Zentrum für Technomathematik

Fachbereich 3 – Mathematik und Informatik

Comparing Different Approaches for Model Parameters Identification in Short Time

Linghan Li
Bastian Kanning
Christian Schenck
Bernd Kuhfuss

Report 11–12

Berichte aus der Technomathematik

Report 11–12 December 2011

Comparing Different Approaches for Model Parameters Identification in Short Time

Linghan Li[1,a], Bastian Kanning[2,b], Christian Schenck[1], Bernd Kuhfuss[1]

[1] bime | Bremen Institute for Mechanical Engineering,
University of Bremen
Badgasteiner Str.1, 28359 Bremen, Germany
[a] Corresponding author: li@bime.de

[2] ZeTeM | Center for Industrial Mathematics,
University of Bremen
Bibliothekstraße 1, 28359 Bremen, Germany
[b] Corresponding author: kanning@math.uni-bremen.de

***Abstract*—Model Parameter values of machine tools change during machining. An optimal process control needs precise knowledge of the actual parameter values. Three different algorithms are introduced to estimate the modal parameter values of system in a short time window with high resolution: least squares estimation (LSE), estimation of signal parameters via rotational invariance (ESPRIT) and orthogonal matching pursuits (OMP) algorithm.**

These algorithms are augmented with a sliding-window operation to reveal the actual system dynamic behavior at every time instance. This paper focuses on comparing the performance and the identification accuracy of the proposed methods and the influence of the applied window size and noise content using numerical examinations. The results show that the sliding-window LSE can estimate transient parameters accurately and suits realtime control processes.

***Keywords*—modal parameter identification; short time analysis; sliding window operation; least squares estimation; ESPRIT; matching pursuits.**

I. INTRODUCTION

Natural frequencies and damping ratios are relevant modal parameters in analyzing mechanical structures and machining processes. In engineering practice, these parameters are often obtained by experimental impact testing. The modal parameters are identified by determining the frequency response function, which is the input-output (excitation-response) ratio in frequency domain, obtained by Fast Fourier Transform (FFT) under the assumption of a linear and steady state vibration system [1].

However, during machining processes, the modal parameters that characterise the dynamic behaviour may vary rapidly in a short time due to variations in the cutting parameters and changes in boundary conditions [2]. In rotating machinery, the dynamic modal parameters shift between the 0 rpm state and the machining operation state [3,4]. In the area of high-speed machining, time- and frequency-varying events and transient and complex harmonic interactions arise from complicated machining processes [5]. The dynamic modal parameters also shift due to changing geometric configuration, such as in robotic devices [6], flexible mechanisms [7], cranes [8] and so on. These time-varying characterise cannot be discovered with FFT.

Recently, many different approaches have been developed and applied in practice, especially output-only identification methods (OOIM). The OOIM are classified as non-parametric or parametric [2].

Operational modal analysis (OMA) [9,10] is one kind of non-parametric OOIM. This method is applied to big civil structures, where it is extremely difficult to measure the excitation forces. The identification procedure is based on experimental modal analysis with the assumption of a stationary white-noise excitation. However, the OMA could not provide transient information of the measured signal. Other non-parametric OOIM are time-frequency analyses like short time Fourier transform (STFT) [11], the Wigner-Ville distribution of Cohen class [12] and the wavelet transform (WT) [13]. These methods are bound to the Heisenberg's uncertainty principle and thus, the resolution in time-frequency domain is restricted. Hilbert-Huang transform (HHT) [14] is an improved time-frequency analysis method for non-stationary signals. However, the identification accuracy is limited by some shortcomings [15] and side effects of empirical mode decomposition (EMD) which is the core of HHT.

Parametric OOIM is based on time-dependent autoregressive moving average (TARMA) representations [16]. The major parametric method consists in the least squares estimation (LSE), which is proposed by Yang et al. [17] and has been successfully applied to low-frequency oscillation parameters estimation for power systems [18]. The modal parameters can be obtained by fitting the time window data with a damped oscillating function using a least squares estimation (LSE). The second one is an estimation of signal parameters via rotational invariance techniques (ESPRIT) [19,20] which is based on the short time subspace method. The original signal can be transferred to the two signal subspaces. The signal parameters are obtained by computing the eigenvector of two sets of linearly independent vectors in the signal subspaces. One of the advantages of the ESPRIT algorithm, compared to the LSE is the applicability to multiple degree of freedom systems (MDOF). Both methods can estimate the dynamic modal parameters with an analyzing short time window. If the window is shifted in time, modal parameters of time-varying systems can be obtained in each analyzing window [21,22].

