Restricted diffusion and related inverse problems

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Abstract

We present recent advances in studying the dynamics of nuclei diffusing in a bounded domain with a (partially) reflecting boundary. Application of inhomogeneous magnetic fields allows one to "encode" stochastic trajectories of the nuclei and therefore to monitor the dynamics experimentally. This so-called diffusion-weighted imaging has found numerous applications in medicine and oil-recovery industry.

In probabilistic terms, this is equivalent to studying an integral functional of reflected Brownian motion. In PDE context, the problem is described by diffusion (or heat) equation with a complex-valued spatially inhomogeneous reaction rate. Using the Laplace operator eigenbasis, we retrieve perturbative results in the short-time and long-time limits. We also consider the related inverse problems and discuss what information about an unknown geometry of the confinement can be extracted from such a diffusion-weighted imaging?

Given the spectral formulation, this inverse problem revives the Kac's question "Can one hear the shape of a drum?" in a new context.

Reference


Alle Interessenten sind herzlich eingeladen!