Zu Blatt 1

Aufg. 1

a) $a_n = n^2$

$\Downarrow$

$a_{n-1} = (n-1)^2$

$\Downarrow$

$\sqrt{a_{n-1}} = (n-1)$

$$a_n = a_{n-1} + 2\underbrace{(n-1)}_{=\sqrt{a_{n-1}}} + 1 = \underline{\underline{a_{n-1} + 2\sqrt{a_{n-1}} + 1}} \quad \text{(ohne } n\text{)}$$

b) $a_n = \frac{n+1}{n}$ $\Rightarrow$ $a_n = 1 + \cfrac{1}{1 + \cfrac{1}{a_{n-1} - 1}}$

$a_1 = \frac{1+1}{1} = \frac{2}{1} = 2$

$a_2 = \frac{2+1}{2} = \frac{3}{2} = 1\frac{1}{2}$

$a_3 = \frac{3+1}{3} = \frac{4}{3} = 1\frac{1}{3}$

$a_4 = \frac{4+1}{4} = \frac{5}{4} = 1\frac{1}{4}$

$a_1 = 2$

$a_2 = 1 + \cfrac{1}{1 + \cfrac{1}{\underbrace{a_1}_{2} - 1}} = 1 + \frac{1}{1+1} = 1\frac{1}{2} \quad (= \frac{3}{2})$

$a_3 = 1 + \cfrac{1}{1 + \cfrac{1}{\underbrace{a_2}_{\frac{3}{2}} - 1}} = 1 + \frac{1}{1+2} = 1\frac{1}{3} \quad (= \frac{4}{3})$

$a_4 = 1 + \cfrac{1}{1 + \cfrac{1}{\underbrace{a_3}_{\frac{4}{3}} - 1}} = 1 + \frac{1}{1+3} = 1\frac{1}{4} \quad (= \frac{5}{4})$

Aufg. 2

a)

F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}
1	1	2	3	5	8	13	21	34	55

b)

$q_1 = \frac{F_2}{F_1} = \frac{1}{1} = 1$

$q_2 = \frac{F_3}{F_2} = \frac{2}{1} = 2$

$q_3 = \frac{F_4}{F_3} = \frac{3}{2} = 1{,}5$

$q_4 = \frac{F_5}{F_4} = \frac{5}{3} = 1{,}666\ldots$

$q_5 = \frac{F_6}{F_5} = \frac{8}{5} = 1{,}6$

$q_6 = \frac{F_7}{F_6} = \frac{13}{8} = 1{,}625$

$q_7 = \frac{F_8}{F_7} = \frac{21}{13} \approx 1{,}6153846$

$q_8 = \frac{F_9}{F_8} = \frac{34}{21} \approx 1{,}6190476$

$q_9 = \frac{F_{10}}{F_9} = \frac{55}{34} \approx 1{,}6176471$

$\downarrow$

geht alternierend gegen den goldenen Schnitt

$\frac{1+\sqrt{5}}{2} \approx 1{,}618034$

c) $S_1 = F_1 = 1$ $\qquad$ $S_1 = F_3 - 1 = 2 - 1 = 1$

$S_2 = F_1 + F_2 = 1 + 1 = 2$ $\qquad$ $S_2 = F_4 - 1 = 3 - 1 = 2$

$S_3 = F_1 + F_2 + F_3 = 1 + 1 + 2 = 4$ $\qquad$ $S_3 = F_5 - 1 = 5 - 1 = 4$

$S_4 = F_1 + \ldots + F_4 = 1 + 1 + 2 + 3 = 7$ $\qquad$ $S_4 = F_6 - 1 = 8 - 1 = 7$

$S_5 = S_4 + F_5 = 7 + 5 = 12$ $\qquad$ $S_5 = F_7 - 1 = 13 - 1 = 12$

$S_6 = S_5 + F_6 = 12 + 8 = 20$ $\qquad$ $S_6 = F_8 - 1 = 21 - 1 = 20$

$S_7 = S_6 + F_7 = 20 + 13 = 33$ $\qquad$ $S_7 = F_9 - 1 = 34 - 1 = 33$

$S_8 = S_7 + F_8 = 33 + 21 = 54$ $\qquad$ $S_8 = F_{10} - 1 = 55 - 1 = 54$

$S_9 = S_8 + F_9 = 54 + 34 = 88$ $\qquad$ $(S_9 = F_{11} - 1 = 89 - 1 = 88)$

$S_{10} = S_9 + F_{10} = 88 + 55 = 143$ $\qquad$ $(S_{10} = F_{12} - 1 = 144 - 1 = 143)$

$\boxed{\text{Regelmäßigkeit: } S_n = F_{n+2} - 1}$

Beweis per Induktion:

I.A.: $n=1$: $\quad S_1 = F_{1+2} - 1 = F_3 - 1 = 2 - 1 = 1$ ✓ (s.o.)

