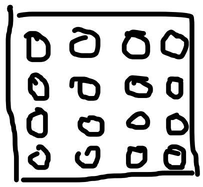
# Figurierte Zahlen

## Quadratzahlen

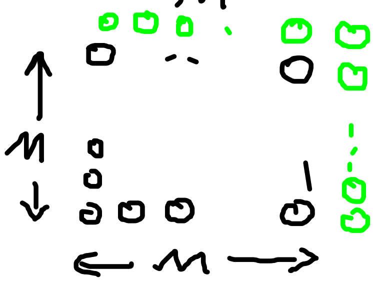
$3^2 = 9 \qquad 11^2 = 121$



$4 \times 4 = 16$

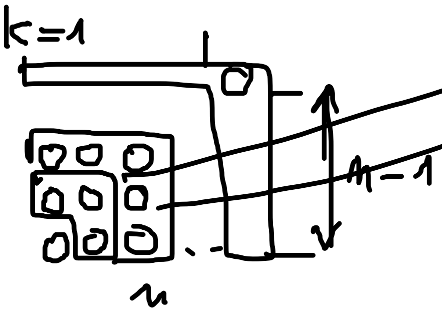
| | | | |
|---|---|---|---|
| 10 | 11 | 12 | 13 |
| 5 | 6 | 7 | 14 |
| 2 | 3 | 8 | 15 |
| 1 | 4 | 9 | 16 |

$$Q_n = n^2$$




$$Q_{n+1} = Q_n + 2n + 1$$
$$= n^2 + 2n + 1$$
$$Q_{n+1} = (n+1)^2$$

$$\sum_{k=1}^{n} (2k-1) = 1 + 3 + 5 + \ldots + (2n-1) = n^2$$




$$2(n-1)+1$$
$$2n \underbrace{-2+1}_{-1}$$

## Dreieckszahlen


$D_1 = 1 \quad D_2 = 3 \quad D_3 = 6$

$D_4 = 10$

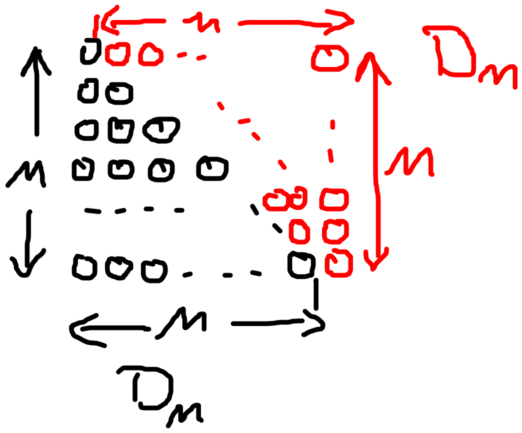
$$D_n = D_{n-1} + n$$

$$\sum_{k=1}^{n} k = 1+2+3+\ldots+n = \boxed{\frac{n(n+1)}{2} = D_n}$$

explizite F

$$D_n = D_{n-1} + n \qquad D_1 = 1$$

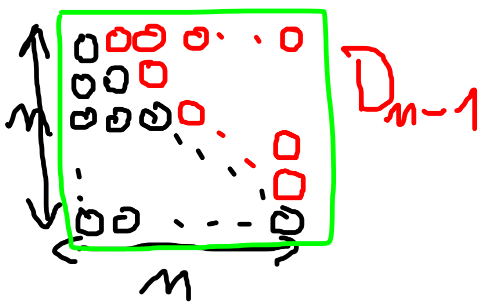
rekursive Formel




$D_n + D_n$ ergibt ein Rechteck $(n+1) \times n$

$$2 D_n = n(n+1)$$

$$D_n = \frac{n(n+1)}{2}$$




$Q_n$

$$Q_n = D_n + D_{n-1}$$

$D_n$

$D_1 = 1 \quad D_2 = 3 \quad D_3 = 6 \quad D_4 = 10 \quad D_5 = 15$

$D_6 = 21 \quad D_7 = 28 \quad D_8 = 36$

$$D_n = \frac{n(n+1)}{2} \qquad D_{n-1} = \frac{(n-1)n}{2}$$

$$D_n + D_{n-1} = \frac{1}{2}\left[n^2 + n + n^2 - n\right] = \frac{1}{2}\left[2n^2\right] = n^2$$

$$= Q_n$$

Fünfeckzahlen

$P_4 = P_3 + 3 \cdot \underline{4} - 2$

$P_5 = P_4 + 3 \cdot \underline{5} - 2$

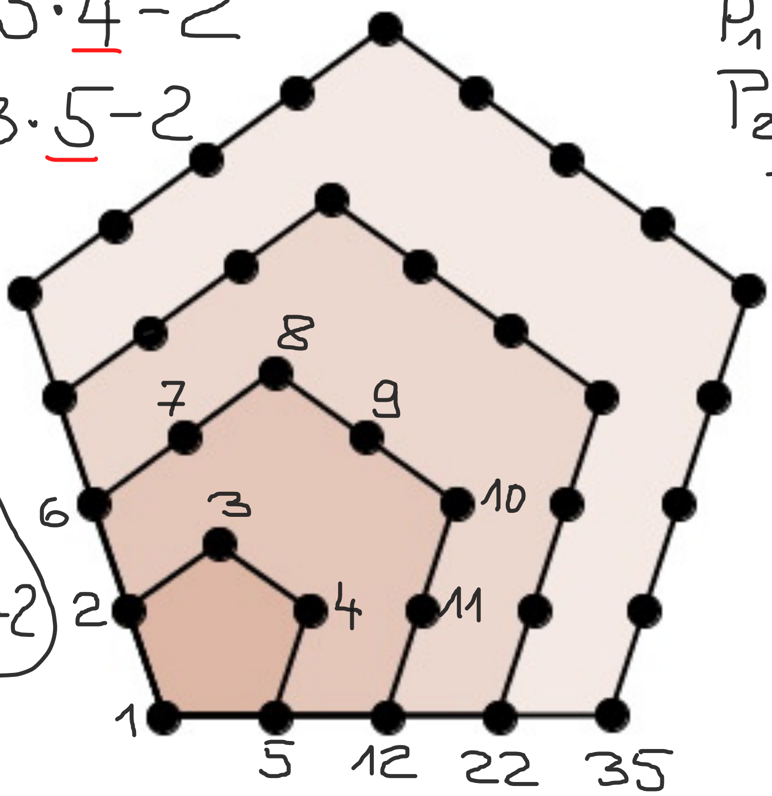
$P_1 = 1$

$P_2 = 5$

$P_3 = 12$

$P_4 = 22$

$P_5 = 35$




$\boxed{P_n = P_{n-1} + 3 \cdot n - 2}$