Advanced Materials Research Vol. 223 (2011) pp 849-858
© (2011) Trans Tech Publications, Switzerland
doi:10.4028/www.scientific.net/AMR.223.849

Simulation Model for Micro-milling Operations and Surface Generation

Iwona Piotrowska-Kurczewski, 1[1,a] , Jost Vehmeyer[1,b]

Bremen University, Center for Industrial Mathematics, Bibliothekstrasse 1, 28359 Bremen, Germany

[a]iwona@math.uni-bremen.de, [b] vehmeyer@math.uni-bremen.de

Keywords: Micro cutting, modelling, surface generation.

Abstract. In this paper we propose a new mathematical model for micro milling operations. To achieve the desired quality of the final product or the desired structure on the product's surface the process kinematics as well as tool-workpiece interaction are considered. The presented model takes into account the relative motion between tool and workpiece. We consider the input infeed rate which is reduced by the elastic deflection of the tool due to the cutting forces appearing during the process. The tool wear and surface texture depend on the cutting force; therefore the analysis of the forces plays an important role in characterizing the cutting process. Moreover, the analyzing these forces during the simulation we can calculate the effective cross-sectional area of the cut in each time step of the process. This gives us a forward model for the full production chain. This model is extended in order to include a surface generation model as well as quality parameters for the resulting micro-milling surface.

Motivation

Micro material removal processes are of great importance in the production of precise micro molds with complex shapes for metal forming processes. Manufacturing of micro components provides a specific surface structure of molds which can enhance the tribological behavior during the forming processes. The surface structure in the forming zone can improve the material flow and consequently the quality of the end product. To optimize the production of the micro molds with respect to their tribological properties the mathematical simulations will be used. Because of the high quality demands of production processes in micro scale the mathematical model takes into account the process kinematics as well as the tool-workpiece interaction. In this paper we generalize the approach introduced in [1], for turning processes, to the micro milling processes. In order to predict the dependence of the surface kinematics of the end product on the process parameters, the surface simulation model was developed.

Matematical model

In this section we present the mathematical methods in the contexts of the modelling of the tool tip position in order to simulate the resulting surface for micro milling operations. In this case we analyze the ball milling cutter which is rotating and moving along the given path through the workpiece. The goal is to describe the tool path including the dynamical changes which occur during the process. We assume that the reader is familiar with kinematics of this process. Otherwise we refer the reader, for example to [2, 3]. We start with a description of the effects occuring during the micro milling process which affect the resulting surface.

Run-out. Mounting the tool into the tool holder usually leads to a small misalignment between the tool axis and the spindle rotation axis. This can easily be modelled with a static run-out vector

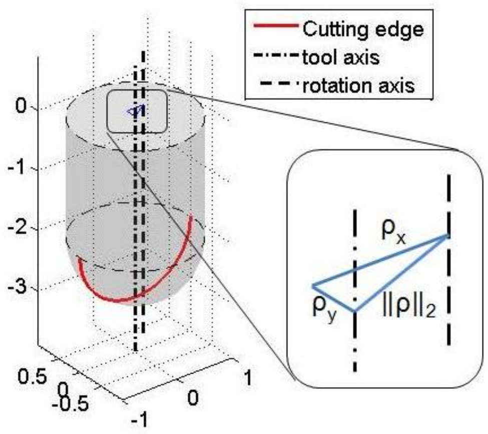

Fig. 1: Model of a 2-fluted ball-end tool, cutting edge and the rotation axis with parallel offset

$$\rho = (\rho_x, \rho_y, 0)^T \tag{1}$$

which describes a parallel tool axis offset of $\|\rho\|_2$ [mm] in specified direction (see Fig. 1). Then, the relative tool center of the tool tip can be written as

$$\begin{cases} m_x &= \|\rho_x\|_2 \sin(\phi - \gamma) \\ m_y &= \|\rho_y\|_2 \cos(\phi + \gamma) \\ m_z &= 0 \end{cases} \tag{2}$$

where ϕ is the rotation angle and γ is the run out angle (see Figure 1). In Figure 6 an example of the trajectory of the tool center is presented.

