the structure of mathematical belief systems. *International Reviews on Mathematical Education* (ZDM), 28 (4), 109--112.