Moreover, the orthogonal matching pursuit (OMP) is a method for adaptive signal reconstruction and was firstly proposed in the field of signal processing [23,24]. Assuming that the signal represents a biased discretization of superposed sinoids, a basis consisting of sinoids with different frequencies and damping ratios is defined. The OMP greedily chooses atoms from a predefined basis, which are mostly correlated with the measured signal. This method is also applicable to MDOF systems.

This paper does not discuss the improved parametric algorithms or novel approaches, but emphasizes the performance or applicability of the proposed algorithms to identify the transient or time-varying modal parameters with high resolution, so that it can be applied in real-time control systems. The methods LSE, ESPRIT and OMP are compared in this paper, their performances are examined with various analyzing window sizes, signal-to-noise ratios and so on using simulations.

II. Theoretical Background

A. LSE

Consider a monocomponent signal, transformed by Euler's theorem, whose evolution is described by:

$$x(k)=(1/2)e^{\beta k\Delta t}(Xe^{j\varphi}e^{2\pi fk\Delta t}+Xe^{-j\varphi}e^{-2\pi fk\Delta t}) \tag{1}$$

where X is the amplitude, β is the decay constant, f is the oscillation frequency, φ is the phase and Δt is the time interval.

Using the following definition:

$$A=(X/2)e^{j\varphi}, a=e^{j2\pi f\Delta t}, b=e^{\beta\Delta t}, z=\mathrm{Re}(a) \tag{2}$$

(1) can be rewritten as $x(k)=b^k(Aa^k+\overline{A}a^{-k})$. After some algebraic manipulations, we obtain

$$-bx(k)+2zx(k+1)-b^{-1}x(k+2)=0 \tag{3}$$

Rearrange (3) as:

$$\begin{bmatrix} x(k) & x(k+1) \\ x(k+1) & x(k+2) \\ \vdots & \vdots \\ x(k+L-3) & x(k+L-2) \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \end{bmatrix}=\begin{bmatrix} x(k+2) \\ x(k+3) \\ \vdots \\ x(k+L-1) \end{bmatrix} \tag{4}$$

where L is the data window size and $p_1=-b_1^2$ and $p_2=2z_1b_1$.

(4) can be simplified as $AP=B$. Using the LSE P can be obtained as:

$$P=(A^TA)^{-1}A^TB \tag{5}$$

Then b_1 and z_1 are calculated by $b_1=\sqrt{-p_1}$ and $z_1=p_2/2b_1$. The parameters b_i and z_i can be obtained by shifting the window through the whole signal.

According to the definitions, the frequency f_i and damping ratio ζ_i of an oscillation can be obtained by the following equations:

$$f_i=\frac{\cos^{-1}z_i}{2\pi\Delta t}, \zeta_i=\frac{-\beta_i}{\sqrt{\beta_i^2+(2\pi f)^2}}\times 100\% \tag{6}$$

where $\beta=\ln(b_i)/\Delta t$.

B. ESPRIT

Consider a damped sinusoid with additive white noise, defined as follows:

$$x(n)=s(n)+w(n)=\sum_{k=1}^{K}A_ke^{(-\beta_k+j\omega_k)n}+w(n) \tag{7}$$

here, *s(n)* is the sinusoidal signal part and *w(n)* is the zero-mean white noise, $A_k=(a_k/2)e^{j\varphi_k}$ is the complex amplitude, β_k is the decay constant, φ_k is the initial phase, $\omega_k=2\pi f_k$ is the angular frequency, K is the number of sinusoids and n is the number of samples.

