arithmetische Reihe

$S_n = a_1 + a_2 + \cdots a_n$, a_i arithm Folge

$a_i = a_1 + (i-1)d$

Beisp: $25, 38, 51, 64, \cdots$

geschlossene Summenformel

$$S_n = a_1 + a_2 + \cdots\cdots + a_{n-1} + a_n$$
$$S_n = a_n + a_{n-1} + \qquad + a_2 + a_1$$
$$2 \cdot S_n = (a_1 + a_n) + (a_2 + a_{n-1}) + \cdots + (a_2 + a_{n-1}) + (a_1 + a_n)$$

$$2S_n = n(a_1 + a_n) \quad | :2$$
$$S_n = \frac{n(a_1 + a_n)}{2}$$

Beisp $\underset{a_1}{11}, 16, 21, 26, 31, 36, \underset{a_7}{41}$

$n = 7$
$$S_7 = \frac{7 \cdot (11 + 41)}{2} = 7 \cdot 26 = 182$$
$$S_1 = 11 = \frac{1(11 + 11)}{2} = 11$$

$11, 16, 21, 26, \dots$

$S_1 = 11,\ S_2 = 27,\ S_3 = 48,\ S_4 = 74$

rekursiv: $S_n = S_{n-1} + a_n$

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geometrische Reihe

$S_n = a_1 + a_2 + \dots + a_n \qquad a_i = a_1 \cdot q^{i-1}$

$$S_n = a_1 + a_1 \cdot q + a_1 q^2 + \dots + a_1 \cdot q^{n-1}$$
$$= a_1 \left(1 + q + q^2 + \dots + q^{n-1}\right)$$

$$K_n = 1 + q + q^2 + \dots + q^{n-1}$$
$$q \cdot K_n = \quad q + q^2 + \dots + q^{n-1} + q^n$$

$$K_n - q \cdot K_n = 1 - q^n$$
$$K_n (1-q) = 1 - q^n \quad \Big| : (1-q) \quad q \neq 1$$
$$K_n = \frac{1-q^n}{1-q} \qquad \boxed{S_n = a_1 \frac{1-q^n}{1-q}}$$

$1, 2, 4, 8, 16, \dots \quad S_{64} = 1 \, \dfrac{1-2^{64}}{1-2} = 2^{64} - 1$

1, 2, 4, 8, 16, 32, 64

0 1 3 7 15 31 63

Fall $0 < q < 1$

Beisp $q = \frac{1}{2}$ $a_1 = 1$ $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

$S_1 = 1,\ S_2 = 1\frac{1}{2},\ S_3 = 1\frac{3}{4},\ S_4 = 1\frac{7}{8},\ S_5 = 1\frac{15}{16}$

$$S_n = 1\,\frac{1-(\frac{1}{2})^n}{1-\frac{1}{2}} = 2\cdot\left(1-(\tfrac{1}{2})^n\right) = 2-(\tfrac{1}{2})^{n-1}$$

$n \to \infty$ $S = 2$

$$S_n = a_1\,\frac{1-\underbrace{q^n}_{\to 0}}{1-q} \qquad 0 < q < 1$$

$n \to \infty$

$$S = a_1\,\frac{1}{1-q} \qquad a_1 = 1 \quad q = \frac{1}{2}$$

$$S = 1\cdot\frac{1}{1-\frac{1}{2}} = 2$$

$0,\overline{9} = 0,9999\dots$

$= \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} \dots$ $a_1 = \frac{9}{10}$ $q = \frac{1}{10}$

$$S = \frac{9}{10}\cdot\frac{1}{1-\frac{1}{10}} = \frac{9}{10}\cdot\frac{1}{\frac{9}{10}} = 1 = 0,9999\dots$$

ebenso $0,42899999\dots$

$= 0,429$

$\frac{1}{3} = 0,3333\dots \quad | \cdot 3$

$\frac{3}{3} = 1 = 0,9999\dots$

harmonische Reihe

$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\cdots\cdots$

keine geometrische Reihe

Die harmonische Reihe wird für $n \to \infty$ unendlich groß.