Force model. For the understanding of the machining process, the knowledge about the cutting forces is one of the most fundamental requirements. The modelling approach for the mechanical cutting force for macro-end milling [2, 4] was adopted in order to model the cutting force in micro milling operation. The force model is based on common approach that the force is proportional to the chip thickness h and the axial depth of cut ap. It can be extended by a second term which takes the frictional force into account, [2, 5, 4]. We have

$$\mathbf{F}^* = \mathbf{K_c} a_p h + \mathbf{K_e} a_p \tag{3}$$

where $\mathbf{F}^* = (F_r, F_t, F_a)^T$ is the vector of the force components in radial-, tangential- and axial-direction (see Fig. 2). The axial force is orthogonal to the (X, Y)-plane. $\mathbf{K_c} = (K_{rc}, K_{tc}, K_{ac})^T$ and $\mathbf{K_e} = (K_{re}, K_{te}, K_{ae})^T$ denote the vectors of cutting constants which are dependent on the material properties and have to be determined with help of measurements. Clearly, the force F^* is the sum of the force components on each cutting tooth. To develop the dynamical model for micro milling operations and calculate the actual model parameters we define the chip thickness $h(t)$ and the depth of cut $a_p(t)$ as the time depending functions such that we can integrate these parameters into the tool position model. Moreover, the forces cause a deflection of the micro tool. This is computed as the fraction of the force over the stiffness in each direction

$$\delta_r(t) = \frac{F_r(t)}{k_r}, \; \delta_t(t) = \frac{F_t(t)}{k_t}, \; \delta_a(t) = \frac{F_a(t)}{k_a} \tag{4}$$

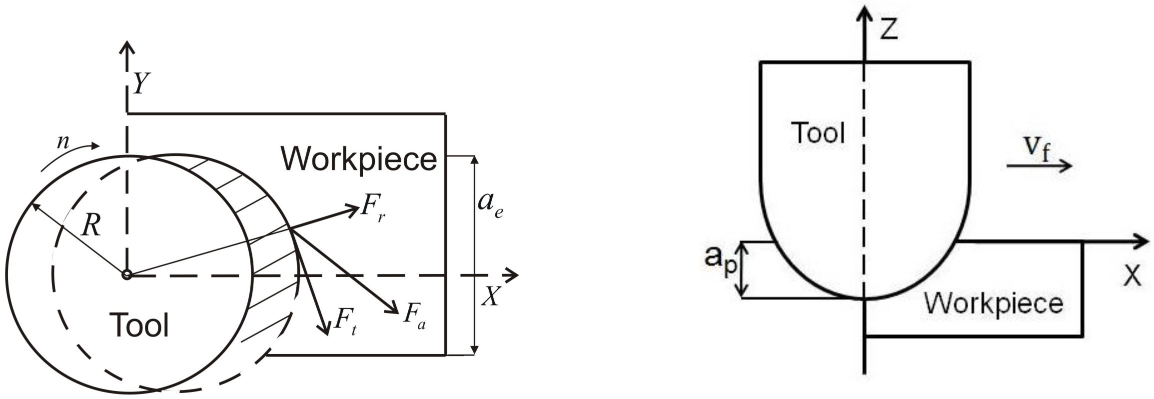

Fig. 2: Koordinatesystem in milling operations: left (X, Z)-plane, right (X, Z)-plane

where k_i represents the stiffness of the tool in direction i for $i \in \{r, t, a\}$. By applying the coordinate transformation

$$R(t) = \begin{pmatrix} \cos(\varphi(t)) & -\sin(\varphi(t)) & 0 \\ \sin(\varphi(t)) & \cos(\varphi(t)) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5}$$

to the formulas (3) and (4) we obtain the resulting forces in three orthogonal directions:

$$\mathbf{F}(t) = (F_x(t), F_y(t), F_z(t))^T \tag{6}$$

and the deflection

$$\boldsymbol{\delta}(t) = (\delta_x(t), \delta_y(t), \delta_z(t))^T \tag{7}$$

on the (X, Y, Z)- plane. We note that presented tool position model is flexible and can easily be generalized, for instance by taking into account other arbitrary chosen model of micro forces. The authors of this paper tested various force models presented in literature [2, 6, 5] and [4]. Outcomes of all the experiments are satisfactory and only negligible differences between final results were observed.