Suppose M to be the length of the signal, then define the signal subspaces X (from 1^{st} to M-1) and Y (2^{nd} to M) as the arrays:

$$X=AS+w_x \tag{8}$$

$$Y=A\Phi S+w_y \tag{9}$$

where $\Phi=diag\{e^{-\beta_1+j\omega_1}e^{-\beta_2+j\omega_2}\cdots e^{-\beta_K+j\omega_K}\}$ is the rotation matrix, $A=\left[A_1A_2\cdots A_k\right]^T$ and S is defined as:

$$S=\begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-\beta_1+j\omega_1} & e^{-\beta_2+j\omega_2} & \cdots & e^{-\beta_k+j\omega_k} \\ \vdots & \vdots & \vdots & \vdots \\ e^{-\beta_1+j(M-1)\omega_1} & e^{-\beta_2+j(M-1)\omega_2} & \cdots & e^{-\beta_k+j(M-1)\omega_k} \end{bmatrix} \tag{10}$$

By computing the auto-correlation matrix R_{xx} and cross-correlation R_{xy}, it follows:

$$R_{xx}=E\left\{XX^H\right\}=SAS^H+\sigma_w^2I \tag{11}$$

$$R_{xy}=E\left\{XY^H\right\}=SA\Phi^HS^H+\sigma_w^2Z \tag{12}$$

where E denotes the expectation, σ_w^2 is the standard deviation of the noise, I is the identity and Z is defined as:

$$Z=\begin{bmatrix} 0 & & & 0 \\ 1 & 0 & & \\ & \ddots & \ddots & \\ 0 & & 1 & 0 \end{bmatrix} \tag{13}$$

According to the eigenvalue decomposition of (11) and (12), we obtain the new matrices:

$$R_1=R_{xx}-\lambda_{\min}I=R_{xx}-\sigma_w^2I=SAS^H \tag{14}$$

$$R_2=R_{xy}-\lambda_{\min}Z=R_{xy}-\sigma_w^2Z=SA\Phi^HS^H \tag{15}$$

here the singular value decomposition (SVD) of R_1 can be expressed as:

$$R_1=U\Sigma V^H=\left[U_1 \quad U_2\right]\begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix}\begin{bmatrix} V_1^H \\ V_2^H \end{bmatrix} \tag{16}$$

then by computing the eigenvalue decomposition of the matrices $\left\{\Sigma_1 \quad U_1^HR_2V_1\right\}$, the eigenvalues Υ for $R_1-\Upsilon R_2$ can be obtained:

$$\Upsilon=e^{-\beta_k+j\omega_k}, k=1,2,\cdots K \tag{17}$$

The frequency and damping ratio are obtained by:

$$f_k=\mathrm{Im}(\ln(\Upsilon_k))/2\pi, \zeta_k=\beta_k\Big/\sqrt{\beta_k^2+\omega_k^2} \tag{18}$$

where $\beta_k = -\mathrm{Re}(\ln(\Upsilon_k))$, $k = 1, 2, \cdots K$, *Re* and *Im* denote the real and imaginary part respectively.

C. OMP

Let the measured signal be g. The method starts with generating a basis of sinusoids, from (7) for different frequencies and damping ratios, defined as $\{\psi_i\}_{i\in N}$. Then, the k-th iteration of the approximation f is computed by:

$$f_k \in \arg\min\{\| f - g \| : f \in G_k\}, \tag{19}$$

with

$$G_k := span\left\{\arg\sup_{\psi_i}\left|\left\langle r^0, \frac{\psi_i}{\|\psi_i\|}\right\rangle\right|, \cdots, \arg\sup_{\psi_i}\left|\left\langle r^{k-1}, \frac{\psi_i}{\|\psi_i\|}\right\rangle\right|\right\} \tag{20}$$

and $r^k := f^k - g$. After a stopping criterion, e.g. $\|r\| < \varepsilon$ is fulfilled, the modal parameters can be identified from the choice of basic elements, used to approximate the measurement.

III. NUMERICAL SIMULATIONS

In this section, we present numerical simulations to illustrate the performance of the different algorithms described in the previous sections.

