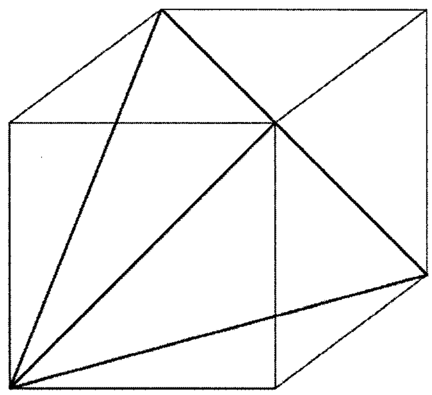
Reimund Albers, Einführung in die Mathematik II

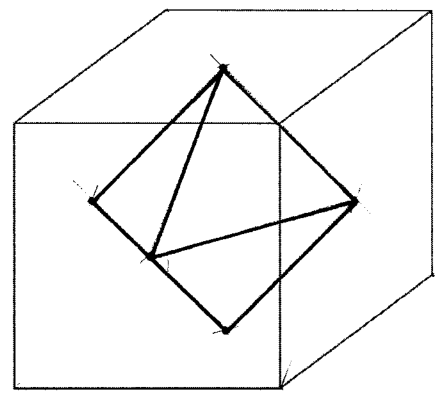
4./5. Übung, Lösungsskizzen

1.

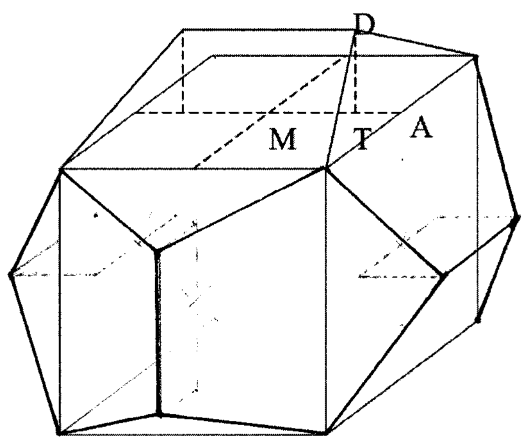
e)



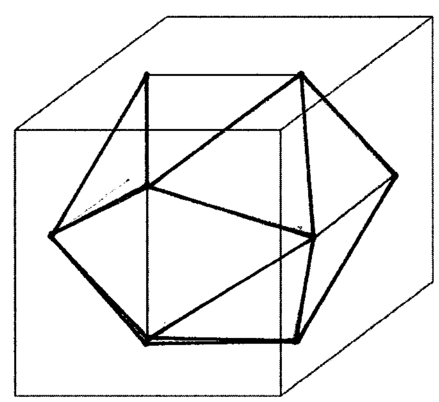
Tetraeder



Oktaeder



Dodekaeder

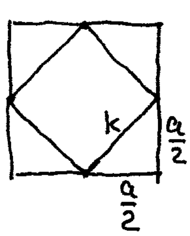


Ikosaeder

f) Tetraeder $\quad k = a\sqrt{2}$ $\quad$ Diagonale im Quadrat

Oktaeder „Draufsicht"

$k^2 = \sqrt{2\left(\frac{a}{2}\right)^2} = \sqrt{\frac{a^2}{2}} = a\sqrt{\frac{1}{2}}$



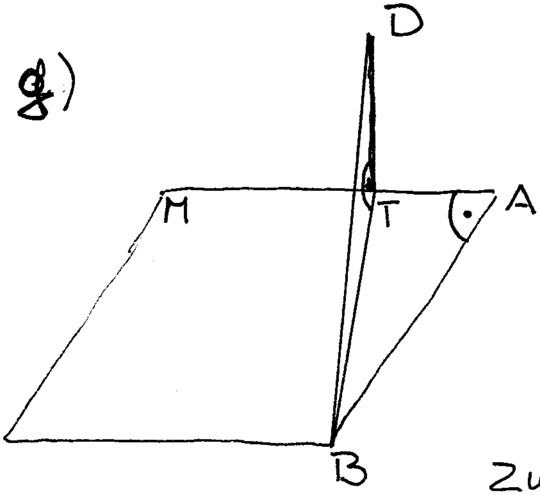
Dodekaeder $\quad$ Berechnung der „Dachkante"