Actual depth of cut. The forces cause deflections of the tool which influence the input depth of cut a_p at the time t (see Fig. 3). On the other hand the displacement of the given depth of cut causes changes of the chip removal rate and consequently the forces. We consider now the active force F (see Fig. 2) which is the resulting force from the tangential and radial force i.e.: $F(t) = F_r(t) + F_t(t)$. Then we calculate the total tool deflection as

$$\delta(t) = \sqrt{\delta_r^2(t) + \delta_t^2(t)} \tag{8}$$

Considering this, the actual depth of cut can be written as

$$a_p^a(t) = a_p - l_h - \sqrt{l_h^2 - \delta^2(t)} \approx \frac{\delta^2(t)}{2l_h} \tag{9}$$

where l_h is the length of the tool. Here we use the Pythagoras theorem and the Taylor approximation of the root. The Fig. 3 shows the geometrical interpretation of the displacements of the tool tip.

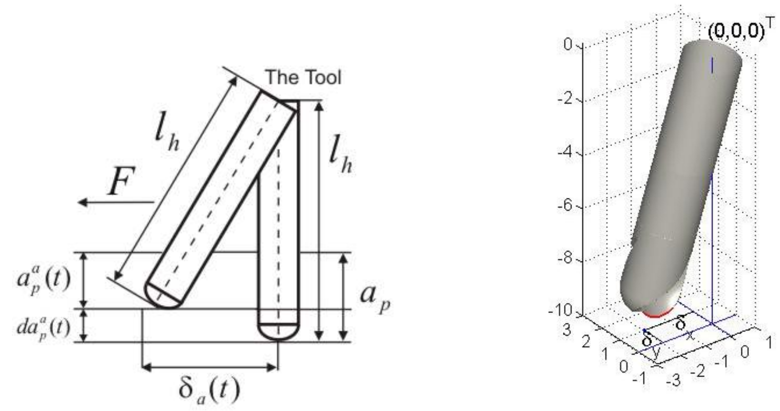

Fig. 3: Deflection of the tool: 2-dimentional view (left), 3-dimentional view with $\delta_x \equiv -2$, $\delta_y \equiv 1$ (right)

Actual tool-tip position model. Now we are in the position to formulate the complete model for the actual position of the tool-tip. In 3-dimensions coordinate system we denote by

$$\mathbf{P_s(t)} = (P_s^x(t), P_s^y(t), P_s^z(t))^T \tag{10}$$

a vector of the position of the tool tip in three plane directions at the time t. We set the vectors $\mathbf{v} = (v_x, v_y, v_z)^T$ with inputs velocities in each directions and $p_0 = (p_0^x, p_0^y, p_0^z)^T$ with the initial positions of the tool at the time t_0. The displacement of the tool on the workpiece at time t can be computed by multiplying the velocity vector $\mathbf{v}$ by time t. We make a standard assumption $v_y = v_z = 0$, $p_0^x = p_0^y = 0$ and $p_0^z = a_p$. The tool-path at the time t in coordinate system (X, Y, Z) is given then by

$$\begin{cases} P_s^x(t) = & v_x t + \delta_x(t) + \rho_x(t); \\ P_s^y(t) = & \delta_x(t) + \rho_y(t); \\ P_s^z(t) = & a_p^a(t) + \delta_x(t). \end{cases} \tag{11}$$

