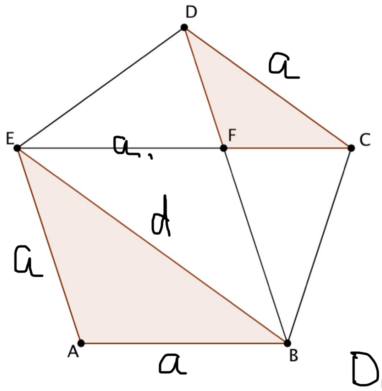
$$\frac{\sqrt{5}-1}{2} = \varphi$$ charakteristische Gleichung

$$\varphi^2 = 1-\varphi \iff \varphi^2+\varphi = 1$$



$|EC| = d$

ABFE ist ein Parallelog.

$|EF| = a$

$\Rightarrow |FC| = d - a$

ΔABE ähnlich ΔFCD

Dann gilt $\dfrac{a}{d} = \dfrac{d-a}{a}$

das ist $\dfrac{\text{Major}}{\text{Ganz}} = \dfrac{\text{minor}}{\text{Major}}$

also gilt $\dfrac{a}{d} = \varphi \Rightarrow a = \varphi \cdot d$

$d = a \cdot \dfrac{1}{\varphi}$ was ist $\dfrac{1}{\varphi}$

$$\varphi^2 + \varphi = 1 \quad | : \varphi \quad \Rightarrow \varphi + 1 = \frac{1}{\varphi} = \phi$$

$\approx 1{,}618$

„goldene Verlängerung"

$$\phi = 1 + \varphi = \cancel{1}\tfrac{2}{2} + \frac{\sqrt{5}-1}{2} = \frac{2+\sqrt{5}-1}{2} = \underline{\underline{\frac{\sqrt{5}+1}{2}}}$$

charakteristische Gleichung

$$\phi^2 = 1 + \phi$$

denn $\varphi^2 + \varphi = 1 \quad \Big| : \varphi^2$

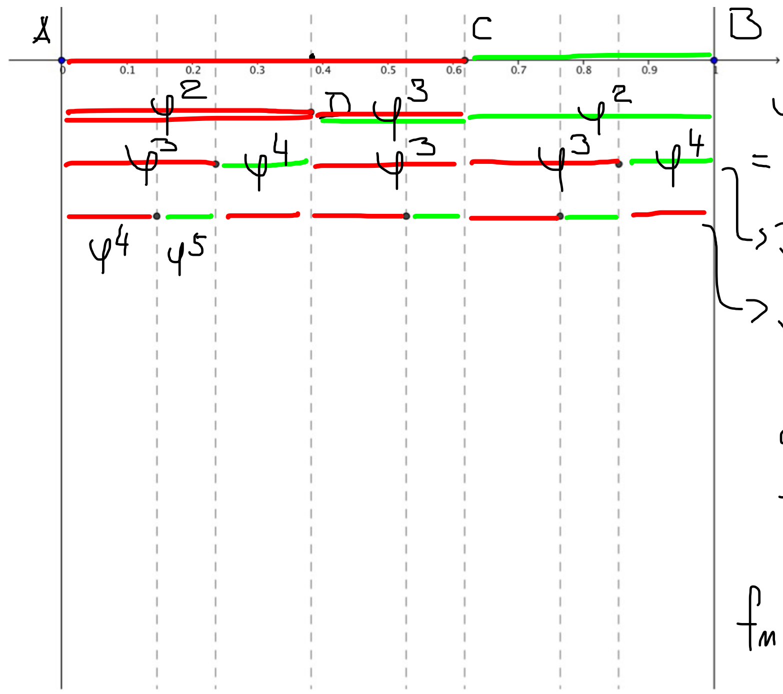
$$1 + \frac{1}{\varphi} = \frac{1}{\varphi^2} = \left(\frac{1}{\varphi}\right)^2 \quad \text{mit } \frac{1}{\varphi} = \phi$$

$$1 + \phi = \phi^2 \qquad \square$$

Der goldene Schnitt als „stetige Teilung"

$|AC| = \varphi \quad |CB| = 1-\varphi$ — Der Major vom Major ist so lang wie der Minor

$|AD| = \varphi \cdot \varphi = \varphi^2$



$\varphi(1-\varphi) = \varphi \cdot \varphi^2 = \varphi^3$ $\qquad 2\varphi^2 + \varphi^3 = 1$

$\to 3\varphi^3 + 2\varphi^4 = 1$

$\to 5\varphi^4 + 3\varphi^5 = 1$

$8\varphi^5 + 5\varphi^6 = 1$

$f_6 \qquad f_5$

. . . .

$f_{n+1}\varphi^n + f_n\varphi^{n+1} = 1$

1 1 2 3 5 8 13 21 34 55

$\frac{1}{1} = 1 \quad \frac{1}{2} = 0{,}5 \quad \frac{2}{3} = 0{,}666.. \quad \frac{3}{5} = 0{,}6$

$\frac{5}{8} = 0{,}6125 \quad \frac{8}{13} = 0{,}615 \quad \frac{13}{21} = 0{,}619\ldots$

$$\lim_{n \to \infty} \frac{f_n}{f_{n+1}} = \varphi$$

$$f_{n+1} = f_n + f_{n-1} \quad \Big| : f_n$$

$$\frac{f_{n+1}}{f_n} = 1 + \frac{f_{n-1}}{f_n}$$

$$\frac{1}{\frac{f_n}{f_{n+1}}} = 1 + \frac{f_{n-1}}{f_n} \qquad \frac{1}{\varphi} = 1 + \varphi$$