$k = 2 \cdot |MT| = 2 \cdot \varphi \cdot |MA|$ $\quad$ mit $\varphi = \frac{\sqrt{5}-1}{2}$ goldener Schnitt „kleine Zahl"

$2 \cdot |MA| = a$ $\quad$ also

$k = \varphi \cdot a = \frac{\sqrt{5}-1}{2}\, a$

Ikosaeder $\quad k = \frac{\sqrt{5}-1}{2}\, a$ $\quad$ laut Erläuterung g)



siehe Dodekaeder

$$|MA| = |AB| = \frac{a}{2}$$

Länge der „Dachkante" ist

$$k = \varphi \cdot a = \frac{\sqrt{5}-1}{2}\, a$$

Zu zeigen ist: $|BD| = k$

$$|MT| = \varphi \cdot |MA| \quad \Rightarrow |TA| = (1-\varphi)|MA|$$

Pythagoras: $|BT|^2 = |BA|^2 + |TA|^2$

$$= \left(\frac{a}{2}\right)^2 + (1-\varphi)^2 |MA|^2$$

$$= \left(\frac{a}{2}\right)^2 + (1-\varphi)^2 \left(\frac{a}{2}\right)^2 = \left(\frac{a}{2}\right)^2 \left[1 + (1-\varphi)^2\right]$$

Pythagoras: $|BD|^2 = |BT|^2 + |TD|^2$

$$= \left(\frac{a}{2}\right)^2 \left[1 + (1-\varphi)^2\right] + \varphi^2 \cdot \left(\frac{a}{2}\right)^2$$

$$= \left(\frac{a}{2}\right)^2 \left[1 + 1 - 2\varphi + \varphi^2 + \varphi^2\right]$$

$$= \left(\frac{a}{2}\right)^2 \left[2 - 2\varphi + 2\varphi^2\right]$$

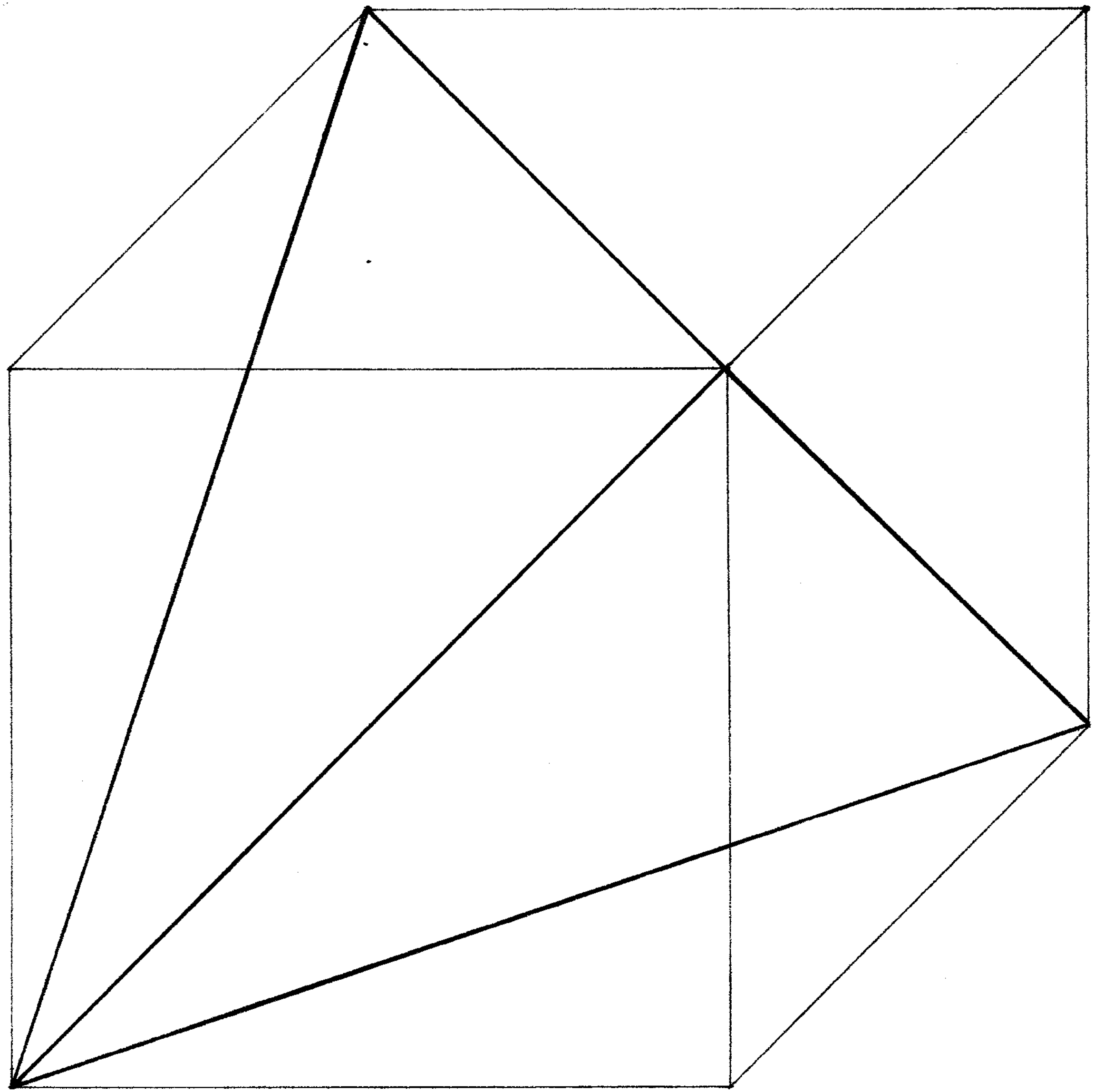
Für den goldenen Schnitt gilt $1-\varphi = \varphi^2$ (siehe 1. Übung)

