The CoShREM Toolbox Parameter Guide

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PARAMETERS OF THE SHEARLET SYSTEM	
waveletEffSupp (see Figure 4)	Length of the effective support in pixels of the Mexican hat wavelet ψ used in the construction the generating shearlet $\psi^{\text{gen}}(x,y) = \psi(x)\phi(y)$, where ϕ is a Gaussian. The effective support is the interval on which the values of ψ significantly differ from 0. It is, however, not a strictly defined property. A good choice for this parameter is often **1/8 of the image width**. If the edges/ridges in the processed image are visible on a large scale, this value should be large relative to the width and height of the processed image.
gaussianEffSupp (see Figure 5)	Length of the effective support in pixels of the Gaussian ϕ used in the construction of the generating shearlet $\psi^{\text{gen}}(x,y) = \psi(x)\phi(y)$, where ψ is a Mexican hat wavelet. Typically, this value is chosen to be **roughly the half of *waveletEffSupp***. However, if the edges/ridges in the processed image consist of smooth curves, it can be chosen larger.
scalesPerOctave (see Figure 6)	Determines the number of intermediate scales for each octave. If *scalesPerOctave* is set to n, for each orientation, there will be n differently scaled shearlets within one octave. **2 is typically a good choice for this parameter.**
shearLevel (orientations) (see Figure 7)	Determines the number of differently oriented shearlets on each scale. If *shearLevel* is set to n, there will be $2^n + 2$ differently sheared shearlets on each scale, completing a 180° semi-circle. **A sufficient choice for this parameter is typically 3.**
alpha (orientations) (see Figures 2 and 8)	This parameter can take any value between 0 and 1 and governs the degree of anisotropy introduced via scaling. Roughly speaking, it determines how much the Gaussian is squeezed relative to the wavelet, when scaling the generating shearlet. Formally, the n-th octave is defined by $\psi_n(x,y) = \psi^{\text{gen}}(2^n x, 2^{\alpha n} y)$. For $alpha = 0$, the degree of anisotropy is maximized while for $alpha = 1$, both directions are treated the same. **The default choice is 0.5**.
octaves	The number of octaves spanned by the shearlet system. When *scalesPerOctave* is greater than 1, this parameter can also take non-integer values. **A standard choice is 3.5**.

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PARAMETERS OF THE EDGE AND RIDGE MEASURES	
minContrast (see Figure 1)	Specifies the minimal contrast for an edge/ridge to be detected. This parameter can also be seen as a soft threshold. That is, in the CoShREM toolbox, the complex shearlet-based edge measure is implemented as $$\mathrm{E}(f,x) = \frac{\left|\sum_{a\in A}\langle f,\psi^{\mathrm{odd}}_{a,x}\rangle\right| - \sum_{a\in A}\left|\langle f,\psi^{\mathrm{even}}_{a,x}\rangle\right|}{|A|\max_{a\in A}\left|\langle f,\psi^{\mathrm{odd}}_{a,x}\rangle\right|} - \frac{\boldsymbol{minContrast}}{\max_{a\in A}\left|\langle f,\psi^{\mathrm{odd}}_{a,x}\rangle\right|},$$ where $A \subset \mathbb{R}$ is a set of scaling parameters implicitly defined by *octaves* and *scalesPerOctave*. **For a 0-255 grayscale image, a standard choice for this parameter is 4**.
offset	This parameter defines a scaling offset between the even- and odd-symmetric shearlets measured in octaves. If *offset* $= x$, the first even-symmetric shearlet used for the computation of the complex shearlet-based edge measure is already x octaves above the first odd-symmetric shearlet considered. In the case of the ridge measure, the converse is true. ***offset* = 1 is often a good choice.**
scalesUsedForPivotSearch (see Figure 3)	This parameter defines which scales of the shearlet system are considered for determining the orientation for which the complex shearlet-based edge/ridge measure is computed at a specific location. It can take the values 'all', 'highest', 'lowest' and any subset $B \subset \{1,\dots,$ *scalesPerOctave*$\cdot$*octaves*$\}$. **The default choice is 'all'.** If *scalesUsedForPivotSearch* is unequal to 'all', the computation of the complex shearlet-based edge measure changes to $$\mathrm{E}(f,x) = \frac{\left|\sum_{a\in A}\langle f,\psi^{\mathrm{odd}}_{a,x}\rangle\right| - \sum_{a\in A}\left|\langle f,\psi^{\mathrm{even}}_{a,x}\rangle\right|}{|A|\max_{a\in B}\left|\langle f,\psi^{\mathrm{odd}}_{a,x}\rangle\right|} - \frac{\boldsymbol{minContrast}}{\max_{a\in B}\left|\langle f,\psi^{\mathrm{odd}}_{a,x}\rangle\right|},$$ where $A \subset \mathbb{R}$ is a set of scaling parameters implicitly defined by *octaves* and *scalesPerOctave* and B is defined by *scalesUsedForPivotSearch*.

