‘Where, what and how’ - specifying gestures as a pathway to mathematics?

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While in many disciplines students deal with objects that are directly accessible, the case is crucially different for mathematics: Mathematical objects are not tangible. Although their handling can only be mediated by representations (Duval 2000, 61) they must not be confused with them (Duval 2006, 107): Mathematical objects are abstract individual constructions. In math classes mathematical objects often are established within social interactions, even without the teacher. This raises the question how a shared understanding of such abstract concepts among students is possible. A shared understanding may be accomplished by establishing ‘conceptual pacts’ (Brennan & Clark 1996), that is, shared conceptualizations in discursive situations. Research has already shown *that g*estures take major part in this (Arzarello, Paola, Robutti & Sabena 2009, Edwards 2005, Radford 2003, Goldin-Meadow 2003, 2010, Goldin-Meadow, Cook & Mitchell 2009, Cook 2011) but little is known about *how* this happens.

In the current study, we research the issue of *how gestures contribute to building mathematical conceptual pacts on a highly abstract level.*

This has been investigated by an empirical study by means of teaching experiments, which induced dense and highly abstract epistemic processes of building mathematical concepts. For this, three couples of high achieving students of grade 10 solved three problems from diverse and abstract mathematical areas. The epistemic processes of solving the problems, each lasting from 60 to 160 minutes, have been videotaped considering three perspectives in split-screen. All data has been transcribed focusing on verbal and non-verbal actions. The students’ mathematical discourse was analyzed using the semiotic sequence analysis (Bikner-Ahsbahs 2006, 161f.) in a fine-grained way, linking speech, representations and the use of gestures. An epistemic action model (Bikner-Ahsbahs & Halverscheid 2014) allowed identifying the crucial moments of building new mathematical entities by the epistemic action of *structure seeing,* the nucleus of conceptualization. The core units of gesture analysis concerned *strokes and post-stroke holds* (McNeill 1992, p.25, Kendon 2004, 112) and their relations to other semiotic resources in the current epistemic process.

Results show that establishing shared conceptualization benefits from specification gestures, i.e. gestures that provide information additional to that conveyed in speech. This is done by gestures specifying aspects of mathematical objects within a certain representation. Our analyses reveal four different kinds of specifying gestures: specifying aspects of mathematical objects through location, sort and style in the representations at hand, and specifying relations between mathematical entities as displayed. These specifying gestures and the mathematical concepts under construction can be shared on three referential levels, the one of inscription, the one of the interaction space with indexical gestural reference to the inscription, and finally the level of the mathematical gesture space in which mathematical meaning is blended from inscription (Yoon, Thomas & Dreyfus 2011).

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