In practice the measured signal, a multi frequency vibration response with noise, is pre-filtered first, in order to obtain a narrowband signal containing one significant frequency. Because of the nonlinear or time-varying system the modal parameters of this signal change in time. The dynamic characteristics can be captured by using the proposed algorithms with a sliding analyzing window. Therefore, the simulation analysis is performed on the following single degree damped sinusoid with narrowband nosie:

$$x(t) = A_{signal} \cdot e^{-\xi 2\pi f_n t} \sin(2\pi f_n t) + y_{noise} \tag{21}$$

where $A_{signal}=1$, $0<t<1$, f_n is the natural frequency of the simulated signal, ξ is the damping ratio and y_{nosie} is a narrowband white Gaussian noise. Here we use an ideal digital frequency filter with a Hanning window as a pre-filter. The noise level is expressed by the signal-to-noise ratio (SNR). The SNR is defined as $SNR_{dB} = 10\log_{10}(A_{signal}^2/\sigma_{noise}^2)$ with σ_{noise}^2 being the standard deviation of the noise. To ensure the noise has the same effect on the different amplitudes of the simulated signal, it is multiplied by an exponential function, which is the upper envelope of the simulation signal, before added to the signal.

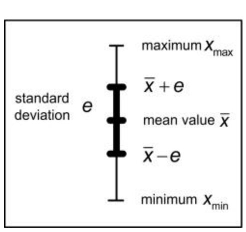

Figure 1. Symbolic representation of numerical results

The simulated signal is analyzed using the proposed algorithms augmented with a sliding window operation. The analyzing window contains short time data and is shifted through the whole simulated signal *x(t)*. The estimated frequencies and damping ratios are computed for each window using the proposed identification algorithms. The experiment is repeated 5 times. The mean, standard deviation, maximum and minimum values are calculated and displayed as shown in Fig.1. The results are compared with each other to examine the performance of the proposed algorithms. Tab.1 lists the preset modal parameters and other optional parameters, relevant for computation.

Table 1. Preset values and computing parameters

Parameter	Value
preset frequency	260 Hz
preset damping ratio	0.2 %
filter pass band	210 Hz-310 Hz
shifting step of window	4 ms
sampling rate	5120 Hz

The identification accuracy of the proposed algorithms will be demonstrated through a number of following numerical examples (A-F).

IV. RESULTS AND DISCUSSION

A. Variation of the window length

In this case, the modal parameters are estimated under different window length. The SNR is set to 5 dB.

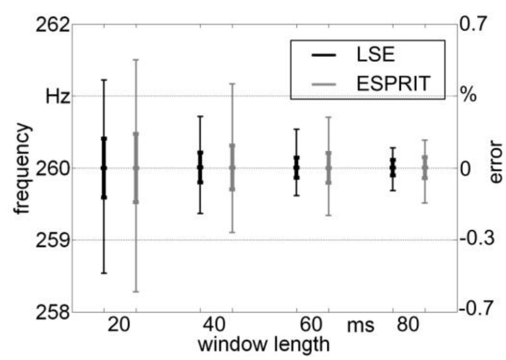

Figure 2. Frequency estimation with varying window length

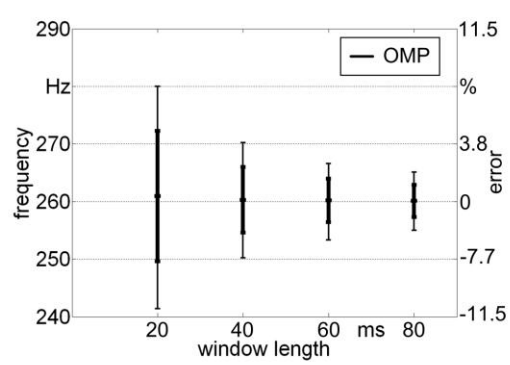

Figure 3. Frequency estimation with varying window length for OMP

Due to a great difference of identification results between LSE, ESPRIT and OMP method, the results of LSE, ESPRIT and the results of OMP are plotted in Fig.2 and 3 respectively. It can be seen that the frequency estimation of the LSE and ESPRIT has a high accuracy (less than 0.5 % error). The increasing window size improves the identification accuracy. In comparison with the LSE and ESPRIT, the identification accuracy of the OMP is obviously too low. Therefore, we will not show the results of OMP in next sections.