POST PROCESSING PARAMETERS	
thinningThreshold	Post processing converts the results of the complex shearlet-based edge/ridge measure to a binary image. Only pixels where the complex shearlet-based edge/ridge measure is greater than *thinningThreshold* will be included. **A good choice is typically 0.1 or 0.2**.

This following figures aim to illustrate the effects different parameter have on the constructed shearlet system and on the detection of edges/ridges. To this end, we consider a shearlet system obtained from parameters

waveletEffSupp:	70
gaussianEffSupp:	25
scalesPerOctave:	2
shearLevel (orientations):	3
alpha:	0.5
octaves:	3.5
minContrast:	4
offset:	1
scalesUsedForPivotSearch:	'all'

for images of size 512×512 and vary each parameter independently.

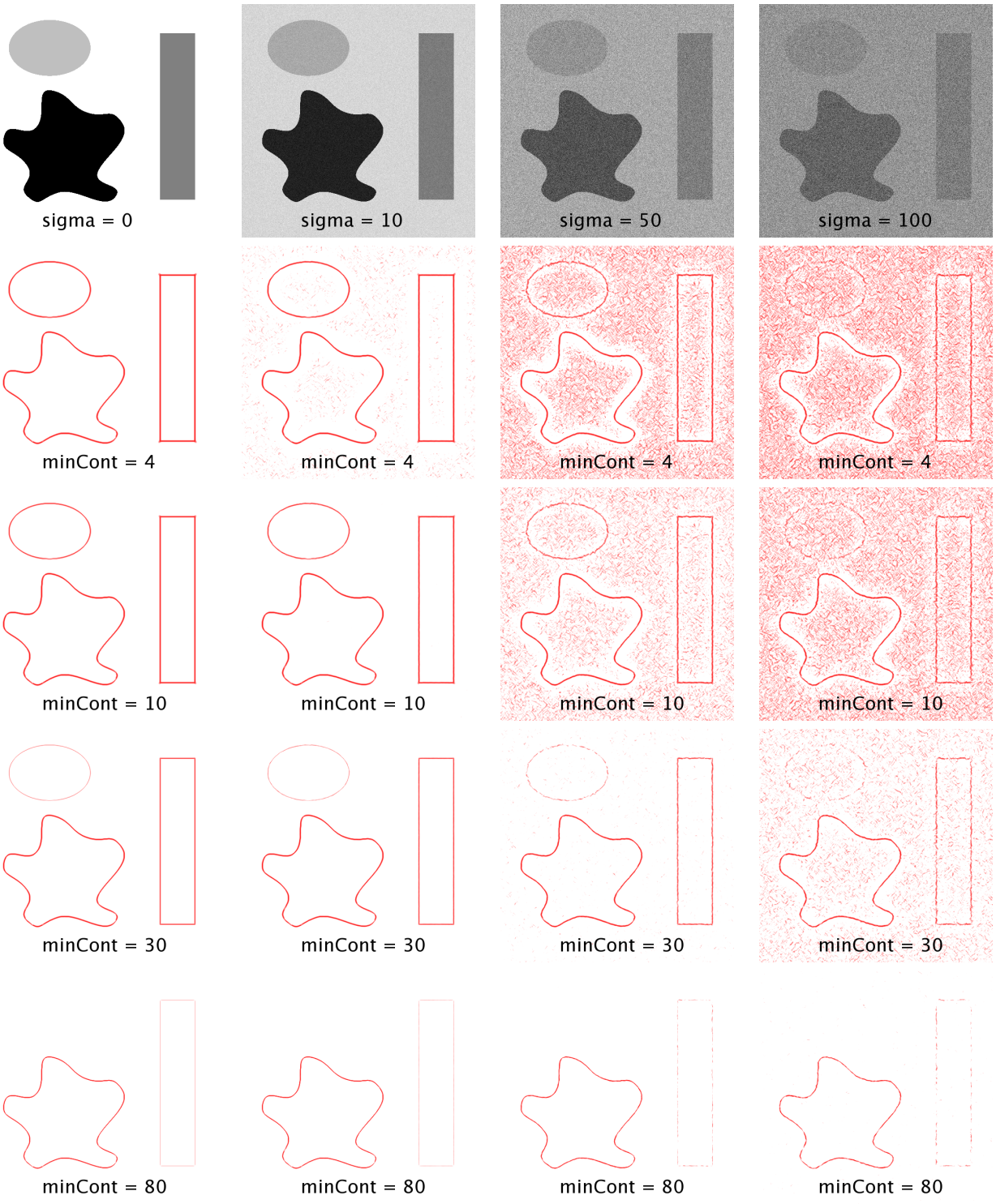

Figure 1: The relationship between *minContrast* and increasing levels of Gaussian noise.