also $2 - 2\varphi = 2\varphi^2$

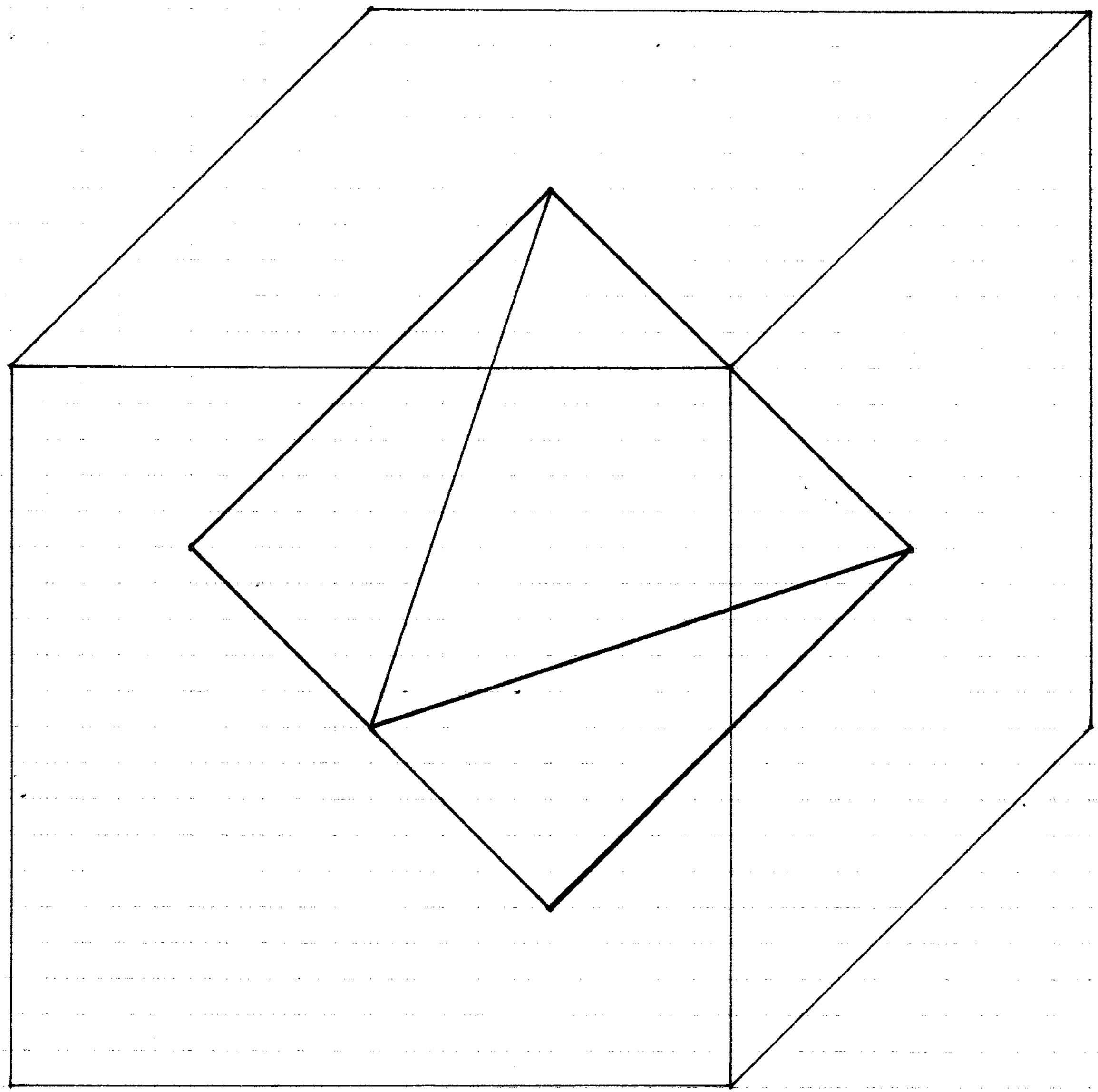
$$|BD|^2 = \left(\frac{a}{2}\right)^2 4\varphi^2$$