The estimation for damping ratio under various window sizes is shown in Fig.4. The estimation accuracy for LSE and ESPRIT is very low. (about 100 % error). The accuracy increases with increasing window size. It is worth noting that the estimated damping ratio attains negative values for a window length shorter than 60 ms. It is well known that real mechanical structures do not have negative damping ratios. This effect is a pure numerical problem arising from the choose algorithms and limits the smallest reasonable window size. Moreover, the accuracy of the LSE is a little bit higher than of the ESPRIT method.

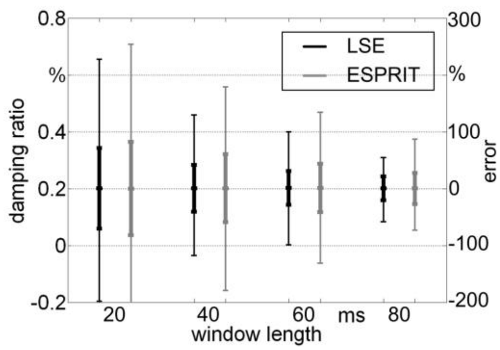

Figure 4. Damping ratio estimation with varying window length

B. Variation of the noise level

The modal parameters are estimated for different noise levels. The window size is set to 20 ms.

The identification results at different noise levels are given in Fig.5 and 6. It can be seen that the accuracy is obviously improved, if the SNR is reduced from 5 dB to 20 dB. The estimation accuracy of both methods sensitively depends on the noise level.

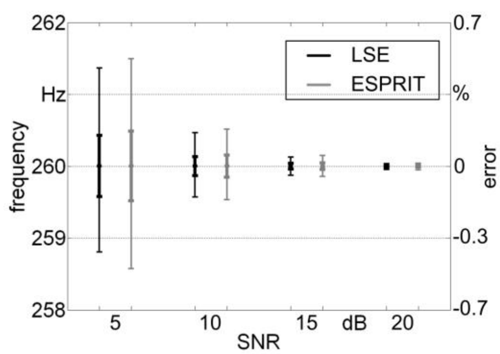

Figure 5. Frequency estimation with varying noise levels

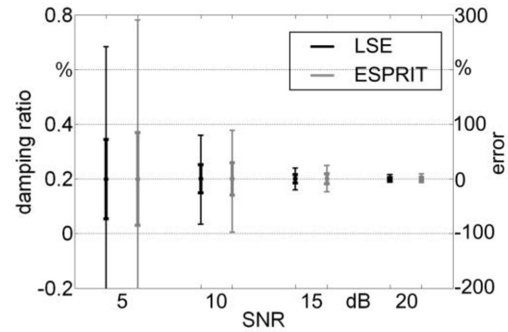

Figure 6. Damping ratio estimation with varying noise levels

C. Variation of the pass band value W

Due to time varying systems, in which the natural frequency shifts, the influence on the identification accuracy of the pass band filter value W, will be investigated in this example. The value W is defined as the distance between the natural frequency f_n of the simulated signal and the center frequency of the pass band, while the total width of the applied frequency filter remains constant at 100 Hz. The window size is set to 20 ms and the SNR is set to 20 dB.