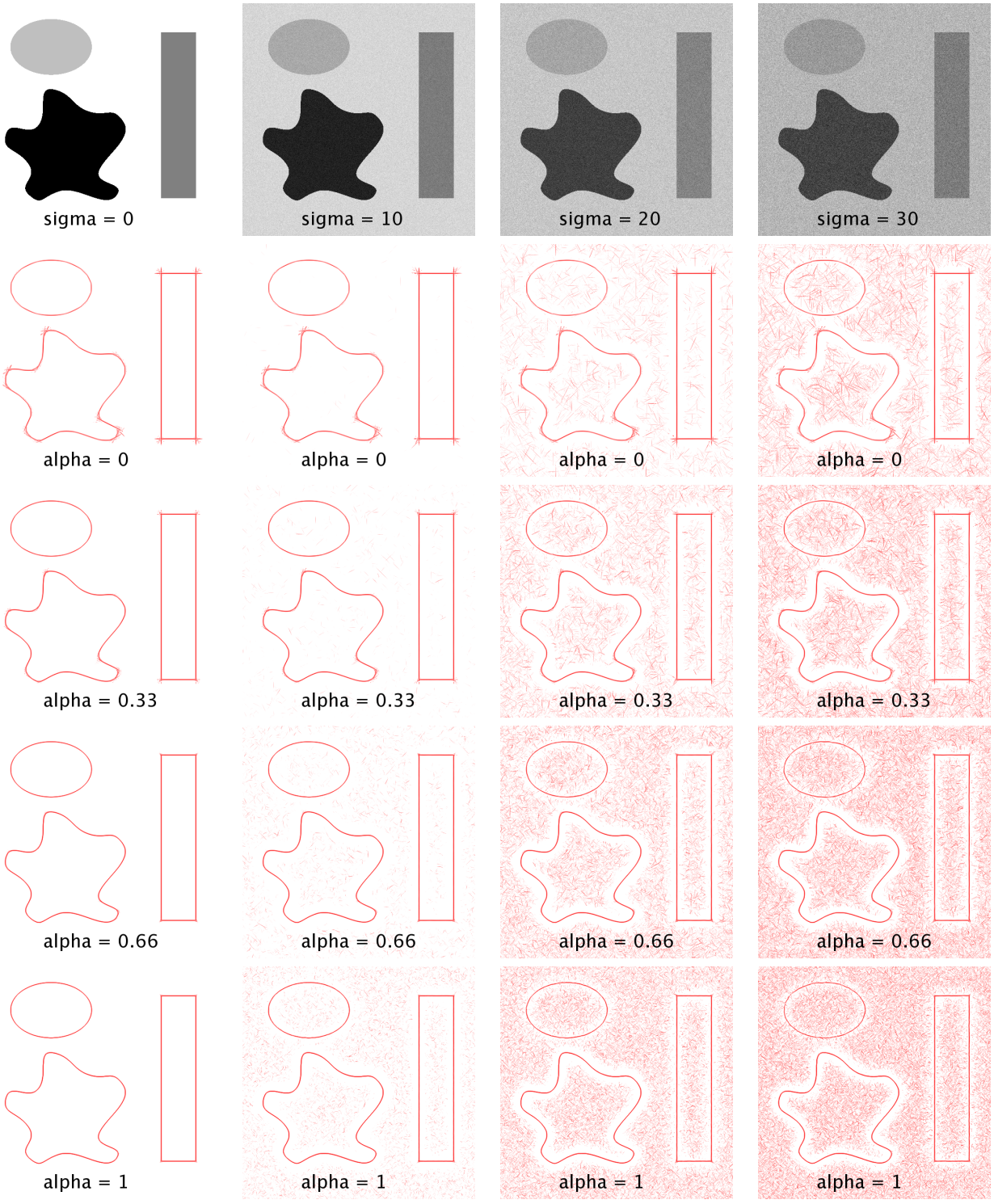

Figure 2: The relationship between *alpha* and increasing levels of Gaussian noise. Please note that to enhance the effect of different alphas, *gaussianEffSupp* was changed to 70.

Figure 3: The relationship between *scalesUsedForPivotSearch* and increasing levels of Gaussian noise.