Here we take into account the deflection of the tool, the actual depth of cut and the run-out size effect. Combining equations (11) describing the tool path at time t, equations (4) modelling the deflections of the micro tool with force model (3) and their derivatives, we obtain the system of ordinary differential equation

$$\begin{cases} \dot{\varphi}(t) & = 2\pi n; \\ \dot{v}_f^a(t) & = v_x - \delta_x(t); \\ \dot{a}_p^a(t) & = -\frac{1}{l_h}\left(\frac{1}{k_r}F_r(t)\dot{F}_r(t) + \frac{1}{k_t}F_t(t)\dot{F}_t(t)\right); \\ \dot{h}(t) & = \frac{v_f}{nk}\cos(\varphi(t))\dot{\varphi}(t); \\ \dot{F}_r(t) & = K_{rc}\left(\dot{a}_p^a(t)h(t) + a_p^a(t)\dot{h}(t)\right) + K_{re}\dot{a}_p^a(t); \\ \dot{F}_t(t) & = K_{tc}\left(\dot{a}_p^a(t)h(t) + a_p^a(t)\dot{h}(t)\right) + K_{te}\dot{a}_p^a(t). \end{cases} \tag{12}$$

Here n is the rotational speed of tool, k is the number of cutting edges and v_f^a describe an actual velocity in feed x-direction. The solution of the ODE system (12) yields us the actual process parameters as well as the actual forces at each time step. With this results we are able to compute the actual removal rate at time t and hence also the resulting surface. We note that the presented model, if it is needed, can be extended by the additional effects which can affect the process, for example, the vibration of the tool or machine (see [7, 8, 9]). Moreover we can extend to any machining operation provided that geometrical similarity conditions are fulfilled. This forward model gives us the exact position and orientation of each cutting edge at each time point what is a starting point for modelling of the generated surface (see section 3).

Parameters	Up milling
Spindle speed n [min^{-1}]	10800
Feedrate v_f [mm/min]	840
Depth of cut a_p [mm]	0.08

Table 1: Machining parameters during experiment.

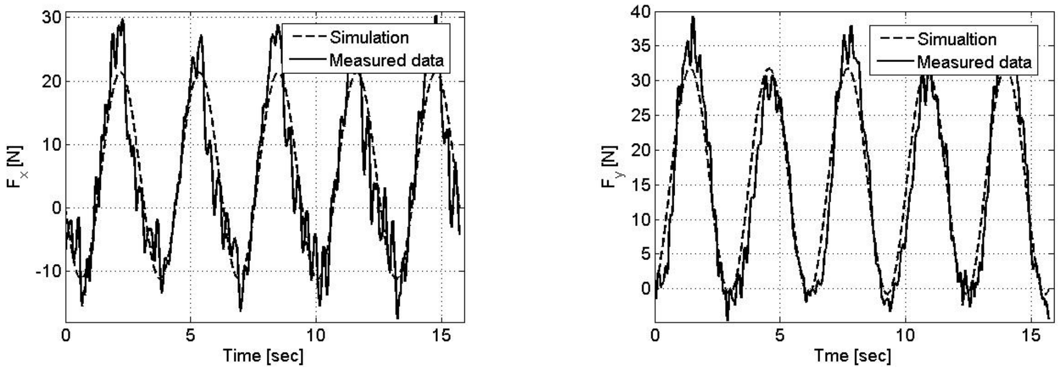

Fig. 4: Comparison between measured and simulated force with optimal force coefficient in: x-direction (left), y-direction (right)

Numerical results. The system of ODE's was solved with help of Matlab. The solution of (12) previous the vector of actual process parameters and actual forces at each time of the process. The developed model was validates in order to derive the parameters consistent with measured cutting force. The comparison of outcomes of the model simulations with measurements was done several for various of values of initial depth of cut a_p, velocity $v_x = v_f$, and rotational speed of the tool n. We analyze two different cutting force models. The first is the model which includes only the cutting part, i.e.:

$$\mathbf{F^*(t)} = \mathbf{K_c} a_p(t) h(t). \tag{13}$$

The results of the comparison between measured and simulated cutting forces in x- , and y-direction is shown in Fig. 4. Here we consider the end-ball milling process with radius $R =$ 1[mm]. The machining parameters are given in Table 1. The validation of the process by means of least square method yields the following cutting force coefficients $K_{rc} = -3.251565e + 003$ and $K_{tc} = 9.942452e + 003$. In second case we take into account the same initial conditions of process. However here we considerate the force model with the friction term, i.e.:

$$\mathbf{F^*(t)} = \mathbf{K_c} a_p(t) h(t) + \mathbf{K_e} a_p(t). \tag{14}$$

Based on experiments with both models of cutting forces (13) and (14) we observe that simulated force match the measured data very accurately. Moreover the difference between outcomes of simulations with cutting forces (13) and (14) is negligible. Hence, we conclude that both models are suitable for experiments and therefore we suggest to choose the cutting force (13) because of its ease of computation