also $|BD| = \frac{a}{2} \cdot 2\varphi = a \cdot \varphi = k$ q.e.d.

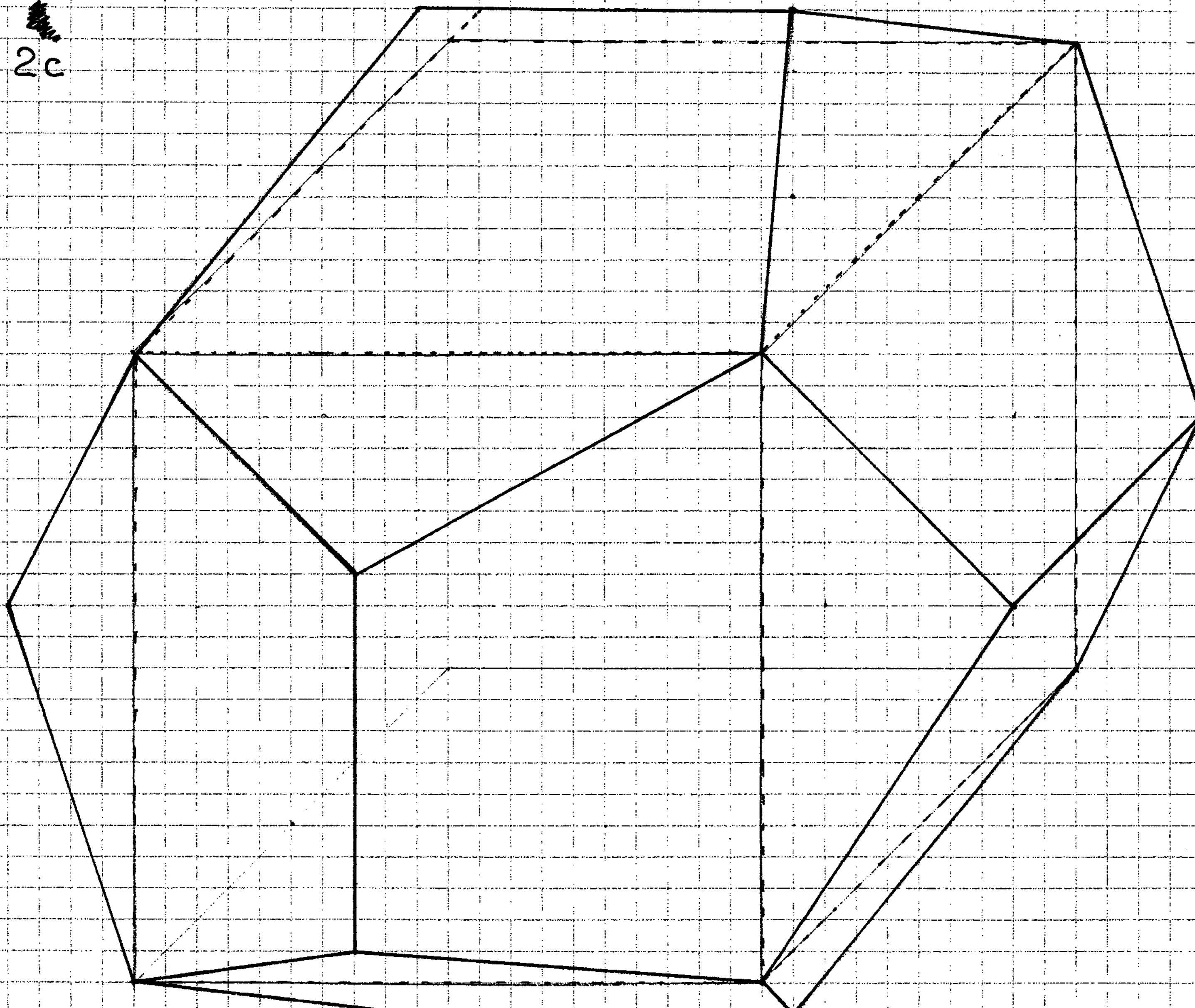
2a)