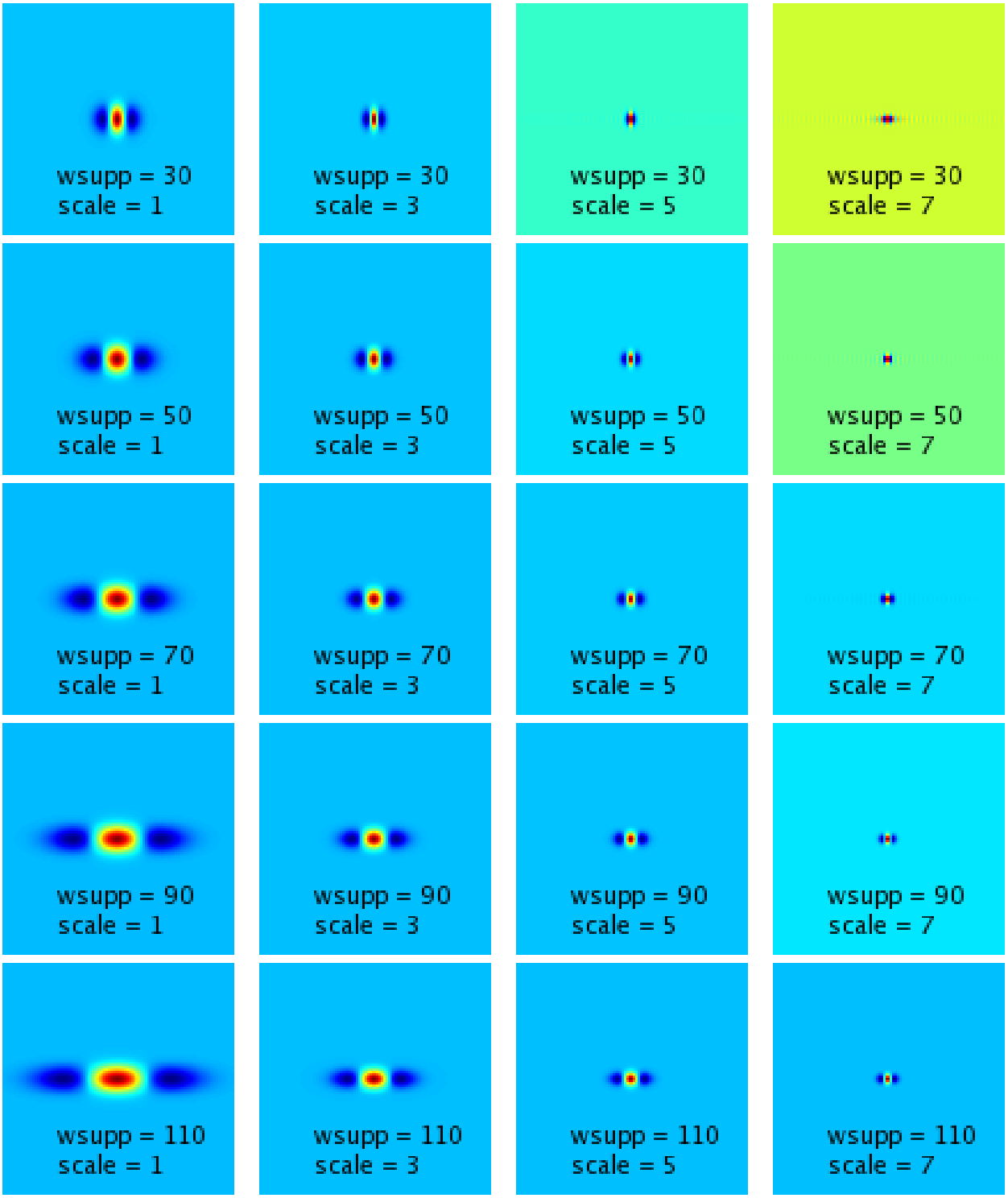

Figure 4: The relationship between *waveletEffSupp* and the even-symmetric elements of the constructed complex shearlet system.