Surface model

In this section we describe the surface generation for the micro-milling process under assumption that the tool dislocation is given by the tool tip position model (11). The surface simulation is a part of a kinematic simulation of the process and is based on a chip removal calculation.

To subtract the removed material from the workpiece we need to know the relative position of each cutting edge to the workpiece. In the context of this paper the tool and the workpiece are positioned orthogonal to each other, but strictly spoken the orthogonality might be lost due to the deflection of the tool. The surface model can deal with an arbitrary inclination angle. Our process chain is as follows:

- Solve the ODE (12) numerically
- Calculate the tool trajectories
- Discrete chip removal
- Calculate normed surface parameters.

Calculation of the 3-dimensional tool trajectories. In milling processes the tool typically rotates around Z-axis in mathematically negative direction and at the same time the tool is translated along X-axis, see Fig. 2 for the coordinate system. The rotational and translational movement can be described by an affine linear mapping, conveniently formulated by a homogeneous matrix. From here we use a homogeneous coordinate system, which is built from the original system by

$$(x, y, z) \mapsto (x, y, z, 1)^T. \tag{15}$$

In general a point $(x, y, z, 1)^T$ is rotated around Z-axis and

$$R_z(\Phi) = \begin{pmatrix} \cos(\Phi) & -\sin(\Phi) & 0 & 0 \\ \sin(\Phi) & \cos(\Phi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and translated by

$$T(p) = \begin{pmatrix} 1 & 0 & 0 & p_x \\ 0 & 1 & 0 & p_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

For a ball-end tool of length l_h and radius r the cutting edge shall be given by

$$\varepsilon(\varphi) = (0, \cos(\varphi), -r\sin(\varphi), -r + l_h, 1)^T, \quad \varphi \in [0, \pi] \tag{16}$$

Consequently we are able to formulate the tool trajectories for an ideal milling process:

$$(x_t, y_t, z_t, 1)^T = T(p)R_z(-\Phi)\varepsilon(\varphi), \quad \varphi \in [0, \pi]. \tag{17}$$

The curve $p : [0, T] \to R^3$ describes the path of the tool holder during simulation time. For the scenario in section 2, we have the path $p = (v_x t, 0, l - a_p)T$. The trajectories in (16) describe the simplest case: a orthogonal cutting process without any disturbances. Changing inclination angle or adding a run-out error is straightforward. Without going into details, we formulate the tool trajectories, under assumption of the run-out and deflection of the tool, by the operator

$$\Phi := T(p)T(m)\left(\arctan\left(\frac{\delta_y}{\delta_x}\right)\right) R_y\left(\arcsin\left(\frac{\sqrt{\delta_x^2 + \delta_y^2}}{l_h}\right)\right) R_z\left(-\arctan\left(\frac{\delta_y}{\delta_x}\right)\right) R_z(-\Phi) \tag{18}$$

Thus, the tool trajectories are given by

$$(x_t, y_t, z_t, 1)^T = \Phi\varepsilon(\varphi) \qquad \varphi \in [0, T]. \tag{19}$$

In Fig. 5 the tool tip locus are plotted, that is the operator Φ is applied to $\varepsilon(\pi/2)$ and to the outer point $\varepsilon(0)$ lying on the edge and the corresponding trajectories are computed. For another examples of the surface models we refer the reader to [10, 11].