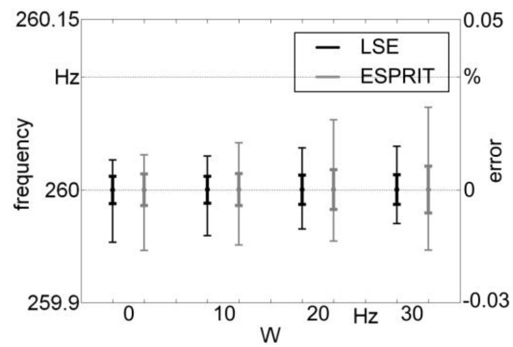

Figure 7. Frequency estimation with varying pass band value W

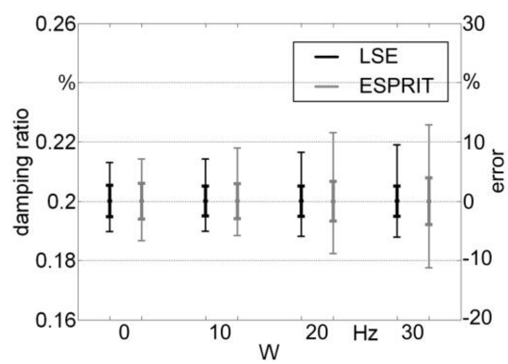

Figure 8. Damping ratio estimation with varying pass band value W

As shown in Fig.7 and 8, shifting the pass band does not have a large influence on the estimated values. In machining processes or engineering structures, the range in which modal parameters change was proved to be relatively small, as in a parallel kinematic machine the change in frequency stays within 2.1 Hz [7]. Therefore, the setting of the pass band has little influence on the accuracy of modal parameter estimation, even for time-varying systems.

D. *Test at low frequency*

In this example, a single degree damped sinusoid with f_n=5Hz, ξ =0.2% is simulated. The identification accuracy will be examined, while the window length is set to less than a quarter of the frequency's period. The analyzing window size is set to 40 ms. The pass band is set from 1 Hz to 9 Hz.
As shown in Fig.9 and 10, the frequency identification accuracy for low-frequent signals is also high and decreases with decreasing SNR. The damping ratio estimates exhibit large errors, but approach the preset value with reducing the noise.

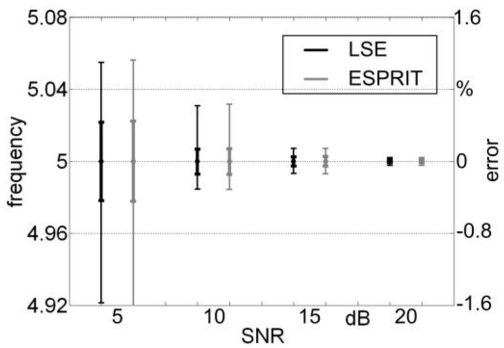

Figure 9. Frequency estimation for low frequency signal with varying noise level

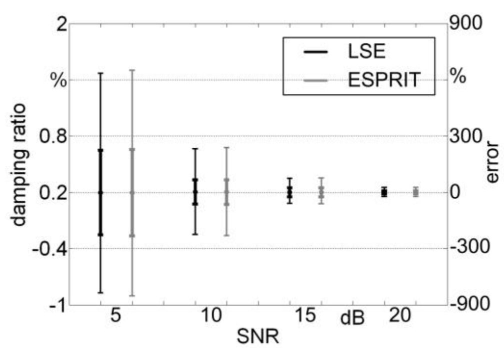

Figure 10. Damping ratio estimation for low frequency signal with varying noise level

E. *Computation effort*

Because the purpose of this work is to find an optimal algorithm for real-time control systems or online monitoring, the computational cost is also a very important parameter. The time, used for the required computations of each proposed algorithm is investigated for the simulated signal with preset values as seen in Tab.1 and a window size of 40 ms. It should be noted that the time of computation depends on the optimization of the programs and the CPU. The algorithms were implemented in MatLab 7.11.0 and run on an Intel Pentium Dual processor with a 2.20 GHz CPU and 2 GB RAM.

The computing time, used for one analyzing window is presented in Tab.2 for each method. With 0.2 ms, the LSE has fastest computing time for one analyzing window. The ESPRIT algorithm requires more computational load than the LSE algorithm, since it requires computing eigenvectors and SVD. The computing time of the OMP algorithm depends mostly on the definition of the basis. By reducing the accuracy, the OMP algorithm converges faster to the preset value.

Table 2 Time required in proposed algorithms

Algorithm	**Used time**
LSE	0.2 ms
ESPRIT	32 ms
OMP	30 ms

F. *Simulation signal to test*

To test the applicability of the proposed algorithms in engineering practice, a time-varying signal was simulated. The natural frequency increases linearly from 255 Hz to 265 Hz with time. Similarly, the damping ratio changes linearly from 0.2 % to 0.3 %. The instantaneous frequency and damping ratio are obtained using a sliding window LSE method. As a result, the identification error is illustrated. The window size is set to 40 ms and the SNR is set to 10 dB.