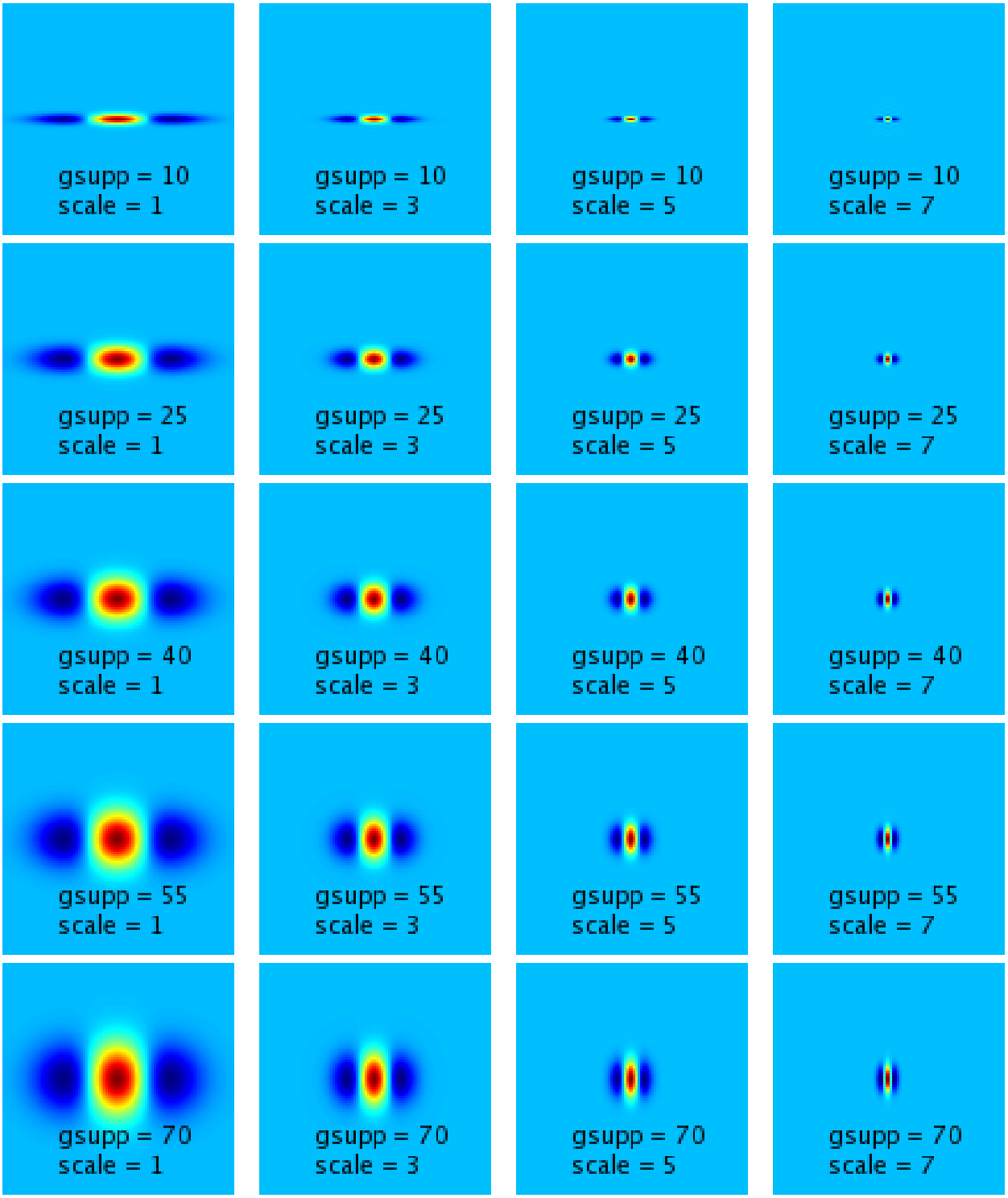

Figure 5: The relationship between *gaussianEffSupp* and the even-symmetric elements of the constructed complex shearlet system.