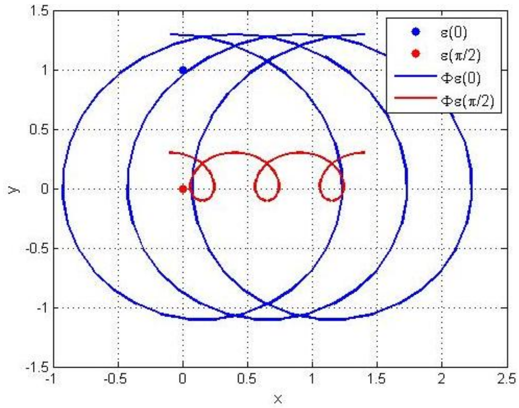

Fig. 5: Trajectories for $\delta_x = -0.1$, $\delta_y = 0.1$ and $\rho = (0, 0.2, 0)^T$.

Parameters for ball-end milling process	Ideal process	One canal process
Tool	2-flute ball end mil	2-flute ball end mil
Tool radius R [mm]	1	1
Depth of cut a_p [mm]	0.05	0.08
Radial depth of cut a_e [mm]	0.03	-
Feedrate v_f [mm/min]	0.03	840
Spindle speed n [1/min]	30000	10800

Table 2: Process parameter for surface simulation under ideal condition and for measurements and simulation of the one canal milling process

Simulated surfaces results. We implemented and tested the tool-tip position model to design the surface for micro-ball milling. Moreover the presented model was verified by the comparing the machined and simulated surfaces on the surface parameter scale. A micro ball mill with radius $R = 1$ [mm] was taken into account. The simulated ideal surface in dependence on geometry of the tool and constant process parameters (given in Table 2) is depicted in Fig. 6. Here we are not only able to obtain the process parameters but we can also distinguish the elements of the structure of the surface such as for example feed scallop or pick scallop (see Fig. 6). In Fig. 7 and Fig. 8 the measured and simulated surface for the process parameters given in Table 2 (right column) are presented. To quantify the final surface, the 3D surface parameters were computed [12]. The resulting surface parameters are stated. Next the surface simulation for more than one canal shall be done. We take here the analogous assumptions as in section 2. In Fig. 6 (right) the simulated surface with 6 canals for ideal initial process parameters is presented.

3D Surface Parameter	Measured Surface	Simulated Surface
S10z [νm]	14.46969	13.08819
Sq [νm]	4.18354	3.89927
Ssk	0.67404	0.64033
Sku	2.19784	2.13805

Table 3: Comparison of the 3D surface parameters between the measured data and simulated surface.

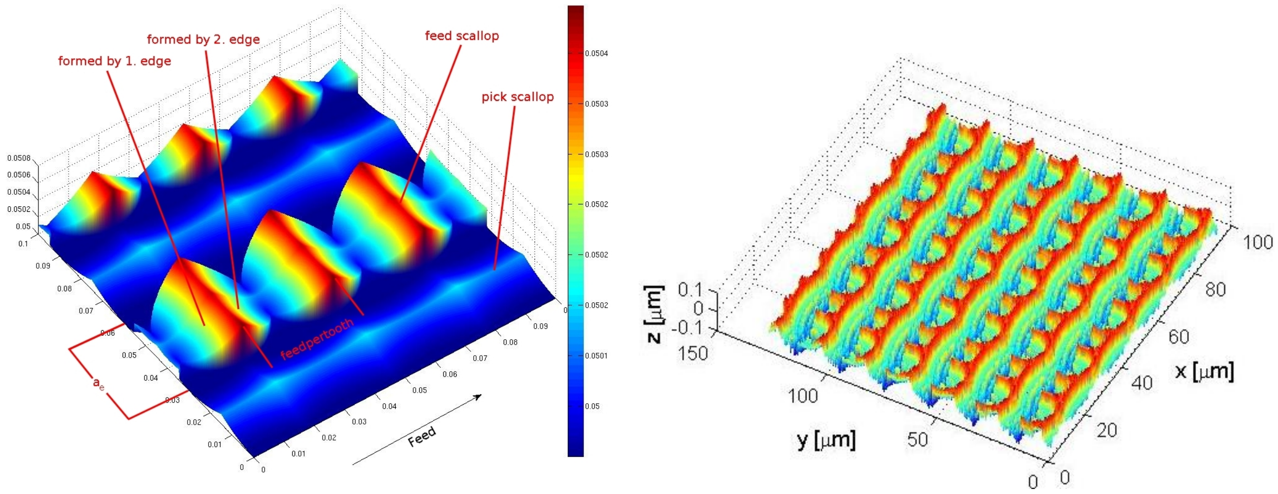

Fig. 6: Simulated surface under ideal process conditions.

