Ind. vorauss

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$$

Ind. behauptung

$$\sum_{k=1}^{n+1} \frac{1}{k(k+1)} = \frac{n+1}{n+2}$$

Beweis

$$\sum_{k=1}^{n+1} \frac{1}{k(k+1)} = \sum_{k=1}^{n} \frac{1}{k(k+1)} + \frac{1}{(n+1)(n+2)}$$

Ind V.

$$= \frac{n}{n+1} + \frac{1}{(n+1)(n+2)}$$

$$= \frac{n(n+2)}{(n+1)(n+2)} + \frac{1}{(n+1)(n+2)} = \frac{n+1}{n+2} \quad \square$$

$$= \frac{n(n+2)+1}{(n+1)(n+2)} = \frac{n^2+2n+1}{(n+1)(n+2)} = \frac{(n+1)^{\cancel{2}}}{\cancel{(n+1)}(n+2)}$$

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{n(n+1)}$$

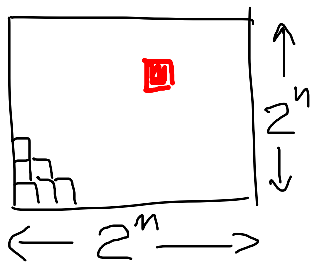
$$= \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \quad + \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

Teleskopsumme

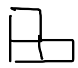
$$= \frac{1}{1} \cancel{- \frac{1}{2}} \cancel{+ \frac{1}{2}} \cancel{- \frac{1}{3}} \cancel{+ \frac{1}{3}} + \quad \cancel{- \frac{1}{n}} \cancel{+ \frac{1}{n}} - \frac{1}{n+1}$$

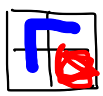
$$= 1 - \frac{1}{n+1} = \frac{n+1-1}{n+1} = \frac{n}{n+1} \quad \square$$

$$= \frac{n+1}{n+1} - \frac{1}{n+1}$$



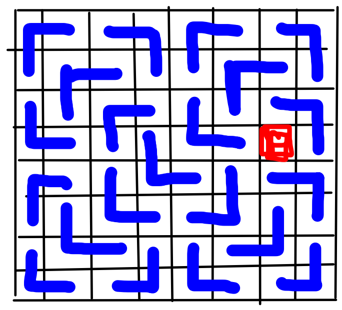
□ ausschneiden

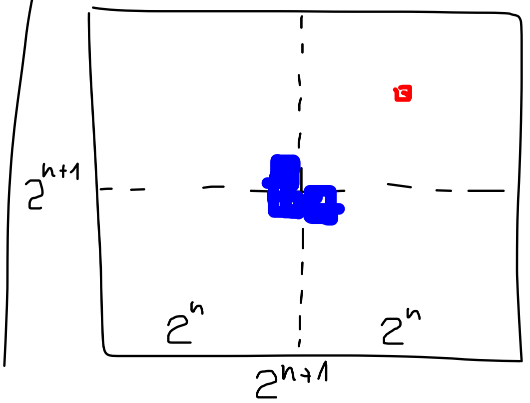




$n = 1$

$2^n \longrightarrow 2^{n+1}$





Hausaufgabe

$$\sum_{k=1}^{n} k^2 = \frac{1}{6}\, n\,(n+1)\,(2n+1) \qquad n \to n+1$$

$$(2(n+1)+1)$$

$$(2n+3)$$

$n=2 \quad 1^2+2^2=5 \qquad \frac{1}{\cancel{6}}\,\cancel{2\;3}\;5 = 5$