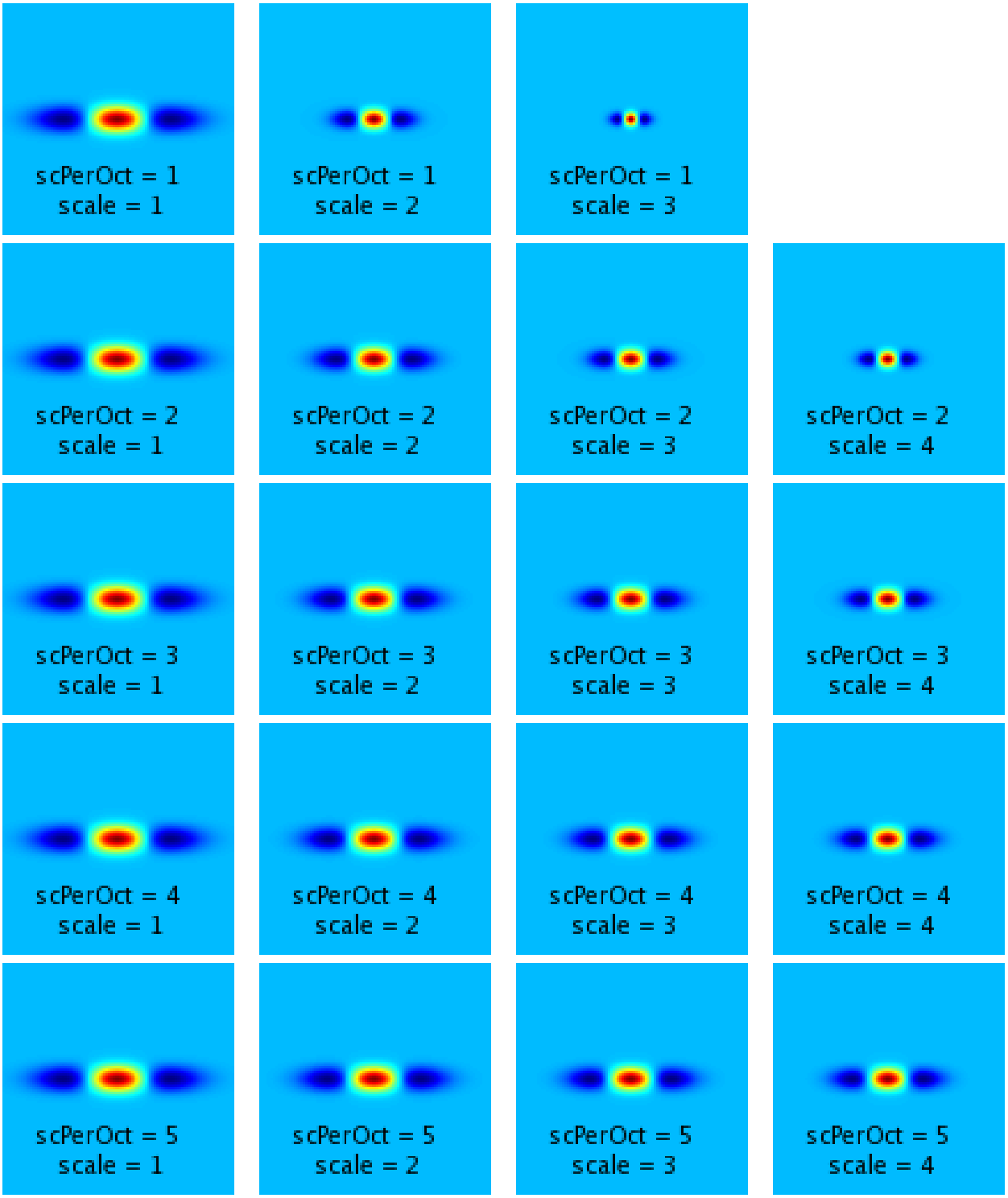

Figure 6: The relationship between *scalesPerOctave* and the even-symmetric elements of the constructed complex shearlet system.

