Wichtige Ableitungen und Integrale. (Prof. B.O. Stratmann)

Ableitungen.

$\frac{d(\sin x)}{dx} = \cos x$
$\frac{d(\cos x)}{dx} = -\sin x$
$\frac{d(\tan x)}{dx} = \frac{1}{\cos^2 x} = 1 + \tan^2 x$ (Erinnerung: $\tan x := \frac{\sin x}{\cos x}$)
$\frac{d(\cot x)}{dx} = \frac{-1}{\sin^2 x} = -(1 + \cot^2 x)$ (Erinnerung: $\cot x := \frac{\cos x}{\sin x}$)
$\frac{d(\sec x)}{dx} = \sec x \tan x$ (Erinnerung: $\sec x := \frac{1}{\cos x}$)
$\frac{d(\csc x)}{dx} = -\csc x \cot x$ (Erinnerung: $\csc x := \frac{1}{\sin x}$)
$\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$
$\frac{d(\cos^{-1} x)}{dx} = \frac{-1}{\sqrt{1-x^2}}$
$\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$
$\frac{d(\sec^{-1} x)}{dx} = \frac{1}{x\sqrt{x^2-1}}$
$\frac{d(\sinh x)}{dx} = \cosh x$
$\frac{d(\cosh x)}{dx} = \sinh x$
$\frac{d(\tanh x)}{dx} = \frac{1}{\cosh^2 x}$ (Erinnerung: $\tanh x := \frac{\sinh x}{\cosh x}$)
$\frac{d(\coth x)}{dx} = \frac{-1}{\sinh^2 x}$ (Erinnerung: $\coth x := \frac{\cosh x}{\sinh x}$)
$\frac{d(\text{sech}\, x)}{dx} = -\text{sech}^2\, x \tanh x$ (Erinnerung: $\text{sech}\, x := \frac{1}{\cosh x}$)
$\frac{d(\text{csch}\, x)}{dx} = -\text{csch}\, x \coth x$ (Erinnerung: $\text{csch}\, x := \frac{1}{\sinh x}$)
$\frac{d(\sinh^{-1} x)}{dx} = \frac{1}{\sqrt{1+x^2}}$
$\frac{d(\cosh^{-1} x)}{dx} = \frac{1}{\sqrt{x^2-1}} \quad (x > 1)$
$\frac{d(\tanh^{-1} x)}{dx} = \frac{1}{1-x^2} \quad (|x| < 1)$
$\frac{d(\text{sech}^{-1} x)}{dx} = \frac{-1}{x\sqrt{1-x^2}} \quad (0 < x < 1)$

Integrale.

$\int \sin x \; dx = -\cos x + c$
$\int \cos x \; dx = \sin x + c$
$\int \frac{1}{\sin^2 x} \; dx = -\cot x + c$
$\int \frac{1}{\cos^2 x} \; dx = \tan x + c$
$\int \sec x \tan x \; dx = \sec x + c$
$\int \csc x \cot x \; dx = -cscx + c$
$\int \tan x \; dx = -\ln|\cos x| + c$
$\int \cot x \; dx = \ln|\sin x| + c$
$\int \sec x \; dx = \ln|\sec x + \tan x| + c$
$\int \csc x \; dx = \ln|\csc x - \cot x| + c$
$\int \sinh x \; dx = \cosh x + c$
$\int \cosh x \; dx = \sinh x + c$
$\int \frac{1}{\cosh^2 x} \; dx = \tanh x + c$
$\int \frac{1}{\sinh^2 x} \; dx = -\coth x + c$
$\int \frac{1}{\sqrt{a^2-x^2}} \; dx = \sin^{-1} \frac{x}{a} + c$
$\int \frac{1}{\sqrt{x^2+a^2}} \; dx = \sinh^{-1} \frac{x}{a} + c (= \ln|x + \sqrt{1+x^2}| + c) \; (a > 0)$
$\int \frac{1}{\sqrt{x^2-a^2}} \; dx = \cosh^{-1} \frac{x}{a} + c (= \ln|x + \sqrt{x^2-a^2}| + c) \; (0 < a < x)$
$\int \frac{1}{x\sqrt{a^2-x^2}} \; dx = -\frac{1}{a}\text{sech}^{-1} \frac{|x|}{a} + c, \; (0 < |x| < a)$
$\int \frac{1}{a^2+x^2} \; dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$
$\int \frac{1}{a^2-x^2} \; dx = \frac{1}{a} \tanh^{-1} \frac{x}{a} + c \; (|x| < a).$