I.S.: $n \to n+1$:

I.V.: Für $n \in \mathbb{N}$ beliebig aber fest gelte: $S_n = F_{n+2} - 1$.

z.z.: $S_{n+1} = F_{n+3} - 1$

Es gilt: $S_{n+1} = \underbrace{F_1 + F_2 + \ldots + F_n}_{=S_n} + F_{n+1} = S_n + F_{n+1} \overset{\text{I.V.}}{=} \underbrace{F_{n+2} - 1 + F_{n+1}}_{F_{n+3}}$

$= F_{n+3} - 1$. ✓ $\square$

Aufg. 3

a)
$$\begin{aligned} x &= 2 \overset{+5}{+} 7 \overset{+5}{+} 12 \overset{+5}{+} 17 + \ldots \overset{+5}{+} 92 + 97 \\ x &= 97 + 92 + 87 + 82 + \ldots + 7 + 2 \quad \text{(andersherum geschrieben)} \\ \hline 2x &= \underbrace{99 + 99 + 99 + 99 + \ldots + 99 + 99}_{20 \text{ mal } = [97:5]+1 \text{ mal } = 95:5+1 \text{ mal } = 19+1 \text{ mal}} = 20 \cdot 99 \end{aligned}$$

also: $2x = 20 \cdot 99 \Rightarrow x = 10 \cdot 99 = \underline{\underline{990}}$

b) $x = 5 + 10 + 15 + 20 + \ldots + 95 + 100 - 20 \cdot 3$ ← (überall, d.h. 20 mal "+3" dazu)

$= 5 \cdot \underbrace{(1 + 2 + 3 + 4 + \ldots + 19 + 20)}_{\frac{20(20+1)}{2} = 10 \cdot 21 = 210} - \underbrace{20 \cdot 3}_{60}$

$= 5 \cdot 210 - 60 = 1050 - 60 = \underline{\underline{990}}$

Aufg. 4

a) c cis d dis e f fis g gis a b h c'

$$c \xrightarrow{1} cis \xrightarrow{2} d \xrightarrow{3} dis \xrightarrow{4} e \xrightarrow{5} f \xrightarrow{6} fis \xrightarrow{7} g \xrightarrow{8} gis \xrightarrow{9} a \xrightarrow{10} b \xrightarrow{11} h \xrightarrow{12} c'$$

$$1 \xrightarrow{\cdot q} 2^{\frac{1}{12}} \xrightarrow{\cdot q} 2^{\frac{2}{12}} \xrightarrow{\cdot q} 2^{\frac{3}{12}} \quad \dots \quad \xrightarrow{\cdot q} 2 = 2^{\frac{12}{12}} = \left(\sqrt[12]{2}\right)^{12}$$

$\cdot q \leftarrow 12$ mal

$\sqrt[12]{2} = 2^{1/12}$

gesucht: q?

$$\boxed{\text{Lösung: } q = \sqrt[12]{2} = 2^{\frac{1}{12}}}$$

b) $c \to g$ 7 Schritte $\Rightarrow \underbrace{\sqrt[12]{2} \cdot \sqrt[12]{2} \cdot \ldots \cdot \sqrt[12]{2}}_{7 \text{ mal}} = \left(\sqrt[12]{2}\right)^7 = 2^{\frac{7}{12}}$

$\approx 1{,}4983071$

Grundton c: 600 Hz

exakte / reine Stimmung: $600\,\text{Hz} \cdot 1{,}5 = 900\,\text{Hz}$

temperierte Stimmung: $\underbrace{\left(\sqrt[12]{2}\right)^7}_{2^{\frac{7}{12}}} \cdot 600\,\text{Hz} \approx 898{,}98425\,\text{Hz}$

$\Rightarrow$ Unterschied: $900\,\text{Hz} - 898{,}98425\,\text{Hz} = 1{,}0157539\,\text{Hz}$

[redacted]