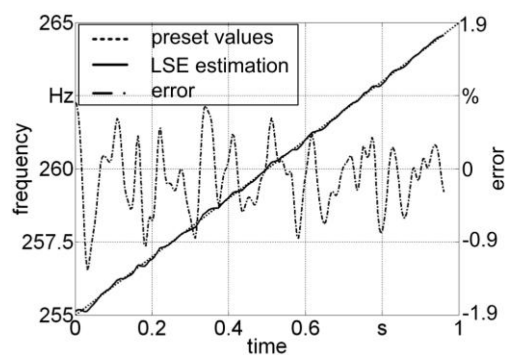

Figure 11. Frequency estimation using sliding-window LSE

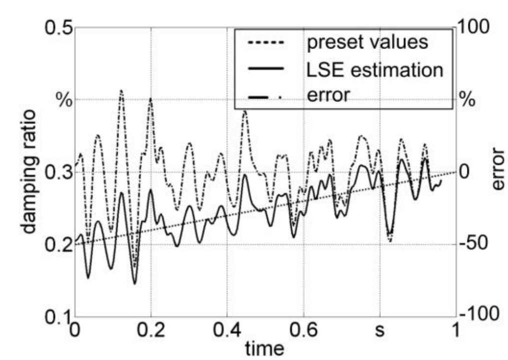

Figure 12. Damping ratio estimation using sliding-window LSE

The instantaneous frequency and damping ratio are obtained and plotted in Fig. 11 and 12, respectively. The shifting of the natural frequency and damping ratio are recovered effectively. The estimation error for the frequency is bounded by 1 % and by 50 % for the damping ratio.

V. CONCLUSION

To review the dynamic behaviour of time varying systems precisely, three algorithms (LSE, ESPRIT and OMP) are introduced and their identification accuracy is investigated using a number of numerical examples. The LSE and ESPRIT can estimate the modal parameters for very short time windowed data with high resolution. Unlike the time-frequency analysis, the accuracy of both methods does not depend on the time- or frequency-resolution. The parameter estimation errors become smaller with increasing the analyzing window length and with reducing the noise level of the original signal. Moreover, the identification accuracy does not depend strongly on the centre frequency of the pass band filter. The algorithms can be applied for short windowed data of low frequency signal. The identification accuracy of the LSE algorithm is higher than the ESPRIT algorithm. Both methods have higher identification accuracy for the natural frequency compared to the OMP.

However, the damping ratio identification has larger error than the frequency identification, because the small values pose serious estimation accuracy problems [25]. The damping ratio estimation is also very sensitive to the SNR. The identification accuracy can be improved by increasing the window size and reducing the noise. Besides, the analyzing window can be divided into several blocks in which the damping ratio is estimated iteratively. By averaging the estimated values of the blocks, the identification accuracy might be further improved.

The multi frequency vibration response case is much more important in engineering practice. The ESPRIT algorithm can also be applied for these signals. In future work, the applicability of the LSE algorithm for this case will be studied.

The computing time of the LSE is shorter than the other proposed methods. Consequently, it is more applicable to the online monitoring of machining processes and real-time control systems. The OMP algorithm may be applied in offline modal analyses.

REFERENCES

[1] D. J. Ewins, Modal Testing: Theory, Practice and Application, 2nd Edition ed. Baldock: Research Studies Press, 2000.

[2] M. D. Spiridonakos and S. D. Fassois, Parametric identification of a time-varying structure based on vector vibration response measurements, Mechanical Systems and Signal Processing, 2009, vol. 23, pp. 2029-2048.

[3] I. Zaghbani and V. Songmene, Estimation of machine-tool dynamic parameters during machining operation through operational modal analysis, International Journal of Machine Tools and Manufacture, 2009, vol. 49, pp. 947-957.

[4] H. Ouyang, Stationary and non-stationary vibration of atomising discs, Journal of Sound and Vibration 308 (2007) 699–708.

[5] S. Gu, J. Ni and J. Yuan, Non-stationary signal analysis and transient machining process condition monitoring, International Journal of Machine Tools and Manufacture, 2002, vol. 42, pp. 41-51.