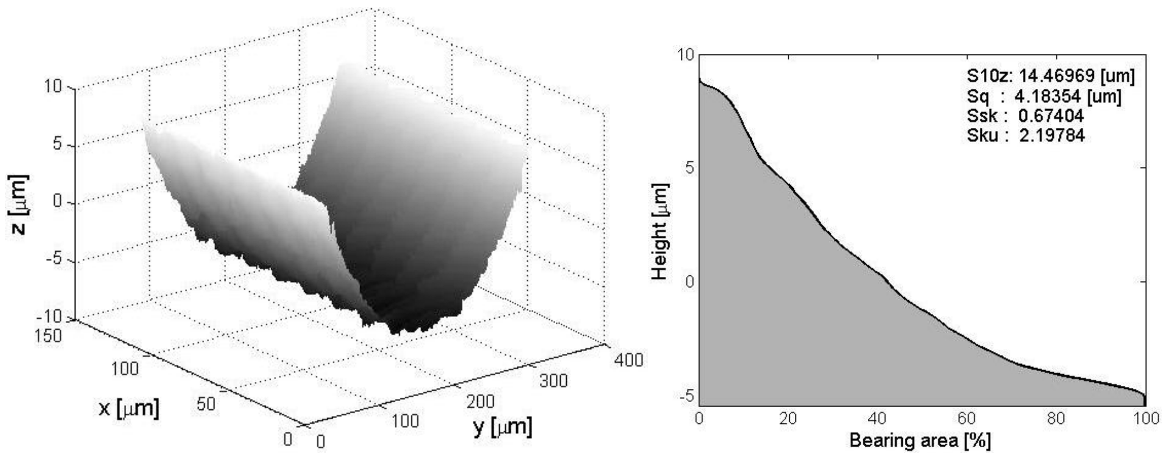

Fig. 7: Measured surface and its abbott curve.

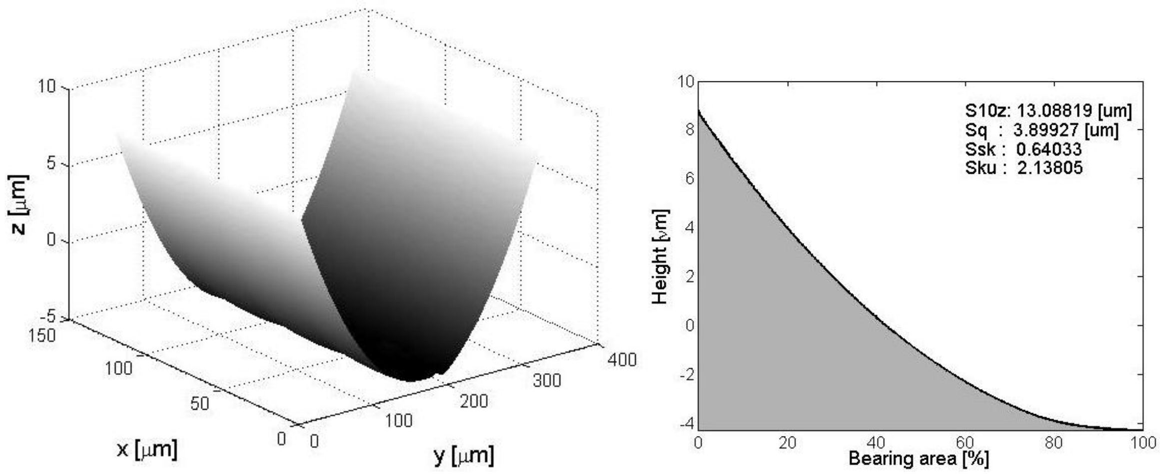

Fig. 8: Simulated surface and its abbott-curve.