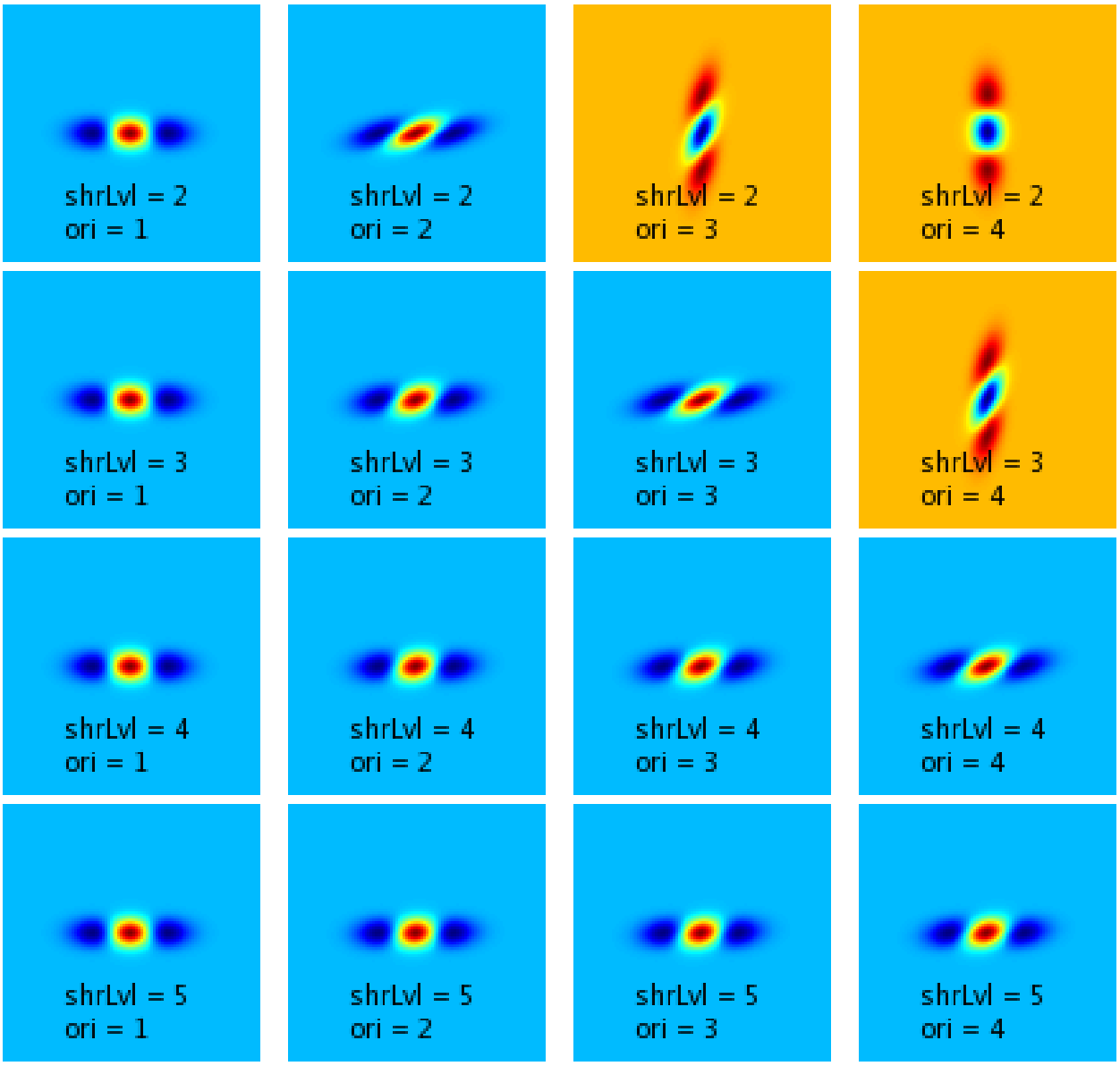

Figure 7: The relationship between *shearLevel* and the even-symmetric elements of the constructed complex shearlet system.