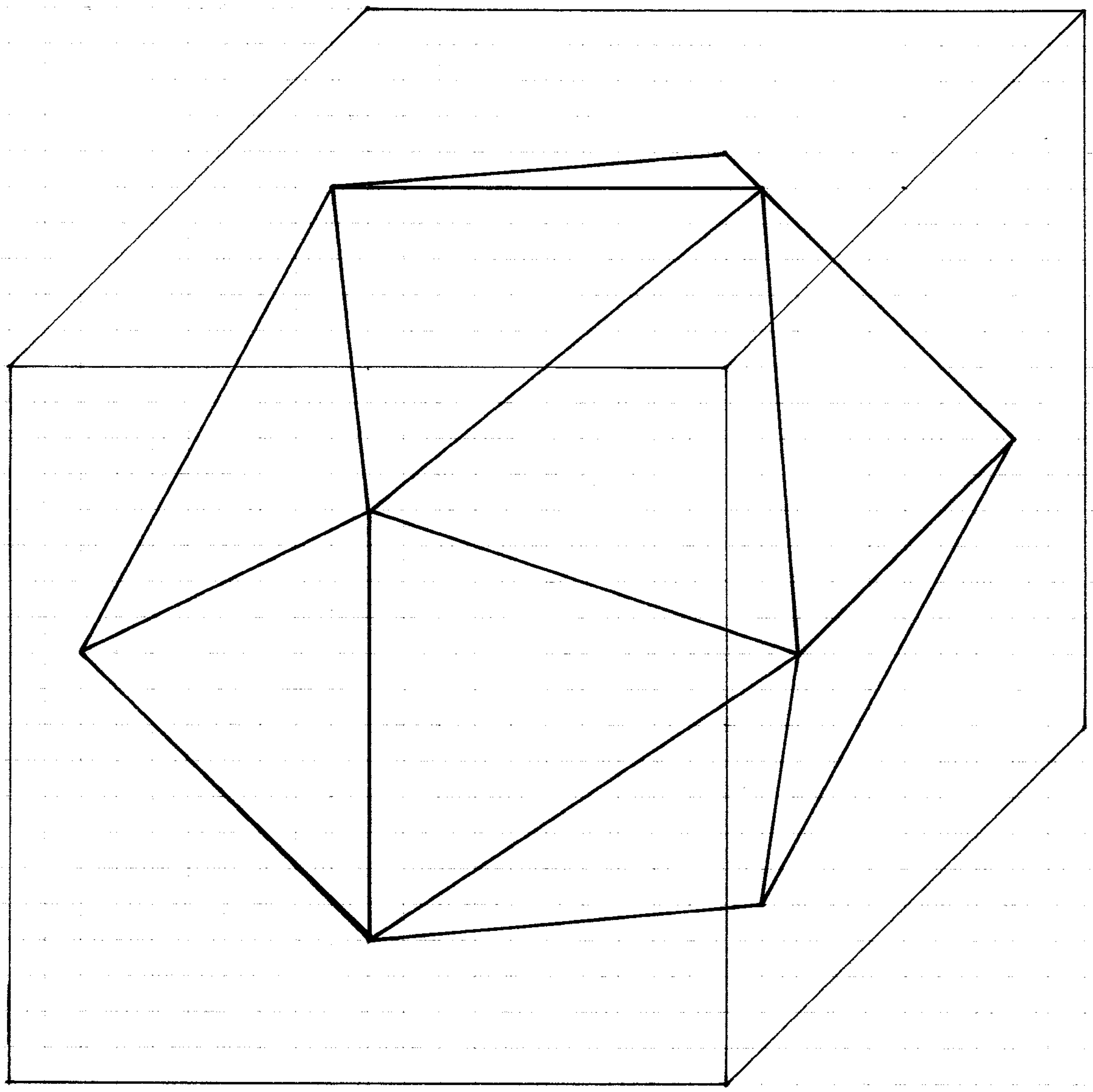
2b



2c

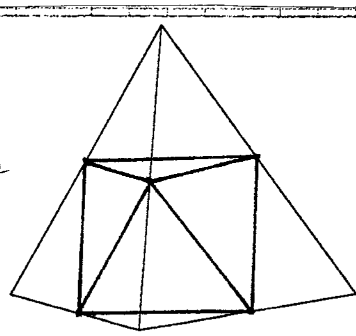


2 d)

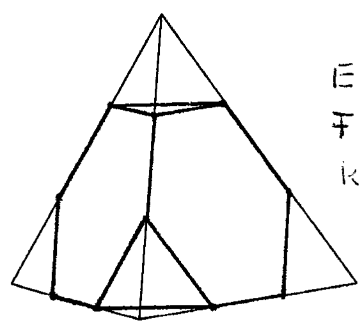


3

E 6
F 8 8 3-Ecke
K 12



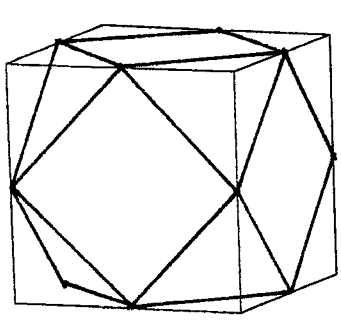
(3,3,3,3) Oktaeder



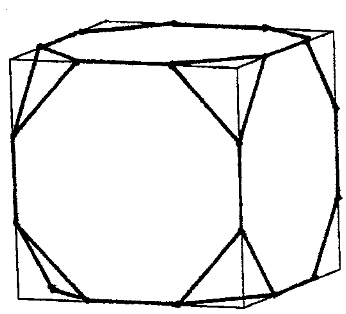
