Einladung zum

MATHEMATISCHEN KOLLOQUIUM


spricht

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über

The Horn problem and its refinements

The Horn problem (although I.M. Gel’fand stated this problem much earlier) deals with the following question: What can we say about the spectrum of the sum of two Hermitian (or real symmetric) matrices if we known the spectra of $A$ and $B$? The spectrum of a Hermitian matrix, that is the set of its eigenvalues, consists of real numbers, which are usually arranged in decreasing order. Let $A$ have spectrum $\alpha = (\alpha(1) \geq \alpha(2) \geq \ldots \geq \alpha(n))$ (where $n$ is the size of the matrices), and let $B$ and $C = A + B$ have spectra $\beta$ and $\gamma$ of a similar form. In 1962, Horn conjectured that the set $H(\alpha, \beta)$ of possible values of $\gamma$ was a convex polyhedron specified by linear inequalities of the form

$$\sum_{k \in K} \gamma(k) \leq \sum_{i \in I} \alpha(i) + \sum_{j \in J} \beta(j),$$

where $I$, $J$ and $K$ are some subsets (of the same size) in $[n]$. Horn suggested a recursive procedure for constructing such triples $(I, J, K)$. The Horn problem was solved in 1996 by Klyachko modulo Saturation Conjecture and the latter was solved by Knutson and Tao in 1998.

I will suggest a somewhat different statement of the solution to the Horn problem and a significantly more elementary proof. Specifically, we prove that the existence of the required triple of matrices $(A, B, C)$ for given $(\alpha, \beta, \gamma)$ is equivalent to the existence of a so-called discretely concave function on the triangular grid $\Delta(n)$ with boundary value-increments $\alpha$, $\beta$, and $\gamma$. Our proof involves only one non-elementary issue, namely, a reference to the convexity of the image of a moment mapping. The Horn original linear inequalities can be obtained from this form of the Horn problem by fairly elementary (but non-trivial) means of linear programming. Several refinements of the Horn problem will be proposed (some are still open problems). Reported results are obtained jointly with V.Danilov.

Der Vortrag findet statt um 17 Uhr c.t. im Raum 7260, 7. Ebene des Mehrzweckhochhauses (MZH) der Universität Bremen, Bibliothekstr.
Zuvor gibt es Kaffee/Tee und Gebäck im Raum 7140.

Alle Interessierten sind herzlich eingeladen.
Marc Keßeböhmer als Kolloquiumsbeauftragter.