Conclusion and discussion

In this paper we presented a dynamical model for prediction of the cutting force. The developed model is described by system of ODE's and takes into consideration the interaction between tool and workpiece. The compression of simulated and measured data has been presented and shows a potential of introduces model for use in the whole production chain. Additionally, surface generation model was established and provides satisfactory results. The model under consideration can be extended to every cutting process fulfils the kinematics and geometry requirements. Moreover, in next steps of the model development the simulation of the complex surfaces shell be taken into account. The goal will be to determine the process parameters for given or desired surface topography. This shall be realized by means of the regularisation method of the inverse model. The starting point here is the forward model of the whole process chain. For this reason it is important to describe the simulated surface topography in dependence on parameters and with the minimal computation time.

Adknowlegement

The work presented in this article was supported by the German Research Foundation (DFG) within the SFB 747, subproject C2 and the grand SPP 1480 German Research Foundation (DFG). The authors would like to thank Dipl.-Ing. Sven Twardy from the Laboratory of Precision Machining (LfM) in Bremen, Germany, for the fruitful cooperation and the allocation of data.

Literature References
References

[1] I. Piotrowska, C. Brandt, H. R. Karimi, P. Maass: *Mathematical model of micro turning process* The International Journal of Advanced Manufacturing Technology, Vol. 45 (2009), p. 33-40

[2] Y. Altintas: *Manufacturing automation: metal cutting mechanic Machine tool vibrations and CNC design*, Cambridge University Press (2000)

[3] H. K. Tönshoff, B. Denkena: *Spanen* (Grundlagen, Springer 2004).

[4] M. Weck, K. Teiple: *Dynamisches Verhalten spanender Werkzeugmachinen* (Springer, Berlin, Heidelberg New York 1977)

[5] O. Rott, D. Hm̈berg, C. Mense: *A comparison of analytical cutting force models* Weierstrass Institut fr Angewandte Analysis und Stochastik, preprint (2006)

[6] I. S. Kang, J. H. Kim, M. C. Kang, Y. W. Seo: *A mechanistic model of cutting force in the micro end milling process* Journal of Materials Processing Technology, Vol. 187-188 (2009), p. 250-255

[7] C. Brandt, J. Niebsch, R. Ramlau, P. Maass: *Modeling the Influence of Unbalances for Ultra Precision Cutting Processes* submitted to Zeitschrift fr Angewandte Mathematik und Mechanik (2010)

[8] C. Brandt, J. Niebsch, P. Maass, R. Ramlau: *Simulation of Process Machine Interaction for Ultra Precision Turning* Proceedings of the 2nd International Conference on Process Machine Interactions, June 10-11, 2010, Vancouver, Canada (2010)

[9] C. Brandt, I. Piotrowska, H. R. Karimi, J. Niebsch, R. Ramlau, A. Krause, O. Riemer, P. Maass: *Process Machine Interaction Model for Turning Processes* International Journal of Control Theory and Applications, Vol. 1(2) (2008), p. 145-153

[10] M.A. Arizmendi, J. Fernandez, L.N.L.d. Lacalle, A. Lamikiz, A. Gil, J.A. Sanchez, F.J. Campa, F. Veiga: *Model development for the prediction of surface topography generated by ball end mills taking into account the tool parallel axis offset* Experimental validation, CIRP Annals - Manufacturing Technology, Vol. 57(1) (2008), p. 101-104

[11] J. S. A. Chen, Y. K. Huang, M. S. Chen: *Study of the surface scallop generating mechanism in the ball end milling process* International Journal of Machine Tools and Manufacture, Vol. 45(9) (2005), p. 1077-1084

[12] X. Luo: *High Precision Surfaces Generation: Modelling, Simulation and Maschining Verification* PhD thesis (2004)