[6] K. Petsounis, S. Fassois, Non-stationary functional series TARMA vibration modelling and analysis in a planar manipulator, Journal of Sound and Vibration 231 (2000) 1355–1376.

[7] Z. Zhou, C. Mechefske, F. Xi, Nonstationary vibration of a fully flexible parallel kinematic machine, Journal of Vibration and Acoustics 129 (2007) 623–630.

[8] J. Kullaa, Structural health monitoring of a crane in variable configurations, in: Proceedings of the ISMA2004 International Conference on Noise and Vibration Engineering, Leuven, Belgium, September 2004, pp. 457–470.

[9] S. Ibrahim and E. Mikulcik, A Method for the Direct Indentifikation of Vibration Parameters from the Free Responses, Shock and Vibration Bulletin, 1977, vol. 47.

[10] P. Mohanty and D. J. Rixen, Operational modal analysis in the presence of harmonic excitation, Journal of Sound and Vibration, 2004, vol. 270, pp. 93-109.

[11] J. Hammond, P. White, The analysis of non-stationary signals using time–frequency methods, Journal of Sound and Vibration 190 (1996) 419–447.

[12] Cohen, L., "Time-frequency analysis: theory and application,". Englewood Cliffs, N J: Prentice Hall, 1995.

[13] C. E. Heil and D. F. Walnut, Continuous and Discrete Wavelet Transforms, SIAM Review, 1989, vol. 31, pp. 628-666.

[14] Huang, N.E., Shen, Z., Long, S.R., et al., "The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis," Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences, vol. 454(1971), pp. 903-995, 1998.

[15] Y. B. Liu, Q. Wu, Z. Y. Ma, and K. G. Yan, "An Improved Hilbert-Huang Transform and Its Application in Faults Signal Analysis," in Mechatronics and Automation, Proceedings of the 2006 IEEE International Conference on, 2006, pp. 2426-2431.

[16] A. G. Poulimenos and S. D. Fassois, Parametric time-domain methods for non-stationary random vibration modelling and analysis -- A critical survey and comparison, Mechanical Systems and Signal Processing, 2006, vol. 20, pp. 763-816.

[17] J. Yang, S. Lin, Identification of parametric variations of structures based on least squares estimation and adaptive tracking technique, Journal of Engineering Mechanics 131 (2005) 290–298.

[18] J. Z. Yang, C. W. Liu and W. G. Wu, "A hybrid method for the estimation of power system low-frequency oscillation parameters," in Power and Energy Society General Meeting - Conversion and Delivery of Electrical Energy in the 21st Century, 2008 IEEE, 2008, p. 1-1.

[19] R. Roy and T. Kailath, ESPRIT-estimation of signal parameters via rotational invariance techniques, Acoustics, Speech and Signal Processing, IEEE Transactions on, 1989, vol. 37, pp. 984-995.

[20] R. Badeau, G. Richard and B. David, "Fast adaptive esprit algorithm," in Statistical Signal Processing, 2005 IEEE/SP 13th Workshop on, 2005, pp. 289-294.

[21] L.Li, C. Schenck, B. Kuhfuss, "Short Time Identification of Modal Parameters," Proc. 22nd International Conference on Computer-Aided Production Engineering (CAPE), Edinburgh, Scotland, April 2011.

[22] I. Y. H. Gu and M. H. J. Bollen, Estimating Interharmonics by Using Sliding-Window ESPRIT, Power Delivery, IEEE Transactions on, 2008, vol. 23, pp. 13-23.

[23] S. G. Mallat, Z. Zhang, Matching pursuits with time frequency dictionaries. IEEE Transactions on Signal Processing, volume 41, number 12, pages 3397{3415, 1993.

[24] Y. Pati, R. Rezaiifar and P. Krishnaprasad, Orthogonal matching pursuit: recursive function approximation with applications to wavelet decomposition. Proceedings of 27th Asilomar Conference on Signals, Systems and Computers, volume 1, pages 40-44, 1993

[25] S. D. FASSOIS, K. F. EMAN and S. M.WU, Sensitivity analysis of the discrete-to-continuous dynamic system transformation. Journal of Dynamic Systems, Measurement and Control, 1990, 112, 1-9.