(3,6,6)

E 12
F 8 { 4 6-Ecke, 4 3-Ecke }
K 18

E 12
F 14 { 6 4-Ecke, 8 3-Ecke }
K 24



(3,4,3,4)

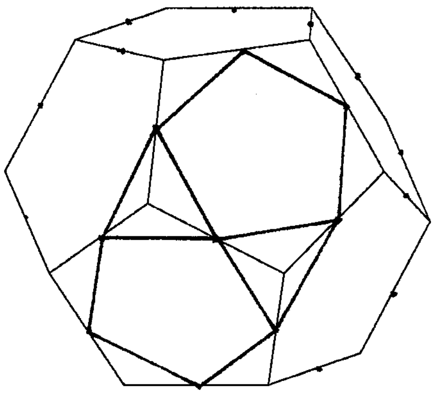


(3,8,8)

E 24
F 14
8 3-Ecke
6 8-Ecke
K 36

E 30
F 32
K 60

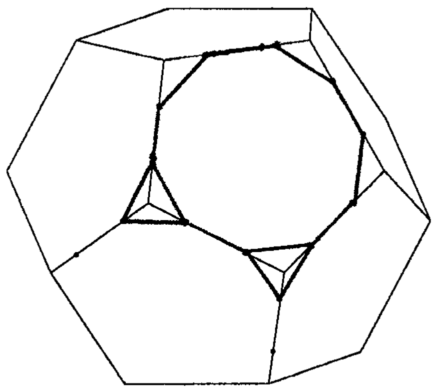
12 5-Ecke
20 3-Ecke



(3,5,3,5)