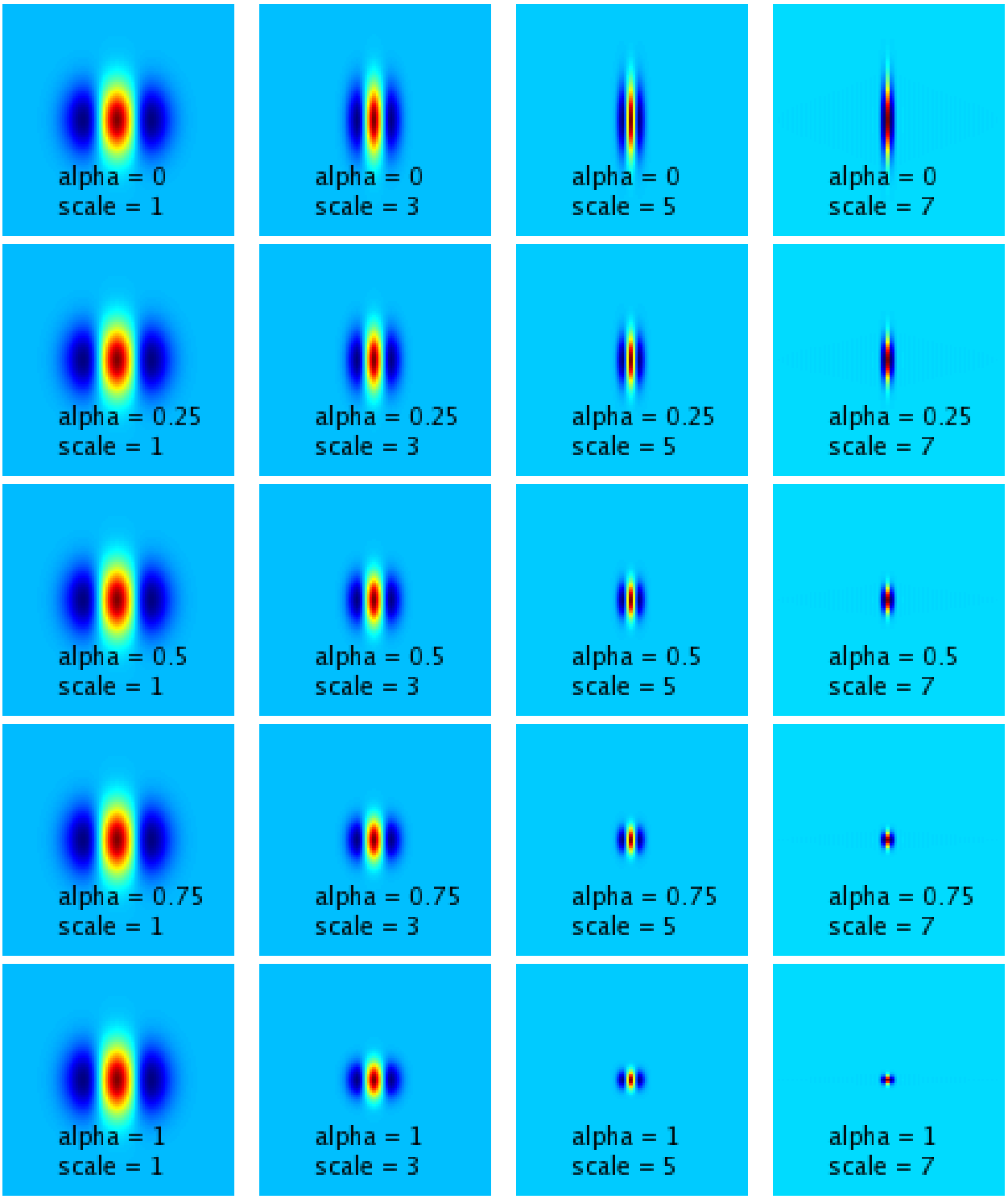

Figure 8: The relationship between *alpha* and the even-symmetric elements of the constructed complex shearlet system. Please note that to enhance the effect of different alphas, *gaussianEffSupp* was changed to 70.