E 60
F 32
K 90

12 10-Ecke
20 3-Ecke

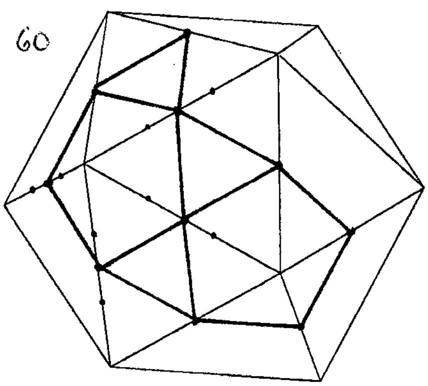


(3, 10, 10)

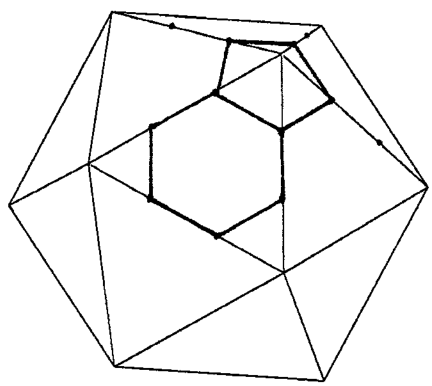
nur teilweise

E 30
F 32
K 60

20 3-Ecke
12 5-Ecke



(3,5, 3,5)



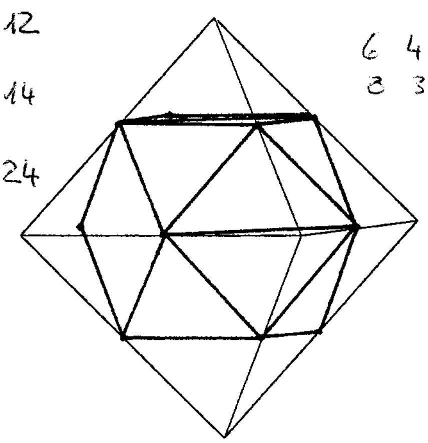
E 60
F 32
K 90

20 6-Ecke
12 5-Ecke

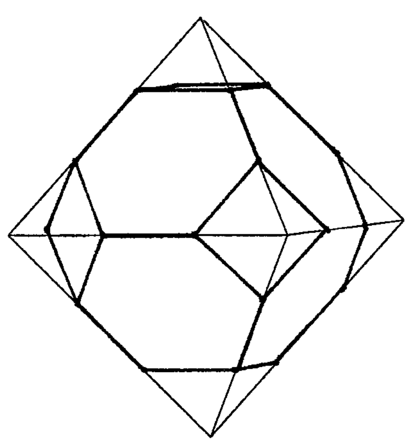
(5, 6, 6) Fußball

E 12
F 14
K 24

6 4-E
8 3-E



(3, 4, 3, 4)



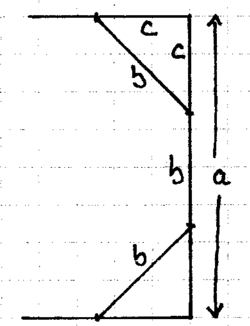
E 24
F 14
K 36

6 4-E
8 6-E

(4, 6, 6)

4. a) $2c + b = a$ ①

$c^2 + c^2 = b^2$ ②



Ziel: b und c als Funktion von a ausdrücken

① $\Rightarrow$ $b = a - 2c$ ② $\Rightarrow$ $2c^2 = b^2$

$$b^2 = a^2 - 4ac + 4c^2 = 2c^2$$

$$2c^2 - 4ac + a^2 = 0 \quad | :2$$

$$c^2 - 2ac + \frac{1}{2}a^2 = 0$$

pq-Formel:

$$c = a \pm \sqrt{a^2 - \frac{1}{2}a^2} = a\left(1 \pm \sqrt{\frac{1}{2}}\right)$$

Die geometrisch sinnvolle Lösung ist

$$c = a\left(1 - \sqrt{\frac{1}{2}}\right) = a\left(1 - \frac{1}{2}\sqrt{2}\right) \approx 0{,}293\,a$$

$$b = a - 2c = a - 2a + a\sqrt{2} = a\left(\sqrt{2} - 1\right) \approx 0{,}414\,a$$

b)